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时滞竞争捕食扩散系统的周期解与概周期解的存在唯一性

Existence and Uniqueness of the Periodic, Almost Periodic Solutions of a Prey-Predator and Competition System with Diffusion

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摘要　讨论在两斑块生态环境下, 具有有关文献中 Cosner型功能性反应的竞争捕食扩散系统.在一定条件下实现了系统的一致持久性.通过构造 V泛函, 得到了正概周期解的存在唯一性、全局吸引性和在壳扰动下的稳定性.

关键词　时滞竞争捕食扩散系统　周期解　概周期解　V泛函　存在唯一性

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Abstract　A competition and prey-predator model with diffusion is considered. By using the V funtional and the definition of almost periodic solution and the stability under the disturbance from the hull, the existence, uniqueness and their stability of periodic and almost periodic solutions are obtained.

Key words　prey-predator and competition system with diffusion, periodic solution, almost periodic solution, V funtional, existence and uniqueness

1　模型及定义

种群的持续生存一直是生物数学中探讨的重要问题.出于季节的考虑, 讨论种群在周期和概周期环境下的变化情况尤为重要.文献 [1]中考虑了扩散对种群的影响,文献 [1~4]考虑了时滞对种群的影响,文献 [4, 5] 考虑功能性反应对种群的影响.但到目前为止综合考虑诸因素和在概周期环境下讨论的文章还很少见,本文拟讨论如下带有扩散的竞争捕食系统

$$\begin{cases} \dot{x}_1(t) = x_1(t)\Big(b_1(t) - a_{11}(t)x_1(t) - a_{12}(t)x_2(t) - \dfrac{a_{13}(t)x_3(t)}{m_1(t)x_3(t) + d_1(t)x_1(t)}\Big) + D_1(t)(x_4(t) - x_1(t)) \triangleq f_1, \\ \dot{x}_2(t) = x_2(t)\Big(b_2(t) - a_{21}(t)x_1(t) - a_{22}(t)x_2(t) - \dfrac{a_{23}(t)x_3^2(t)}{m_2(t)x_3^2(t) + d_2(t)x_2^2(t)}\Big) \triangleq f_2, \\ \dot{x}_3(t) = x_3(t)\big(b_3(t) + a_{31}(t)x_1(t-\tau_1)/(m_1(t)\cdot x_3(t-\tau_1) + d_1(t)x_1(t-\tau_1)) + a_{32}(t)\cdot x_2(t-\tau_2)x_3(t-\tau_2)/(m_2(t)x_3^2(t-\tau_2) + d_2(t)x_2^2(t-\tau_2))\big) \triangleq f_3, \\ \dot{x}_4(t) = x_4(t)(b_4(t) - a_{44}x_4(t)) + D_2(t)(x_1(t) - x_4(t)) \triangleq f_4, \end{cases} \tag{1}$$

其中,$x\in \mathrm{Int}R_+^4 = \{x_i\in R_+^4; x_i > 0, i = 1, 2, 3, 4\}$,设 $C^+ = C([-\tau, 0], R_+^4)$, $\tau = \max\{\tau_1, \tau_2\}$.

令系统(1)的初始函数空间为 C^+.容易看出,满足初始条件

$$\varphi = (\varphi_1, \varphi_2, \varphi_3, \varphi_4)\in C^+, \varphi_i(0) > 0, (i = 1, 2, 3, 4). \tag{2}$$

系统(1)一定存在解 $x(t,\varphi)$, $t\in[-\tau, \infty)$且 $x(t,\varphi) > 0$, (当 $t\in[-\tau, \infty)$时).我们称这样的解为系统(1)的正解.$x_i(t)$表示第 i个种群在时刻 t的密度;在系统(1)中种群 x_1与 x_2相互竞争共存于同一斑块中,并且在此斑块中存在共同的捕食者 x_3,其中仅有 x_1与 x_4可在斑块之间相互扩散;捕食与被捕食关系中不同的 Cosner功能性反应体现了空间因素和种群密度间比率的作用关系.不同的时滞体现了捕食者种群 x_3消化吸收被捕食者以及 x_3对种群增长的影响.

