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A New Method for Analysis of the Continuous Box-girder Bridges
连续箱型梁桥分析的新方法*

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Abstract QR-method, based on cubic B-spline function and the principle of minimum total potential energy, for analysis of the continuous box-girder bridges is presented. This method has great advantages over the finite element method and the finite strip method. It can be easily carried out on a microcomputer.

Key words box-girder, straight continuous box-girder bridges, curved continous box-girder bridges, QR-method, spline function method

摘要 提出以 3次 B样条函数及最小势能原理建立的连续箱型梁桥分析的 QR法。该法比有限元法及有限条法优越,很容易在微机上实现。

关键词 箱型截面梁 连续直箱型梁桥 连续曲箱型梁桥 QR法 样条函数方法
中图法分类号 O 344.3

The continuous box-girder bridges contain both straight and curved continuous box-girder bridges, which play an important part in bridge engineeings. Numerical methods may be used to analyze continuous box-girder bridges. If the finite element method is used for analyzing the continuous box-girder bridges, the unknown numbers are so large that a large computer is need and computing costs so expensive. In this paper, we present a new method, called QR-method, for analysis of the continuous box-girder bridges. It is based on spline function methods. This method has not only advantages of the finite elemnent method and the finite strip method, but also is superior to them. The main advantages of this method are few number of unknowns, simple program, few input data, short time of computing, low storage, higher accuracy and wide applicability, and it can be easily carried out with a microcomputer to solve the large complex problems.

This paper aims to introduce briefly the fundamentals and applications of QR-method in the analysis of continuous box-girder bridges. This paper contains static analysis, dynamic analysis and stability analysis.

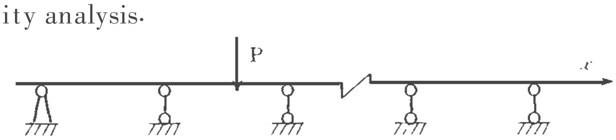

Fig. 1 Continuous box-girder bridge

1 Displacement funtions

Fig. 1 is a computational pattern of continuous box-girder bridges, its cross section is box section as shown in Fig. 2. If u, v, w are respectively the displacements in directions x, y, z, then displacement functions of this structure may be expressed by

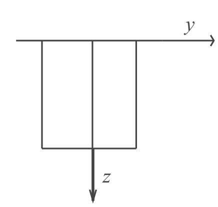

Fig. 2 Box-section

$$u = \sum_{m=1}^{r} [H(x)]\{u\}_m X_m(y) X_m(z)$$

$$v = \sum_{m=1}^{r} [H(x)]\{v\}_m Y_m(y) Y_m(z)$$

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$$w = \sum_{m=1}^{r} [H(x)]\{w\}_m Z_m(y) Z_m(z) \qquad (1)$$

where $[H(x)] = [H_0(x), H_1(x), \cdots, H_N(x)]$

$$\{u\}_m = [u_0, u_1, u_2, \cdots, u_N]_m^T$$

$$\{v\}_m = [v_0, v_1, v_2, \cdots, v_N]_m^T$$

$$\{w\}_m = [w_0, w_1, w_2, \cdots, w_N]_m^T$$

in which

$$H_i(x) = \frac{10}{3} h_3\left(\frac{x}{h} - i\right) - \frac{4}{3} h_3\left(\frac{x}{h} - i + \frac{1}{2}\right) - \frac{4}{3} h_3\left(\frac{x}{h} - i - \frac{1}{2}\right) + \frac{1}{6} h_3\left(\frac{x}{h} - i + 1\right) + \frac{1}{6} h_3\left(\frac{x}{h} - i - 1\right) \qquad i = 0, 1, 2, \cdots, N \qquad (2)$$

$$H_i(x_k) = \begin{cases} 1 & i = k \\ 0 & i \neq k \end{cases} \qquad (3)$$

where $h_3(x)$ is cubic B-spline function[1], $h = x_{i+1} - x_i$; X_m, Y_m, Z_m are the orthogonal functions satisfied the boundry conditions at two opposite sides in y-direction or z-direction. From the above mentioned, we can obtain

$$\{V\} = [N]\{W\} \qquad (4)$$

where $\{V\} = [u, v, w, \theta_x, \theta_y]^T$

$$[N] = [[N]_1, [N]_2, \cdots, [N]_r] \qquad (5)$$

$$\{W\} = [\{W\}_1^T, \{W\}_2^T, \cdots, \{W\}_r^T]^T$$

$$\{W\}_m = [\{u\}_m^T, \{v\}_m^T, \cdots, \{w\}_m^T]$$

$$[N]_m = \begin{bmatrix} [H]X_m(y)X_m(z) & [0] & [0] \\ [0] & [H]Y_m(y)Y_m(z) & [0] \\ [0] & [0] & [H]Z_m(y)Z_m(z) \\ [0] & [0] & [H]Z'_m(y)Z_m(z) \\ [0] & [0] & [j]Z_m(y)Z_m(z) \end{bmatrix} \qquad (6)$$

