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$\tilde{\rho}$相依序列加权和的强收敛性
Almost Sure Convergence of Weighted Sums of $\tilde{\rho}$ Sequences

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摘要　给出$\tilde{\rho}$相依序列加权和的强收敛性的两个充分条件。
关键词　$\tilde{\rho}$相依序列　加权和　强收敛性
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Abstract　Two sufficient conditions for almost sure convergence of weighted sums of $\tilde{\rho}$ sequences were given.
Key words　$\tilde{\rho}$ sequences, weighted sums, almost sure convergence

1　引理和结论

设$\{\xi_i: i\in N\}$是概率空间(Ω, B, P)中的随机变量序列，$F_S=\sigma\{\xi_i: i\in S\subset N\}$为$\sigma$-域，在$B$中给定$\sigma$-域$F, R$，令

$\rho(F,R)=\sup\{\mathrm{corr}(\xi,\eta): \xi\in L_2(F), \eta\in L_2(R)\}$，

Bradley[1,2]，Bryc和Smolenski[3]使用如下相依系数：对$k\geqslant 0$，令

$$\tilde{\rho}(k)=\sup\{\rho(F_S,F_T): \text{有限子集 } S,T\subset N \text{ 且 } \mathrm{dist}(S,T)\geqslant k\}, \quad (1)$$

显然，$0\leqslant\tilde{\rho}(k+1)\leqslant\tilde{\rho}(k)\leqslant 1$且$\tilde{\rho}(0)=1$。这种相依系数$\tilde{\rho}(k)$与通常的$\rho$混合系数有一定的类似，但也不完全相同。事实上，在通常的ρ混合系数中，式(1)中的S, T分别是$[1,n]$和$[n+k,\infty]$中的子集。文献[4]在$\tilde{\rho}$相依序列中得到了与独立情形一致的矩不等式：

引理[4]　设$\tilde{\rho}=\tilde{\rho}(1)<1$，$q>1$，$X_i$为$\sigma(\xi_i)$可测且$EX_i=0$，$E|X_i|^q<\infty$，$(i=1,2,\cdots)$，则存在仅依赖于$\tilde{\rho}, q$的正常数$C$，使$1<q\leqslant 2$时，有：

$$E\Big|\sum_{i=a+1}^{a+n}X_i\Big|^q\leqslant c\sum_{i=a+1}^{a+n}E|X_i|^q, \forall n\geqslant 1, a\geqslant 0,$$

$q>2$时，有：

$$E\Big|\sum_{i=a+1}^{a+n}X_i\Big|^q\leqslant c\Big\{\sum_{i=a+1}^{a+n}E|X_i|^q+\Big(\sum_{i=a+1}^{a+n}EX_i^2\Big)^{q/2}\Big\},$$

$\forall n\geqslant 1, a\geqslant 0$.

利用此引理，本文在$\tilde{\rho}$相依下讨论加权和的强收敛性，得到如下结论：

定理 1　设$\tilde{\rho}=\tilde{\rho}(1)<1$，$p\geqslant 2$，$X_i$为$\sigma(\xi_i)$可测、均值为零，$\sup_i E|X_i|^p<\infty$ $(i=1,2,\cdots)$，如果存在$0<\theta<2/p$和常数$k>0$，$0<\rho<1-\theta$，使：

$$\sum_{i=1}^n a_{ni}^2\leqslant kn^{\theta}, |a_{ni}|\leqslant kn^{-\rho}, i=1,2,\cdots,n, n\geqslant 1.$$

则$T_n\triangleq\sum_{i=1}^n a_{ni}X_i/n^{1/p}\to 0$, a. s.

定理 2　设$\tilde{\rho}=\tilde{\rho}(1)<1$，$p\geqslant 2$，$X_i$为$\sigma(\xi_i)$可测、均值为零，$\sup_i E|X_i|^p<\infty$ $(i=1,2,\cdots)$，如果存在$\theta\geqslant 1(>1/p)$和$k>0$使$|a_{ni}|\leqslant kn^{-\theta}$，则$T_n\triangleq\sum_{i=1}^n a_{ni}X_i\to 0$, a. s.