相关记号: $\bar{g} = \sup\{g(t)\}$, $\underline{g} = \inf\{g(t)\}$, $f_i = f_i(t, x_1(t), x_2(t), x_3(t), x_4(t))$, $(i = 1, 2, 3, 4)$.系统(1)中 $b_i(t)$, $a_{ij}(t)$, $D_i(t)$, $(i, j = 1, 2, 3, 4)$均为正的连续函数,且满足:

$0 < \min\{\underline{a_{ij}}, \underline{b_i}, \underline{D_i}\} \leqslant \max\{\overline{a_{ij}}, \overline{b_i}, \overline{D_i}\} < +\infty$, $(i, j = 1, 2, 3, 4)$.

2　一致持久性

定义　对系统(1)而言,如果存在一个紧集 $S\subset$

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$\mathrm{Int}R_+^4$，是系统(1)的每一个满足初始条件(2)的正解，$x(t)$在t充分大之后都将进入并保持在S中，此时则称系统(1)是一致持久的.

引理 1 $\mathrm{Int}R_+^4=\{(x_1,x_2,x_3,x_4)\mid x_i>0,i=1,2,3,4\}$为系统(1)的不变集.

证明 由系统(1)知，

$$\dot{x}_1|_{x_1=0}=D_1(t)x_4(t)>0,\dot{x}_4|_{x_4=0}=D_2(t)x_2(t)>0,$$

$$x_2(t)=x_2(0)\exp\{\int_0^t[b_2(s)-a_{21}(s)x_1(s)-a_{22}(s)x_2(s)-a_{23}(s)x_3^2(s)/(m_2(s)x_3^2(s)+d_2(s)x_2^2(s))]\mathrm{d}s\}>0,$$

$$x_3(t)=x_3(0)\exp\{\int_0^t[-b_3(s)+a_{31}(s)x_1(s-f_1)/(m_1(s)x_3(s-f_1)+d_1(s)x_1(s-f_1))+a_{32}(s)x_2(s-f_2)x_3(s-f_2)/(d_2(s)x_2^2(s-f_2))]\}>0.$$

所以$\mathrm{Int}R_+^4$为系统(1)的不变集.证毕.

引理 2 设$x(t)$为系统(1)满足初始条件(2)的任一正解，且

$$U\triangleq\frac{\overline{a_{21}}}{\underline{d_1}}+\frac{\overline{a_{32}}}{2\underline{m_2d_2}}-\underline{b_3}<0. \qquad (3)$$

则存在$T^*>0$，使$t>T^*$时，使得$x_i(t)\leqslant M_i,i=1,2,3,4$.其中，

$$M_1=M_4>M_1^*=\max\{\frac{\overline{b_1}}{\underline{a_{11}}},\frac{\overline{b_4}}{\underline{a_{44}}}\}>0,M_2>M_2^*=\frac{\overline{b_2}}{\underline{a_{22}}},M_3>M_3^*=(M_1\overline{m_1}\exp\{Uf_2\}+\underline{m_2}M_1\cdot\overline{a_{31}}\exp\{Uf_1\})/\underline{m_1m_2b_3}>0.$$

证明 根据系统(1)的第一式和第四式，得到

$$\dot{x}_1(t)\leqslant x_1(t)(\overline{b_1}-\underline{a_{11}}x_1(t))+\overline{D_1}(x_4(t)-x_1(t)),$$

$$\dot{x}_4(t)\leqslant x_4(t)(\overline{b_4}-\underline{a_{44}}x_4(t))+\overline{D_2}(x_1(t)-x_4(t)).$$

设$V(t)=\max_{t\geqslant 0}\{x_1(t),x_2(t)\}$.沿着系统(1)的解，分情况讨论$V(t)$的上右Dini导数为

(P_1)若$x_1(t)>x_4(t)$或$x_1(t)=x_4(t)$且$\dot{x}_1(t)\geqslant\dot{x}_4(t)$，则$D^+V(t)=\dot{x}_1(t)\leqslant\dot{x}_1(t)(\overline{b_1}-\underline{a_{11}}x_{11}(t))$，

(P_2)若$x_1(t)<x_4(t)$或$x_1(t)=x_4(t)$且$\dot{x}_1(t)<\dot{x}_4(t)$，则$D^+V(t)=\dot{x}_4(t)\leqslant\dot{x}_4(t)(\overline{b_4}-\underline{a_{44}}x_4(t))$，