in which $[j] = [j_0(x), j_1(x), \cdots, j_N(x)]^T$

$$j_0 = -\frac{1}{h}\left(H_0 + \frac{1}{2}H_1\right)$$

$$j_1 = \frac{1}{h}\left(H_0 - \frac{1}{2}H_2\right)$$

$$j_i = \frac{1}{2h}\left(H_{i-1} - H_{i+2}\right) \qquad i = 2, 3, \cdots, N-2$$

$$j_{N-1} = \frac{1}{h}\left(\frac{1}{2}H_{N-2} - H_N\right)$$

$$j_N = \frac{1}{h}\left(\frac{1}{2}H_{N-1} + H_N\right) \qquad (7)$$

where H_i can be determined by eq. (2). From eq. (1) and eq. (3), we can obtain following forms:

$$u_i = \sum_{m=1}^{r} u_{im} X_m(y) X_m(z)$$

$$v_i = \sum_{m=1}^{r} v_{im} Y_m(y) Y_m(z)$$

$$w_i = \sum_{m=1}^{r} w_{im} Z_m(y) Z_m(z) \qquad i = 0, 1, 2, \cdots, N \qquad (8)$$

where u_i, v_i and w_i are the displacements at $x = x_i$. From eq. (8), we may obtain

$$\{V\}_i = [N]_i\{W\} \qquad (9)$$

where $\{V\}_i = [u_i, v_i, w_i, \theta_{xi}, \theta_{yi}]^T$

$$\{W\} = [\{W\}_1^T, \{W\}_2^T, \cdots, \{W\}_r^T]^T$$

$$[N]_i = [[N]_1, [N]_2, \cdots, [N]_r] \qquad (10)$$

in which $\{W\}_m = [\{u\}_m^T, \{v\}_m^T, \{w\}_m^T]^T$

$$[N]_m = \begin{bmatrix} [X]_{im} & [0] & [0] \\ [0] & [Y]_{im} & [0] \\ [0] & [0] & [Z]_{im} \\ [0] & [0] & [Z']_{im} \\ [0] & [0] & [Z]_{im}[Q] \end{bmatrix}$$

$$[Q] = \begin{bmatrix} \frac{-1}{h} & \frac{1}{h} & 0 & 0 & 0 \\ \frac{-1}{2h} & 0 & \frac{1}{2h} & 0 & 0 \\ 0 & \frac{-1}{2h} & 0 & \frac{1}{2h} & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & \frac{-1}{2h} & 0 & \frac{1}{2h} \\ 0 & 0 & 0 & \frac{-1}{h} & \frac{1}{h} \end{bmatrix}_{(N+1)(N+1)} \qquad (11)$$

where $[A]_{im} = [0, \cdots, A_{im}(y)A_{im}(z), 0, \cdots]$,

$$A = X, Y, Z \qquad (12)$$

$$[Z']_{im} = [0, \cdots, Z'_{im}(y)Z_{im}(z), 0, \cdots] \qquad (13)$$

2 QR-method

If we make the plates of box-girder bridge into rectangular mesh or triangular mesh (Fig. 1 and Fig. 2), then the functional of total potential energy of elemets is

$$\prod_e = \frac{1}{2}\{V\}_e^T[k]_e\{V\}_e - \{V\}_e^T\{f\}_e \qquad (14)$$

where $[k]_e$ and $\{f\}_e$ are respectively the stiffness matrix and the load vector of plane-shell element, $\{V\}_e$ is the nodal displacement vector of the element, or

$$[V]_e = [N]_e\{W\}_e \qquad (15)$$

in which

$$[N]_e = [[N]_A^T, [N]_B^T, [N]_C^T, [N]_D^T]^T \qquad (16)$$

for the rectangular element with 4-nodes (Fig. 3).

$$[N]_e = [[N]_A^T, [N]_B^T, [N]_C^T]^T \qquad (17)$$

for the triangular element with 3-nodes (Fig. 4).