本文约定：记号“$\ll$”表示通常的大“O”；c是与n无关的正常数，不同之处可取不同的值；I_A表示集合A上的示性函数。

2　定理的证明

定理 1的证明

令$b_{ni}=a_{ni}/n^{1/p}$，$B_n=\sum_{i=1}^n b_{ni}^2$，则$T_n=\sum_{i=1}^n b_{ni}X_i$，且$B_n\ll kn^{\theta-2/p}$，$|b_{ni}|\ll kn^{-\rho-1/p}$，

令

$$\begin{cases} X_{ni}(1)=X_iI(|b_{ni}X_i|\leqslant n^{-\rho}), \\ X_{ni}(2)=X_iI(|b_{ni}X_i|>n^{-\rho}), \\ \bar{X}_{ni}(j)=X_{ni}(j)-EX_{ni}(j), \\ T_n(j)=\sum_{i=1}^n b_{ni}\bar{X}_{ni}(j), j=1,2. \end{cases}$$

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则 $T_n = T_n(1) + T_n(2)$.

要证 $T_n \to 0$, a. s. 只要证 $T_n(j) \to 0$, a. s. $j = 1, 2$.

1) 先证 $T_n(1) \to 0$, a. s.

设 $M > p$,由引理,有:

$P(|T_n(1)| > X) = P(|\sum_{i=1}^{n} b_{ni}\bar{X}_{ni}(1)| > X) \ll$

$E|T_n(1)|^M \ll \sum_{i=1}^{n} E|b_{ni}\bar{X}_{ni}(1)|^M +$

$(\sum_{i=1}^{n} E(b_{ni}\bar{X}_{ni}(1))^2)^{M/2}\} = \sum_{i=1}^{n} E(b_{ni}\bar{X}_{ni}(1))^2 \cdot$

$|b_{ni}\bar{X}_{ni}(1)|^{M-2} + (\sum_{i=1}^{n} E(b_{ni}\bar{X}_{ni}(1))^2)^{M/2} \ll n^{\theta - 2/p} \cdot$

$n^{-d(M-2)} + n^{(\theta - 2/p)\cdot M/2} \leqslant n^{-d(M-2)} + n^{(\theta - 2/p)\cdot M/2}$,

$\because 0 < \theta < 2/p$, $\therefore$ 取 M 充分大即有 $\sum_{n=1}^{\infty} P(|T_n(1)| > X) < \infty$, $\therefore T_n(1) \to 0$, a. s.

2) 证 $T_n(2) \to 0$, a. s.

令 $T_n'(2) = \sum_{i=1}^{n} b_{ni}X_{ni}^{(2)}$, 则 $T_n(2) = T_n'(2) - ET_n'(2)$, (2)

$|ET_n'(2)| = |E\sum_{i=1}^{n} b_{ni}X_{ni}(2)| \leqslant \sum_{i=1}^{n} E|b_{ni}X_{ni}(2)|$

$\leqslant \sum_{i=1}^{n} E|b_{ni}X_{ni}(2)| [\frac{|b_{ni}X_i|}{n^{-d}}]^{p-1} = \sum_{i=1}^{n} E|b_{ni}X_{ni}(2)|^p n^{d(p-1)}$

$\ll n^{\theta - 2/p}(n^{-d-1/p})^{p-2} n^{d(p-1)} = n^{\theta - 1 + d} \to 0, (n \to \infty)$. (3)

又

$\because [T_n'(2)]^2 = [\sum_{i=1}^{n} b_{ni}X_{ni}(2)]^2 \leqslant \sum_{i=1}^{n} b_{ni}^2 \sum_{i=1}^{n} X_{ni}^2(2) \ll$

$n^{\theta - 2/p}\sum_{i=1}^{n} X_i^2 I_{(|b_{ni}X_i| > n^{-d})} = n^{\theta - 2/p}\sum_{i=1}^{n} X_i^2 I_{(|X_i| > \frac{1}{k}n^{1/p})}$.

