

12个经典二色 Ramsey数 $R(k, l)$ 的新下界*

罗海鹏 苏文龙 张正铀 李桂清

二色经典 Ramsey数 $R(k, l)$ 是指具有下述性质的最小正整数 r : 用两种颜色把 r 阶完全图 K_r 的边任意染色后, K_r 中一定存在单色的 K_k 或 K_l , 其存在性的证明并不困难, 但具体的 Ramsey 数的计算却是组合数学中非常困难的问题^[1]. 当今学术界关于 Ramsey 数研究的最新进展详见文献 [2] 动态综述论文. 本文沿用文献 [3~ 7] 的方法, 构造 12 个素数阶循环图, 得到 12 个二色经典 Ramsey 数的新下界. 研究简报如下.

定义 1 给定素数 p , 记 $Z_p = \{(1-p)/2, \dots, -1, 0, 1, \dots, (p-1)/2\}$, $Z_p^+ = \{1, 2, \dots, (p-1)/2\}$. 对于任意非空的参数集合 $S \subset Z_p^+$, 设图 G 的顶点集 $V = Z_p$, 边集

$$E = \{\{x, y\}: x, y \in Z_p \text{ 且 } |x - y| \in S\}.$$

我们称图 G 为关于参数集 S 的 p 阶循环图并记为 $G_p(S)$.

这里 $|x - y|$ 表示先把 $x - y$ 取模 p 同余归结到 Z_p , 再取绝对值归结到 Z_p^+ .

按定义 1 构造下述 12 个 p 阶循环图:

1) 取素数 $p_1 = 212$, 参数集

$S = \{1, 4, 6, 9, 20, 23, 30, 38, 45, 55, 57, 71, 73, 92, 104\}$.

2) 取素数 $p_2 = 197$, 参数集

$S = \{1, 6, 7, 14, 31, 37, 40, 48, 49, 51, 52, 60, 64, 73, 74, 76, 77, 79, 81, 84, 89, 93, 95, 98\}$.

3) 取素数 $p_3 = 421$, 参数集

$S = \{1, 10, 12, 20, 21, 29, 31, 33, 35, 46, 48, 54, 57, 66, 68, 70, 71, 73, 75, 78, 92, 94, 97, 107, 109, 111, 115, 118, 123, 124, 129, 131, 133, 134, 137, 142, 149, 151, 154, 156, 157, 159, 165, 166, 169, 173, 181, 182, 183, 184, 188, 191, 193, 195, 196, 197, 199, 200, 207, 210\}$.

4) 取素数 $p_4 = 433$, 参数集

$S = \{1, 5, 9, 11, 13, 16, 19, 20, 24, 27, 29, 31, 42, 43, 45, 49, 50, 51, 59, 63, 66, 69, 73, 76, 78, 80, 82, 83, 84, 89, 101, 103, 107, 113, 116, 124, 125, 129, 131, 135, 136, 137, 138, 139, 142, 144, 146, 150, 153, 165, 171, 176, 177, 178, 181, 183, 190, 194, 195, 198, 199, 208, 215\}$.

5) 取素数 $p_5 = 461$, 参数集

$S = \{1, 2, 16, 20, 25, 28, 31, 34, 35, 38, 39, 43, 48, 51, 52, 56, 60, 61, 65, 70, 72, 75, 78, 83, 84, 88, 90, 91, 93, 96, 105, 107, 110, 114, 117, 118, 120, 122, 132, 133, 137, 141, 143, 146, 147, 150, 154, 159, 162, 164, 165, 167, 169, 171, 176, 179, 183, 186, 188, 191, 196, 205, 209, 212, 216, 219, 220, 226, 228, 229\}$.

6) 取素数 $p_6 = 577$, 参数集

$S = \{1, 4, 6, 9, 11, 16, 17, 24, 26, 31, 35, 36, 38, 39, 44, 46, 47, 50, 54, 55, 57, 58, 59, 64, 66, 67, 68, 69, 75, 81, 82, 85, 87, 91, 96, 98, 99, 102, 104, 106, 123, 124, 127, 130, 140, 142, 144, 147, 152, 153, 154, 156, 158, 159, 161, 163, 166, 167, 169, 175, 176, 181, 184, 185, 186, 188, 193, 195, 200, 202, 203, 206, 210, 213, 214, 216, 218, 220, 223, 226, 228, 229, 230, 231, 232, 234, 235, 236, 237, 238, 242, 247, 249, 250, 253, 256, 262, 263, 264, 268, 272, 274, 275, 276, 277, 279, 282, 287\}$.