$[N]_A$ is the value at point A of $[N]_i$, it can be determined by eq. (4) or eq. (9), or $[N]_A =$ [N (A)], $[N]_A = [N(A)]_i$. Substituting eq. (15) into eq. (14) may obtain

$$\prod_e = \frac{1}{2}\{W\}^T [G]_e \{W\} - \{W\}^T \{F\}_e \tag{18}$$

where $[G]_e = [N]_e^T ([T]^T [k]_e [T]) [N]_e$,

$$\{F\}_e = [N]_e^T [T]^T \{f\}_e \tag{19}$$

where $[T]$ is coordinate transformation matrix of elements. Since the functional of total energy of the box-girder bridge (Fig. 1 and Fig. 2) may be determined by

$$\prod = \sum_{e=1}^{M} \prod_e \tag{20}$$

Substituting eq. (18) into eq. (20) may obtain

$$\prod = \frac{1}{2}\{W\}^T [K]\{W\} - \{W\}^T \{f\} \tag{21}$$

$$\text{where} \quad [K] = \sum_{e=1}^{M} [G]_e \quad \{f\} = \sum_{e=1}^{M} \{F\}_e \tag{22}$$

Using the variation principle, we obtain

$$[G]\{W\} = \{f\} \tag{23}$$

This is the stiffness equation of continuous box-girder bridge (Fig. 1 and Fig. 2).

From the abvoe mentioned we may know that eq. (23) is built up from the elements of continuous box-girder bridge, nevertheless its number of unkowns has nothing to do with the number of nodes of mesh of box-girder bridge, it is only relevant to the collocation points and r. Thus, the unknown numbers of eq. (23) are very small. Eq. (22) can directly superpose, with no extension. The above method is called QR-method.

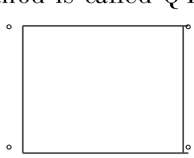

Fig. 3 Rectangular element

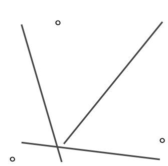

Fig. 4 Triangular element

3 Dynamic analysis of structures

The dynamic equations of continuous box-girder bridge may be established by QR-method. From here we can obtain

$$[M]\{\ddot{W}\} + [C]\{\dot{W}\} + [K]\{W\} = \{P\} \tag{24}$$

This is the dynamic equation of continuous box-girder bridge, where $[M]$, $[C]$ and $[K]$ are respectively the mass matrix, damping matrix and stiffness matrix of the structures. $\{\ddot{W}\}$, $\{\dot{W}\}$, $\{W\}$ and $\{P\}$ are respectively the acceleration vector, velocity vector, displacement vector and disturbed force vector of the structures, and they are all the time's function. If $\{P\} = \{0\}$, $[C] = [0]$, then, eq. (24) becomes

$$[M]\{\ddot{W}\} + [K]\{W\} = \{0\} \tag{25}$$

This is the dynamic equation of free vibration of the structrues. Let

$$\{W\} = \{\bar{W}\}\sin(kt + T) \tag{26}$$

Substuting eq. (25), we can obtain

$$[K]\{\bar{W}\} - k^2 [M]\{\bar{W}\} = \{0\} \tag{27}$$

where k and $\{\bar{W}\}$ are respectively the natural frequency and the vector of amplitudes of vibration modes of structures, they can be determined by eq. (27).

The dynamic response of structures can be determined by the solution of eq. (24). The solution can be solved by spline weighted residual method. From here we had established the computational schemes of conditional stability and unconditional stability[2]. They are all adaptable for analysis of the linear problems and the nonlinear problems of the sructures.

4 Stability analysis

The stability equation of continuous box-girder bridge may be established by QR-method from here, we can obtain

$$([K] - \lambda [H])\{W\} = \{0\} \tag{28}$$

This is the eigen equation of stability of continuous box-girder bridge, where is the eigenvalue, $[K]$ and $[H]$ are respectively stiffness matrix and geometrical stiffness matrix of continuous box-girder bridge.

The critical load of continuous box-girder bridge can be determined by eq. (28).

5 Curved box-girder bridge

Fig. 5 is a computational patttern of curved continuous box-girder bridge, its displacement functions may be expressed by

$$u = \sum_{m=1}^{r} [H(s)]\{u\}_m X_m(y) X_m(z)$$

$$v = \sum_{m=1}^{r} [H(s)]\{v\}_m Y_m(y) Y_m(z)$$

$$w = \sum_{m=1}^{r} [H(s)]\{w\}_m Z_m(y) Y_m(z) \tag{29}$$

where

$$H_j(s) = \frac{10}{3}h_3\left(\frac{s}{h} - i\right) - \frac{4}{3}h_3\left(\frac{s}{h} - i - \frac{1}{2}\right) - \frac{4}{3}h_3\left(\frac{s}{h} - i - \frac{1}{2}\right) + \frac{1}{6}h_3\left(\frac{s}{h} - i + 1\right) + \frac{1}{6}h_3\left(\frac{s}{h} - i - 1\right) \qquad i = 0, 1, 2, \cdots, N \tag{30}$$

in which $h = s_{i+1} - s_i$ is curved coordinate. From the above memtioned we may know that curved continuous box-girder bridge may be analysed by eq.(1) to eq.(28), if we replace x by s