从而，$i=1$或4时，

$D^+V(t)\leqslant\dot{x}_i(t)(\overline{b_i}-\underline{a_{ii}}x_{ii}(t))$，

(A)如果$\max\{x_1(0),x_4(0)\}\leqslant M_1$，那么，$\max_{t\geqslant 0}\{x_1(t),x_4(t)\}\leqslant M_1$;

(B)如果$\max\{x_1(0),x_4(0)\}>M_1$，设$-T=\max_{i=1,4}\{M_1(\overline{b_i}-\underline{a_{ii}}M_1)\}<0$，

有下面3种情况:

(a)$V(0)=x_1(0)>M_1,(x_1(0)>x_4(0))$;

(b)$V(0)=x_4(0)>M_1,(x_4(0)>x_1(0))$;

(c)$V(0)=x_1(0)=x_4(0)>M_1,(x_1(0)>x_4(0))$.

假设(a)成立，则存在$X>0$，当$t\in[0,X)$时，$V(t)=x_1(t)>M_1$，有

$$D^+V(x_1(t),x_4(t))=\dot{x}_1(t)<-T<0;$$

假设(b)成立，则存在$X>0$，当$t\in[0,X)$时，$V(t)=x_4(t)>M_1$，有

$$D^+V(x_1(t),x_4(t))=\dot{x}_4(t)<-T<0;$$

假设(c)成立，则存在$X>0$，当$t\in[0,X)$时，$V(t)=x_i(t)>M_1,i=1,4$.有

$$D^+V(x_1(t),x_4(t))=\dot{x}_i(t)<-T<0;$$

从上面的讨论可得，当$V(0)>M_1$时，$V(t)$是一个至少以速度T单调递减的函数，从而存在$T_1>0$，当$t\geqslant T_1$时有

$$V(t)=\max\{x_1(t),x_4(t)\}\leqslant M_1.$$

由系统(1)的第二式有

$$\dot{x}_2(t)\leqslant x_2(t)(\overline{b_2}-\underline{a_{22}}x_2(t))\leqslant M_2(\overline{b_2}-\underline{a_{22}}M_2)<0.$$

仿照上面讨论可知，

若$x_2(0)>M_2$，且$\dot{x}_2(t)\leqslant 0$，则存在$T_2>0$.使$t\geqslant T_2$时，$x_2(t)\leqslant M_2$.

若$x_2(0)\leqslant M_2$，则$t\in[0,+\infty)$，$x_2(t)\leqslant M_2$.

由系统(1)的第三式，得到，

$$\dot{x}_3(t)\leqslant\overline{a_{31}}x_1(t-f_1x_3(t))/(\underline{m_1}x_3(t-f_1)+\underline{d_1}x_1(t-f_1))+\overline{a_{32}}x_2(t-f_2x_3(t))x_{32}(t-f_2)/(\underline{m_2}x_3^2(t-f_2)+\underline{d_2}x_2^2(t-f_2))-\underline{b_3}x_3(t))$$

$$\leqslant x_3(t)\left(\frac{\overline{a_{31}}}{\underline{d_1}}+\frac{\overline{a_{32}}}{2\underline{m_2d_2}}-\underline{b_3}\right)<0.$$

对上式两端积分可得

$$x_3(t)\leqslant x_3(t-f_i)\exp\{Uf_i\},(i=1,2).$$

即为，$x_3(t-f_2)\geqslant x_3(t)\exp(-Uf_2\}$.从而，

$$\dot{x}_3(t)\leqslant x_3(t)(-\underline{b_3}+\overline{a_{31}}M_1/(m_1x_3(t)\exp\{-Uf_1\})+\overline{a_{32}}M_1/m_2x_3(t)\exp\{-Uf_2\})=x_3(t)[-\underline{b_3m_1m_2}x_3(t)+\underline{m_2}\,\overline{a_{31}}M_1\exp\{Uf_1\}+\overline{a_{32}}\,\underline{m_1}M_1\exp\{Uf_2\}]/\underline{m_1m_2}x_3(t).$$

记$r=\underline{m_2}\,\overline{a_{31}}M_1\exp\{Uf_1\}+\overline{a_{32}}\,\underline{m_1}M_1\exp\{Uf_2\}>0$，有

$$\dot{x}_3(t)\leqslant(-\underline{m_2m_1b_3}x_3^2(t)+rx_3(t))/\underline{m_1m_2}x_3(t)=-\underline{b_3}x_3(t)+\frac{r}{\underline{m_1m_2}}.$$

从而存在$T_3>0$，当$t>T_3$时，有

$$x_3(T)\leqslant M_3^*=\frac{r}{\underline{m_1m_2}}<M_3.$$

取 $T^{*} = \max\{T_i, i = 1, 2, 3\}$,则当 $t > T^{*}$ 时,有引理 2结论成立.证毕.