7) 取素数 $p_7 = 617$, 参数集

$S = \{1, 8, 16, 19, 23, 27, 28, 36, 37, 41, 43, 45, 55, 56, 58, 62, 64, 66, 67, 69, 74, 76, 90, 92, 94, 96, 98, 101, 106, 107, 114, 115, 116, 117, 120, 121, 125, 126, 129, 131, 132, 135, 142, 143, 146, 150, 151, 153, 155, 158, 163, 164, 165, 166, 167, 173, 174, 176, 177, 179, 181, 184, 187, 188, 192, 194, 196, 197, 198, 202, 203, 209, 211, 212, 214, 215, 217, 218, 220, 224, 226, 227, 228, 230, 231, 234, 236, 237, 242, 244, 246, 259, 262, 266, 268, 269, 271, 274, 275, 276, 281, 288, 292, 295, 296, 297, 299, 300, 302, 303, 305, 306\}$.

8) 取素数 $p_8 = 617$, 参数集

$S = \{1, 4, 6, 9, 12, 14, 15, 20, 21, 22, 23, 29, 30, 36, 39, 40, 42, 43, 44, 47, 48, 49, 50, 51, 52, 53, 55, 57, 59, 62, 64, 68, 70, 73, 76, 77, 78, 87, 100, 102, 103, 104, 105, 109, 113, 116, 120, 122, 123, 125, 126, 127, 129, 130, 135, 136, 137, 139, 140, 141, 142, 143, 144, 147, 154, 158, 159, 160, 162, 164, 166, 167, 168, 170, 171, 172, 175, 176, 178, 181, 182, 185, 187, 190, 192, 194, 197, 198, 201, 206, 207, 209, 211, 212, 216, 217, 219, 222, 224, 228, 235, 236, 238, 245, 248, 251, 253,$

256, 260, 261, 266, 267, 268, 271, 273, 276, 277, 278, 279, 290, 292, 293, 296, 303, 304, 305}.

9) 取素数 $p_9 = 677$, 参数集

$S = \{1, 5, 9, 10, 18, 21, 24, 29, 31, 36, 38, 39, 40, 41, 42, 43, 45, 48, 49, 50, 51, 52, 55, 56, 58, 60, 62, 69, 71, 78, 81, 82, 86, 91, 98, 99, 100, 102, 103, 104, 105, 107, 111, 113, 114, 115, 116, 119, 124, 127, 130, 132, 134, 136, 138, 139, 142, 144, 145, 155, 161, 162, 163, 166, 168, 169, 177, 179, 180, 183, 189, 194, 198, 200, 204, 206, 209, 211, 214, 216, 218, 219, 220, 221, 223, 226, 228, 231, 232, 235, 236, 238, 239, 241, 246, 247, 248, 250, 251, 254, 255, 263, 264, 265, 272, 275, 277, 283, 286, 287, 289, 290, 293, 294, 299, 308, 310, 311, 312, 313, 315, 316, 317, 319, 326, 330, 333, 334, 336, 337\}$.

10) 取素数 $p_{10} = 739$, 参数集

$S = \{1, 2, 4, 5, 8, 10, 13, 16, 20, 25, 26, 27, 32, 37, 40, 47, 50, 52, 53, 54, 57, 61, 63, 64, 65, 74, 80, 89, 93, 94, 97, 99, 100, 104, 106, 108, 109, 114, 122, 125, 126, 127, 128, 129, 130, 133, 135, 137, 147, 148, 151, 153, 160, 169, 173, 178, 185, 186, 188, 191, 194, 198, 199, 200, 201, 207, 208, 209, 212, 216, 217, 218, 219, 223, 227, 228, 231, 235, 237, 239, 244, 250, 251, 252, 254, 256, 258, 260, 261, 265, 266, 269, 270, 274, 277, 283, 285, 293, 294, 296, 301, 302, 303, 305, 306, 307, 315, 320, 321, 323, 325, 337, 338, 339, 341, 343, 346, 351, 356, 357, 363, 367, 369\}$.

11) 取素数 $p_{11} = 947$, 参数集

$S = \{1, 2, 3, 7, 13, 14, 20, 21, 22, 25, 30, 32, 33, 34, 38, 40, 41, 43, 44, 45, 47, 48, 49, 51, 52, 57, 58, 60, 62, 66, 67, 72, 73, 74, 78, 82, 86, 87, 90, 91, 93, 94, 98, 99, 108, 109, 111, 115, 116, 117, 122, 123, 125, 127, 128, 129, 131, 135, 140, 141, 142, 147, 152, 154, 157, 162, 170, 174, 175, 178, 183, 184, 187, 191, 192, 193, 197, 199, 210, 211, 213, 221, 224, 228, 230, 231, 238, 239, 243, 253, 254, 255, 260, 261, 262, 266, 267, 271, 276, 277, 278, 280, 281, 283, 284, 286, 287, 288, 289, 293, 296, 298, 299, 301, 307, 308, 310, 311, 315, 317, 325, 326, 329, 334, 336, 337, 338, 341, 342, 343, 345, 347, 357, 362, 364, 365, 373, 381, 388, 390, 393, 397, 399, 400, 401, 403, 404, 406, 409, 412, 414, 417, 420, 424, 426, 429, 432, 434, 436, 440, 442, 443, 444, 446, 447, 454, 458, 462, 463, 465, 469, 472\}$.