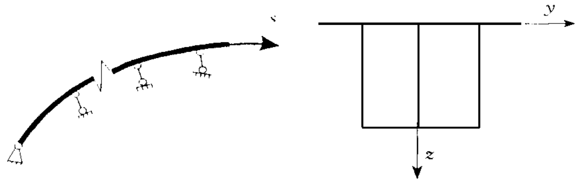

Fig. 5 Curved continuous box-girder bridge

6 Computational example

Fig. 6 is a continuous box-girder bridge. The results are analysed by QR-method (table 1).

Table 1 Vertical displacement w

Nodes	w	Nodes	w
1	1.92	5	1.92
2	2.12	6	1.94
3	2.84	7	1.91
4	2.12	8	1.94

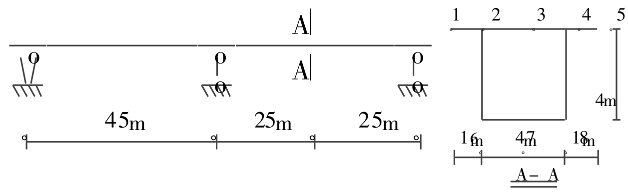

Fig. 6 Box-girder bridge with two spans

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用的载体是光子,强作用力载体是胶子,弱作用力的载体是W粒子和Z粒子。但是,还没有人能够把引力纳入这一理论框架之内。据推测,引力必须由叫做引力子的粒子加以运载,但是事实已经证明,要让这些粒子符合量子场论是不可能的。

不过,引力可以利用超弦论 即M理论,以完全不同的弦和膜的词汇来加以描述。马尔达塞纳的推测认为,在量子场和弦论这两种貌似互不相容的世界观之间,可能存在着一种深藏不露的联系。理论物理学家们近期内发表的100多篇论文对他的这一推测进行了阐述。

新泽西州普林斯顿高级研究所的理论物理学家内森°西伯格说:“这是一种极其激动人心的说法。”马尔达塞钠提出的假设性联系,不仅拉近了引力和其他作用力的距离,而且可能会提供一种新的有力的计算工具,从而帮助解开粒子物理学的难题。

弦论的提出与发展

弦论最早是在60年代末70年代初提出的,当时是想用它解释强作用力,但没有成功。后来提出的量子色动力学(QCI)成功地解释了强作用力,并被纳入“标准模型”。“标准模型”是一系列量子理论的集合,对强作用力、弱作用力及电磁作用进行了统一的描述,但是对引力的解释却仍然没有着落。

不过并不是每个人都对弦论感到绝望。70年代中期,施瓦茨和约埃尔°舍克两位物理学家设法把弦论的一个破绽变成了一个优点:或许方程中始终存在的引力子并非出于偶然。或许他们所看到的并不是强作用力的模型,而是引力的模型——一种以公式表达爱因斯坦相对论的新方法。如果引力可以用弦论来解释,那么也许其他作用力也可以用这一方式重新表达。这样,所有的作用力就会统一在同一个理论体系下。

在此前后,从事弦论研究的物理学家发现,他们可以把这一理论所需的时空维数从原先的26个减少到10个。在这一过程中,当弦论被赋予一种叫做超对称性的假定性质时,它的名字便变成了超弦论。超对称性是指运载力的粒子(如胶子)和构成物质的粒子(如夸克)紧密地结合在一起。

10个维对于物理学家来说仍嫌过多。利用这么多的维,看来物理学家有可能构造出数目无限的各种10维弦论假想。他们又如何知道其中哪一种假想可以对宇宙作出解释呢?弦论研究在80年代中期(第一次革命)取得了重大突破。当时,物理学家证明在所有可能的弦论假想中,只有5种在数学上是站得住脚的,其余的因种种漏洞而不攻自破,但即使这样也还是多出了4种假想。更糟的是,想把6个额外的维“卷起来”,以便得到描述4维世界的假想,其方法仍然有成千上万种之多。

少数弦论研究者仍然顽强地保持着乐观。弦论界新秀爱德华°威滕浪漫地称这门学科“本应属于21世纪的物理学,却意外地掉到了20世纪”。在经过很长一段时间的埋头研究后,他们迎来了90年代中期的第二革命。他们发现,在众多掩盖多

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