引理 3 对系统(1)的每个正解 $x(t) = (x_1(t), x_2(t), x_3(t), x_4(t))$,若满足下列条件

$$\left.\begin{array}{l} 2\underline{b_1} - 2\overline{D_1} - 2\overline{a_{12}}M_2 - 2\dfrac{\overline{a_{13}}}{\underline{m_1}} > 0,\ 2\underline{b_2} - 2\overline{a_{21}}M_1 - 2\dfrac{\overline{a_{23}}}{\underline{m_2}} > 0,\ 2\underline{b_4} - 2\overline{D_2} > 0, \\ (\underline{a_{31}}h_1\overline{d_2}M_2 - \overline{b_3d_1}h_1\overline{m_2}\exp\{-2\overline{b_3}\tau_2\}M_3^2 - 2\overline{b_3}\,\overline{m_1m_2}\exp\{\overline{U_3}\tau_1\}\exp\{-2\overline{b_3}\tau_2\}M_3^2 - \\ 2\overline{b_3d_1d_2}\dfrac{h_1}{2}M_2)/(\overline{a_{32}}h_2\exp\{-U_2\tau_2\}\overline{d_1}\dfrac{h_1}{2} - \overline{b_3}\,\overline{m_1}\exp\{\overline{b_3}\tau_1\}\overline{d_2}M_2) > 0, \end{array}\right\} \quad (4)$$

则存在充分大的 $T^{**} > 0$,使 $t > T^{**}$ 时有,$x_i(t) \geqslant H_i, i = 1, 2, 3, 4$.其中,

$$H_1 \stackrel{\triangle}{=} \frac{2\underline{b_1} - 2\overline{D_1} - 2\overline{a_{12}}M_2 - 2\dfrac{\overline{a_{13}}}{\underline{m_1}}}{\overline{a_{11}}} > 0,$$

$$H_2 \stackrel{\triangle}{=} \frac{2\underline{b_2} - 2\overline{a_{21}}M_1 - 2\dfrac{\overline{a_{23}}}{\underline{m_2}}}{\overline{a_{22}}} > 0,$$

$$H_3 \stackrel{\triangle}{=} (\underline{a_{31}}h_1\overline{d_2}M_2 - \overline{b_3d_1}h_1\overline{m_2}\exp\{-2\overline{b_3}\tau_2\}M_3^2 - 2\overline{b_3}\,\overline{m_1m_2}\exp\{\overline{U_3}\tau_1\}\exp\{-2\overline{b_3}\tau_2\}M_3^2 - 2\overline{b_3d_1d_2}\frac{h_1}{2}M_2)/(\overline{a_{32}}h_2\exp\{-U_2\tau_2\}\overline{d_1}\frac{h_1}{2} - \overline{b_3}\,\overline{m_1}\exp\{\overline{b_3}\tau_1\}\overline{d_2}M_2) > 0,$$

$$H_4 \stackrel{\triangle}{=} \frac{2\underline{b_4} - 2\overline{D_2}}{\overline{a_{44}}} > 0,$$

证明 由系统(1)第一式,得到

$$\dot{x}_1(t) \geqslant x_1(t)(\overline{b_1} - \overline{a_{11}}x_1(t) - \overline{D_1} - \overline{a_{12}}M_2 - (\overline{a_{13}}/\overline{m_1})),$$

易知,存在充分大的 $T_1' > 0$,当 $t > T_1'$ 时,$x_1(t) \geqslant H_1$ 成立.同理可得存在充分大的 $T_2' > 0$,当 $t > T_2'$ 时,有 $x_2(t) \geqslant H_2$;存在 $T_4' > 0$,使 $t > T_4'$ 时,$x_4(t) \geqslant H_4$.

