DOI: 10.13656/j.cnki.gxkx.1997.03.008
广西科学 Guangxi Sciences 1997, 4 (3): 186~187

New Lower Bounds of Classical Ramsey Numbers R (4, 15), R (4, 16) and R (4, 17)

经典 Ramsey数 R (4, 15), R (4, 16) 和 R (4, 17) 的新下界*

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Abstract Three new cyclic graphs were constructed by using computer. The following results were obtained: $R(4,15)\geqslant 138$, $R(4,16)\geqslant 150$, $R(4,17)\geqslant 158$, in which the first one is better than the known result $R(4,15)\geqslant 134$, the last two fill in two blanks in the table of bounds of Ramsey numbers.

Key words Ramsey number, lower bound, cyclic graph

摘要 通过计算机构造了 3个新的循环图, 从而得到了 3个 Ramsey数新的下界: $R(4,15)\geqslant 138$, $R(4,16)\geqslant 150$, $R(4,17)\geqslant 158$. 其中第一个结果超过目前已知最好的 $R(4,15)\geqslant 134$, 后两个结果填补了 Ramsay数下界表的 2个空白.

关键词 Ramsey数 下界 循环图
中图法分类号 O157.5

It is a difficult problem in graph theory to determine Ramsey numbers. So far there is no sign of break-through in both theory and methods. Consequently in the last few years scholars of different nations used special methods to estimate some specific Ramsey mumbers[1], assisted by computers. K. Piwakowski obtained lower bounds of Ramsey numbers $R(4,10)$, $R(4,11)$, $R(4,12)$, $R(4,13)$, $R(4,14)$ in [2], and $R(4,15)\geqslant 134$ in [3]. But the calculation for finding new lower bounds becomes more and more complicate with the increase of Ramsey numbers. So we made a new attemp in [4]: Utilizing the transition, rotation and other properties of cyclic graphs of prime order to improve the generation of parameters so that the computation became more efficient. In this paper we use the way in [4], and obtain other three new lower bounds of Ramsey numbers.

For a given prime number p, let $Z_p=\{0,1,2,\cdots,p-1\}$. Choose a parameter set $S\subset\{1,2,\cdots,(p-1)/2\}$. Let G be a graph whose vertex set is $V_G=Z_p$ and the edge set is defined as follows: two vertices x and y are connected by an edge if and only if $\min\{|x-y|,p-|x-y|\}\in S$. The graph G is called the cyclic graph of order p with respect to the parameter set S and denoted by $G_p(S)$.

From this approach we constructed 3 cyclic

1997-04-01收稿。
* 广西科学基金资助项目。

graphs of prime order as follows:

1) $p = 137$ and parameter set of graph G

$S = \{1, 3, 4, 9, 11, 12, 14, 24, 26, 30, 32, 33, 37, 39, 43, 49, 53, 59, 64, 66\}$.

2) $p = 149$ and parameter set of graph G

$S = \{1, 2, 4, 7, 10, 14, 16, 20, 27, 29, 32, 41, 42, 44, 49, 60, 61, 65, 67, 70\}$.

3) $p = 157$ and parameter set of graph G

$S = \{1, 2, 5, 9, 17, 18, 20, 21, 25, 28, 32, 33, 36, 40, 42, 46, 56, 62, 66, 68, 72, 77\}$.

We have verified by computer the following facts: The cyclic graph $G_{137}(S)$ in 1) contains neither 4-point clique K_4 nor subset $\bar{K}_{15}$ of 15 isolated points; the cyclic graph $G_{149}(S)$ in 2) contains neither 4-point clique K_4 nor subset $\bar{K}_{16}$ of 16 isolated points; the cyclic graph $G_{157}(S)$ in 3) contains neither 4-point K_4 nor subset $\bar{K}_{17}$ of 17 isolated points. Therefore by Ramsey's Theorem we proved

Theorem 1 $R(4, 15) \geqslant 138, R(4, 16) \geqslant 150, R(4, 17) \geqslant 158$.

Note that our first example is better than the known result $R(4, 15) \geqslant 134$. The last two fill in two blanks in the table of bounds of Ramsey numbers, and much better than the results in [5]: $R(4, 16) \geqslant 120, R(4, 17) \geqslant 128$ and $R(4, 18) \geqslant 135$.

Acknowlegdment

We would like to express our sincere gratitude to Prof. S. P. Radziszowski for informing us the results in inference [5], Ph. D. thesis, through internet.

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4(a)位置,等效点坐标见表 2. DyNiSn的晶体结构数据见表 3.

表 3 **DyNiSn**的晶体结构数据

Table 3 Crystal structure data for DyNiSn

晶系 Crystal system	正交 Orthorhombic
空间群 Space group	$Pna2_1$
点阵参数 Lattice parameter	$a = 7.1018(1)$Å, $b = 7.6599(2)$Å, $c = 4.4461(2)$Å
单位晶胞中的化合式量 Number formula unit	$z = 4$
单胞体积 Volume of the unit cell	$v = 241.86$ Å^3
计算密度 Calculated density	$Dx = 9.334$ g/cm^3

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