12) 取素数 $p_{12} = 1051$, 参数集

$S = \{1, 2, 6, 9, 10, 11, 13, 17, 19, 22, 24, 29, 30, 32, 36, 39, 42, 43, 45, 48, 51, 55, 56, 59, 62, 63, 69, 73, 74, 76, 81, 83, 84, 92, 95, 101, 106, 108, 110, 114, 115,$

$116, 118, 120, 121, 122, 128, 131, 133, 134, 137, 138, 142, 143, 145, 149, 150, 152, 154, 156, 158, 159, 160, 161, 162, 163, 172, 173, 178, 179, 184, 186, 187, 189, 194, 195, 198, 200, 203, 204, 208, 210, 213, 215, 218, 222, 224, 225, 229, 233, 237, 240, 241, 242, 246, 249, 251, 254, 255, 256, 257, 260, 261, 263, 264, 266, 267, 269, 272, 273, 277, 278, 279, 280, 282, 283, 284, 291, 292, 294, 295, 300, 301, 307, 309, 312, 314, 315, 316, 317, 320, 326, 327, 328, 330, 331, 333, 334, 336, 337, 338, 339, 340, 341, 342, 344, 348, 349, 352, 353, 357, 358, 359, 361, 362, 364, 365, 366, 369, 373, 376, 380, 382, 383, 386, 388, 390, 392, 396, 402, 404, 405, 407, 408, 409, 412, 413, 414, 416, 417, 420, 423, 424, 429, 430, 432, 433, 437, 438, 442, 446, 447, 448, 449, 452, 453, 454, 460, 461, 462, 463, 467, 472, 473, 475, 476, 487, 490, 492, 494, 495, 497, 501, 502, 503, 505, 507, 508, 509, 510, 511, 513, 521, 522, 523\}$.

根据上述素数 p_i 与参数集 S ($1 \leq i \leq 12$), 由定义 1 就构造了 12 个素数阶循环图. 按照文献 [3] 的方法, 就可以计算得图 $G_{p_i}(S_i)$ 的团数和独立数, 并且得到

定理 1 $R(3, 32) \geq 212^*$, $R(4, 19) \geq 198^*$, $R(5, 22) \geq 422^*$, $R(5, 23) \geq 434^*$, $R(5, 24) \geq 462$, $R(7, 17) \geq 578^*$, $R(7, 18) \geq 618$, $R(8, 15) \geq 618$, $R(8, 17) \geq 678^*$, $R(8, 18) \geq 740$, $R(8, 20) \geq 948^*$, $R(10, 16) \geq 1052$.

其中带“*”号的 7 个结论都超过文献 [2] 综述论文的相应记录, 其余 5 个结论是本文首次报道的.

参考文献

- 1 李乔. 组合数学基础. 北京: 高等教育出版社, 1993, 11: 225~228.
- 2 Radziszowski S P. Small Ramsey numbers. The Electronic Journal of Combinatorics, 1994, DS1, updated on 7/5/1999.
- 3 Su Wenlong, Luo Haipeng, Zhang Zhengyou et al.. New Lower Bounds of Fifteen Classical Ramsey Numbers. Australasian Journal of Combinatorics, 1999, 19: 91~99.
- 4 Su Wenlong, Luo Haipeng, Shen Yunqiu. New lower bounds for classical Ramsey numbers $R(5, 13)$ and $R(5, 14)$. Applied Mathematics Letters, 1999, 12 (6): 121~122.
- 5 苏文龙, 罗海鹏, 李乔. 多色经典 Ramsey 数 $R(q, q, \dots, q)$ 的下界. 中国科学 (A 辑), 1999, 29 (5): 408~413.
- 6 罗海鹏, 苏文龙, 李乔. 经典 Ramsey 数 $R(6, 12)$, $R(6, 14)$ 和 $R(6, 15)$ 的新下界. 科学通报, 1998, 43 (12): 1336~1337.
- 7 苏文龙, 罗海鹏, 李乔. 经典 Ramsey 数 $R(4, 12)$, $R(5, 11)$ 和 $R(5, 12)$ 的新下界. 科学通报, 1997, 42 (22): 2460.

(第一作者单位: 广西科学院)