由系统(1)第三式,得到

$\dot{x}_3(t) \geqslant -\overline{b_3}x_3(t)$,可得,$x_3(t - \tau_i) \leqslant x_3(t)\exp\{\overline{b_3}\tau_i\}$,$i = 1, 2$.从而有

$$\dot{x}_3(t) \geqslant x_3(t)(-\overline{b_3} + \overline{a_{31}}\frac{h_1}{2}/(\overline{d_1}\frac{h_1}{2} + \overline{m_1}x_3(t)\cdot\exp\{\overline{b_3}\tau_1\}) + \underline{a_{32}}\frac{h_2}{2}x_3(t)\exp\{-U\tau_2\}/(\overline{d_2}M_2 + \overline{m_2}x_3^2(t)\exp\{-2\overline{b_3}\tau_2\}),$$

进一步整理可得,存在 $T_3' > 0$,当 $t > T_3'$ 时,$x_3(t) \geqslant H_3 > 0$.所以,存在 $T^{**} = \max\{T_i', i = 1, 2, 3, 4\}$,使当 $t > T^{**}$ 时,得到引理 3成立.证毕.

定理 1 系统(1)满足条件(2),当条件(3)与(4)成立时,系统(1)是一致持久的.

证明 由引理 1,2和 3可知,当条件(3)和(4)成立时,存在 $T = \max\{T^{*}, T^{**}\}$,当 $t > T$ 时,$H_i \leqslant x_i(t) \leqslant M_i, i = 1, 2, 3, 4$.在根据前面的定义知,系统(1)是一致持久的,即存在一个紧集 $S = \{(x_1, x_2, x_3, x_4) | 0 < H_i \leqslant x_i \leqslant M_i < +\infty, i = 1, 2, 3, 4\} \subset \mathrm{Int}R_+^4$,当 $t > T$ 时,集合 S 为系统(1)的最终有界不变集.证毕.

3 周期解和全局渐近稳定性

由定理 1和文献[6]中定理 2知

定理 2 若系统(1)为 k-周期系统,且满足条件(2),(3)和(4),则系统(1)一定存在正 k-周期解.

引理 4[7] 若存在非负函数 $f(t)$ 在 $[0,\infty)$ 上可积且一致连续,则 $\lim\limits_{t\to+\infty} f(t) = 0$.

定理 3 设系统(1)满足条件(2),(3)和(5),且满足

$$\left.\begin{array}{l} \overline{D_2}/H_4 - \underline{a_{11}} - \underline{D_1}H_4 + \overline{a_{13}}\,\overline{d_1}M_3/(\underline{m_1}H_3 + \underline{d_1}H_1)^2 + \overline{a_{12}} + N_3 < 0, \\ \overline{a_{12}} - \underline{a_{22}} + 2\overline{a_{23}}\,\overline{d_2}M_2M_3^2/(\underline{m_2}H_3^2 + \underline{d_2}H_2^2)^2 + N_2 < 0,\ -\underline{a_{44}} - \underline{D_2}H_1/M_4^2 + \overline{D_1}/H_1 < 0, \\ \overline{a_{13}}\,\overline{d_1}M_1/(\underline{m_1}H_3 + \underline{d_1}H_1^2)^2 + 2\overline{d_{23}d_2}M_2M_3/(\underline{m_2}H_3^2 + \underline{d_2}H_2^2)^2 - \underline{a_{32}m_2}H_3^2H_2/(\overline{m_2}M_3^2 + \overline{d_2}M_2^2)^2 - \underline{b_3} + N_1 + N_4 < 0, \end{array}\right\} \quad (5)$$

其中,

$$N_1 = \max\{\overline{a_{32}d_2}M_3M_2^3/(\underline{m_2}M_3^2 + \underline{d_2}M_2^2)^2, \overline{a_{32}m_2}M_2M_3^3/(\underline{m_2}H_3^2 + \underline{d_2}H_2^2)^2\} > 0,$$

$$N_2 = \max\{\overline{a_{32}d_2}M_3^2M_2^2/(\underline{m_2}M_3^2 + \underline{d_2}M_2^2)^2, \overline{a_{32}m_2}M_3^4/(\underline{m_2}H_3^2 + \underline{d_2}H_2^2)^2\} > 0,$$

$$N_3 = \overline{a_{31}m_1}M_1M_3/(\underline{m_1}H_3 + \underline{d_1}H_1)^2 > 0,$$

$$N_4 = \overline{a_{31}m_1}M_3^2/(\underline{m_1}H_3 + \underline{d_1}H_1)^2 > 0,$$

则系统(1)的每个满足(2)正解对位于 R_+^4 的其它解而言是全局渐近稳定的.