而 $\sup_i E|X_i|^p < \infty$, $(p \geqslant 2)$,

$\therefore \sum_i P(|X_i| > n^{1/p}/k) < \infty$.

由 Borel-Cantelli 引理有:

$S = \sum_{i=1}^{\infty} X_i^2 I_{(|X_i| > n^{1/p}/k)} < \infty$, a. s.

$\therefore T_n'(2) \to 0$, a. s.

即 $T_n'(2) \to 0$, a. s. (4)

由 (2), (3), (4) 有: $T_n(2) \to 0$, a. s. 证毕.

定理 2 的证明

令
$$\begin{cases} X_{ni}(1) = X_i I_{(|a_{ni}X_i| \leqslant n^{-d})}, \\ X_{ni}(2) = X_i I_{(|a_{ni}X_i| > n^{-d})}, \\ \bar{X}_{ni}(j) = X_{ni}(j) - EX_{ni}(j), \\ T_n(j) = \sum_{i=1}^{n} a_{ni}\bar{X}_{ni}(j), j = 1, 2. \end{cases}$$

其中 $d > 0$,且为充分小的待定常数.

1) 证 $T_n(1) \to 0$, a. s.

$\forall X > 0$ 和 $M > p \geqslant 2$,由引理,有:

$P(|T_n(1)| > X) \ll E|T_n(1)|^M \ll$

$\sum_{i=1}^{n} E|a_{ni}\bar{X}_{ni}(1)|^M + (\sum_{i=1}^{n} E(a_{ni}\bar{X}_{ni}(1))^2)^{M/2} \ll n^{-\theta(M-1)}$

$+ n^{-\theta\frac{M}{2}}$,

取 M 充分大,即有

$\sum_{n=1}^{\infty} P(|T_n(1)| > X) < \infty$,

从而, $T_n(1) \to 0$, a. s.

2) 证 $T_n(2) \to 0$, a. s.

令 $T_n'(2) = \sum_{i=1}^{n} a_{ni}X_{ni}(2)$,则

$T_n(2) = T_n'(2) - ET_n'(2)$, (5)

$|ET_n'(2)| \leqslant \sum_{i=1}^{n} E|a_{ni}X_{ni}(2)| \leqslant \sum_{i=1}^{n} E|a_{ni}X_{ni}(2)|$

$[\frac{|a_{ni}X_{ni}|}{n^{-d}}]^{p-1} = \sum_{i=1}^{n} E|a_{ni}X_{ni}^p(2)||a_{ni}|^{p-1} \cdot n^{d(p-1)} \ll$

$n^{-\theta(p-1)+d(p-1)} \to 0, (n \to \infty$ 时). (6)

$[T_n'(2)]^2 = [\sum_{i=1}^{n} a_{ni}X_{ni}(2)]^2 \leqslant \sum_{i=1}^{n} a_{ni}^2 \sum_{i=1}^{n} X_{ni}^2(2) =$

$\sum_{i=1}^{n} |a_{ni}||a_{ni}| \sum_{i=1}^{n} X_i^2 I_{(|a_{ni}X_i| > n^{-d})} \ll n^{-\theta}\sum_{i=1}^{n} X_i^2 I_{(|X_i| > n^{\theta - d}/k)}$

$\leqslant n^{-\theta}\sum_{i=1}^{n} X_i^2 I_{(|X_i| > i^{\theta - d}/k)}$, (7)

记 $Y_i = X_i^2 I_{(|X_i| > i^{\theta - d}/k)}$, $Q_k = n^{-\theta}\sum_{i=1}^{n} Y_i$, 由 (5) ~ (7) 式知,为证 $T_n(2) \to 0$, a. s.

仅需证:

$Q_n \to 0$, a. s. (8)

又由 Kronecker 引理知: 要证 (8) 成立,只需证

$\sum_{n=1}^{\infty} Y_i / i^{\theta} < \infty$, a. s.

令 $S_n = \sum_{i=1}^{n} Y_i i^{-\theta}$,下面只要证明 S_n, a. s. 收敛即可.我们用子序列法证明这一点.