证明 设 $x(t), y(t)$ 为系统(1)位于 R_+^4 中的任何两解.取 V 泛函 $W(t)$ 为

$$W(t) = \sum_{i=1,2,4}|\ln x_i(t) - \ln y_i(t)| + |x_3(t) - y_3(t)| + N_1\int_{t-\tau_2}^{t}|x_3(s) - y_3(s)|\mathrm{d}s + N_2\int_{t-\tau_2}^{t}|x_2(s) - y_2(s)|\mathrm{d}s + N_3\int_{t-\tau_1}^{t}|x_1(s) - y_1(s)|\mathrm{d}s + N_4\int_{t-\tau_1}^{t}|x_3(s) - y_3(s)|\mathrm{d}s,$$

沿着 $x(t), y(t)$ 计算 $W(t)$ 的右上 Dini导数为

$$D^{+}W(t) \leqslant (\overline{D_2}/H_4 - \underline{a_{11}} - \underline{D_1}H_4 + \overline{a_{13}d_1}M_3/(\underline{m_1}H_3 + \underline{d_1}H_1)^2 + \overline{a_{12}} + N_3)|x_1(t) - y_1(t)| + (\overline{a_{12}} - \underline{a_{22}} + \overline{a_{23}d_2}M_2M_3^2/(\underline{m_2}H_3^2 + \underline{d_2}H_2^2)^2 + N_2)|x_2(t) -$$

$y_2(t)| + (\overline{a_{13}d_1}M_1/(\underline{m_1}H_3 + \underline{d_1}H_1^2)^2 + \overline{d_{23}d_2}M_2M_3/(\underline{m_2}H_3^2 + \underline{d_2}H_2^2)^2 - \underline{a_{32}m_2}H_3^2H_2/(\underline{m_2}M_3^2 + \underline{d_2}M_2^2)^2 - \underline{b_3} + N_1 + N_4)|x_3(t) - y_3(t)| + (-\underline{a_{44}} - \underline{D_2}H_1/M_4^2 + \overline{D_1}/H_1)|x_4(t) - y_4(t)|$,

从而一定存在正常数 T, 满足 $D^+W(t) \leqslant -T\sum_{i=1}^{4}|x_i(t) - y_i(t)|$. 对上式两边积分可得 $W(t) + T\int_T^t\sum_{i=1}^{4}|x_i(s) - y_i(s)|\,ds \leqslant W(t), t \geqslant T$. 由 $W(t) \geqslant 0, W(t) = \text{const.} > 0$, 有 $\int_T^t\sum_{i=1}^{4}|x_i(s) - y_i(s)|\,ds \leqslant \frac{1}{T}W(T) < +\infty$. 又因 $x(t), y(t)$ 对于 $t \in R_+$ 时有界, 由系统 (1) 知 $\frac{d}{dt}(x_i(t) - y_i(t)), (i = 1,2,3,4)$, 在 R_+ 有界. 因而 $\sum_{i=1}^{4}|x_i(t) - y_i(t)|$ 在 R_+ 上一致连续, 由引理 4 知. 当 $t \to +\infty$ 时 $\sum_{i=1}^{4}|x_i(t) - y_i(t)| \to 0$. 定理 3 证毕.

推论 1 若系统 (1) 为 k-周期系统, 在定理 3 的条件下, 可知每个正周期解相对于 R_+^4 中其他解而言为全局渐近稳定的.

定理 4 若系统 (1) 满足条件 (2), (3), (4) 和 (5), 则系统 (1) 存在唯一正 k-周期解且相对于 R_+^4 中的其它解而言为全局渐近稳定的.

证明 由定理 2 和推论 1 可知系统存在正的周期解, 且为全局渐近稳定的. 不妨设系统 (1) 存在 2 个正周期解 $x(t), y(t)$ (周期未必相同). 于是, $x_i(t) - y_i(t), (i = 1,2,3,4)$ 均为概周期函数, 且 $t \to +\infty$ 时, $(x_i(t) - y_i(t)) \to 0$. 由此可知, $x_i(t) \equiv y_i(t)$. 证毕.