取 $d > 0$ 和 $W > 0$ 使 $(\theta - d)p > 1$ 且

$0 < W < \min\{1/2, ((\theta - d)p - 1)/(2 + \theta - 2d)\}$,

则 $\theta W + (\theta - d)(p - 2W) > 1 + 2W$.

于是, $\forall m \geqslant n \geqslant 1$,有

$E|S_m - S_n|^W = E|\sum_{i=n+1}^{m} Y_i i^{-\theta}|^W \leqslant E\sum_{i=n+1}^{m}(Y_i^W i^{-\theta W})$

$\leqslant \sum_{i=n+1}^{m} E|Y_i|^W / i^{\theta W} = \sum_{i=n+1}^{m} E|X_i^2 I_{(|X_i| > i^{\theta - d}/k)}|^W / i^{\theta W} \leqslant$

$\sum_{i=n+1}^{m} EX_i^{2W}(\frac{|X_i| \cdot k}{i^{\theta - d}})^{p-2W} / i^{\theta W} \leqslant \sum_{i=n+1}^{m} \sup_i EX_i^p \cdot k^{p-2W} \cdot$

$i^{-\theta W - (\theta - d)(p - 2W)} \ll \sum_{i=n+1}^{m} i^{-\theta W - (\theta - d)(p - 2W)} \leqslant \sum_{i=n+1}^{m} i^{-(1+2W)} \leqslant$

$n^{-W} \to 0$, (当 $n \to \infty$ 时),

因此 $\{S_n; n \geqslant 1\}$ 是 L^W 中 Cauchy 序列,从而存在 r. v. S

使 $E|S|^{w}<\infty$, 且 $E|S_n - S|^{w}\to 0$, (当 $n\to\infty$ 时), 又 $\forall \varepsilon > 0$,

$$P(|S_{2^k} - S| > \varepsilon) \ll E|S_{2^k} - S|^{w} \leqslant \limsup_{n} E|S_{2^k} - S_n|^{w} \ll \sum_{i=2^k+1}^{\infty} i^{-(1+2w)} \ll 2^{-kw}, \quad (9)$$

$$P(\max_{2^{k-1}<n<2^k} |S_n - S_{2^{k-1}}| > \varepsilon) \ll E \max_{2^{k-1}<n<2^k} |S_n - S_{2^{k-1}}|^{w} \ll E \max_{2^{k-1}<n<2^k} \left|\sum_{i=2^{k-1}+1}^{n} Y_i i^{-\theta}\right|^{w} \leqslant E \sum_{i=2^{k-1}+1}^{2^k} |Y_i i^{-\theta}|^{w} \ll \sum_{i=2^{k-1}+1}^{2^k} E|Y_i i^{-\theta}|^{w} \ll 2^{-kw}, \quad (10)$$

由 (9) 和 (10) 有 $S_n \to S$, a.s.

从而定理获证. 证毕.

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$\therefore \exists\, t \in (0,1)$, $f(x_k + \bar{\lambda}_k d_k) = f_k + \bar{\lambda}_k g_k^T d_k + \frac{1}{2}\bar{\lambda}_k^2 d_k^T \nabla^2 f(x_k + t\bar{\lambda}_k d_k) d_k \leqslant f_k - \frac{(g_k^T d_k)^2}{M\|d_k\|^2} + \frac{1}{2}\frac{(g_k^T d_k)^2}{M^2\|d_k\|^4} M\|d_k\|^2 \leqslant f_k - \frac{1}{2M}V_k^2.$

$f_{k+1} \leqslant f(x_k + \lambda_k^* d_k) + \varepsilon_k \leqslant f(x_k + \bar{\lambda}_k d_k) + \varepsilon_k \leqslant f_k - \frac{1}{2M}V_k^2 + \varepsilon_k.$

$\therefore f_{k+1} \leqslant f_1 - \frac{1}{2M}\sum_{i=1}^{k} V_k^2 + \sum_{i=1}^{k} \varepsilon_k.$

Let $k \to \infty$, we obtain

$$\sum_{k=1}^{\infty} V_k^2 < +\infty.$$

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