4 概周期解和在壳扰动下的稳定性

定理 5 设 $b_i(t), a_{ij}(t), m_i(t), d_i(t), D_i(t)$ 均为概周期函数, 系统 (1) 满足条件 (2) ~ (5) 且满足:

$$
\begin{aligned}
&\underline{a_{31}}\,\overline{m_1}M_3 + \underline{a_{31}}\,\overline{d_1}M_1 > H_1\underline{d_1}\,\overline{a_{31}},\\
&\overline{m_2}M_3^2\underline{a_{32}} + \overline{d_2}M_2^2\underline{a_{32}} > H_2H_3\overline{a_{32}}(\overline{d_2}^2 + \overline{m_2}^2),\\
&-\underline{a_{11}} - H_4D_1/M_1^2 + \overline{a_{13}}\,\overline{d_1}/E_1 + \overline{a_{21}} + \overline{D_1}/H_4 < 0,\\
&\overline{a_{12}} - \underline{a_{22}} + 2\overline{a_{23}}\,\overline{d_2}M_3^2M_2/E_2 < 0,\\
&\overline{D_1}/H_1 - \underline{a_{44}} + \overline{D_2}M_1/H_4^2 < 0,
\end{aligned}
\qquad (6)
$$

则系统 (1) 存在唯一的正概周期解 $(u(t), v(t), Y(t), Z(t))$, 它在 R_+^4 中为全局渐近稳定的, 又 $\text{Rang}(u(t), v(t), Y(t), Z(t)) \subset S$, $\text{mod}(u(t), v(t), Y(t), Z(t)) \subset \text{mod}(f_1, f_2, f_3, f_4), t \in R$.

证明 由 $b_i(t), a_{ij}(t), m_i(t), d_i(t), D_i(t)$ 的概周期性, 存在数列 $\{f_k\}$, $f_k \to +\infty, (k \to +\infty)$. 使对于 $t \in R$ 成立.

$$
\left.
\begin{aligned}
&b_i(t + f_k) \rightrightarrows b_i(t), a_{ij}(t + f_k) \rightrightarrows a_{ij}(t), m_i(t + f_k) \rightrightarrows m_i(t),\\
&d_i(t + f_k) \rightrightarrows d_i(t), i = 1,2.\ D_i(t + f_k) \rightrightarrows D_i(t), (i = 1,2,3,4).
\end{aligned}
\right\}
\qquad (7)
$$

必要时, 可选取其子序列, 可认为 f_k 是单调递增的, 于是, 对任意实数 U_1, 存在 $k = k(U_1)$ 使对 $k > K$ 时, $f_k + U_1 \geqslant 0$, 于是 $k \geqslant K, t \geqslant U_1$ 时, 有 $t + f_k > f_i > 0, (i = 1,2)$. 由 S 是不变且最终有界集. 对系统 (1) 的任何解 $(x_1(t), x_2(t), x_3(t), x_4(t))$ 而言, 将有 $(x_1(0), x_2(0), x_3(0), x_4(0)) \in S, t \geqslant U_1, k \geqslant K \Rightarrow (x_1(t + f_k), x_2(t + f_k), x_3(t + f_k), x_4(t + f_k)) \in S$.

现证明函数序列 $\{(x_1(t + f_k), x_2(t + f_k), x_3(t + f_k), x_4(t + f_k))\}$ 在 $[U_1, +\infty)$ 的每个紧子集 I 上是一致收敛的. 为此令

$$V(t) = \sum_{i=1,2,4}|\ln x_i(t + f_k) - \ln x_i(t + f_m)| + |x_3(t + f_k) - x_3(t + f_m)|,$$

其中 f_1, f_2 为系统 (1) 中的时间滞量, 记 $x_i(t + f_m) \triangleq x_{im}$. 按微分中值定理, 得

$$l\sum_{i=1}^{4}|\ln x_i(t + f_k) - \ln x_i(t + f_m)| \leqslant V(t) \leqslant L\sum_{i=1}^{4}|\ln x_i(t + f_k) - \ln x_i(t + f_m)|, \qquad (8)$$

其中, $L = \max\{\frac{1}{H}, 1\}$, $H = \min\{H_1, H_2, H_3, H_4\} > 0$, $l = \min\{\frac{1}{M}, 1\}$, $M = \max\{M_1, M_2, M_3\} > 0$. 由条件 (3) ~ (7) 知, 任意给定 $X > 0$, 存在 $N = N(X, U_1) \geqslant k$, $T > 0$, 使对 $m \geqslant k \geqslant N, t \in R$ 有

$$
\left.
\begin{aligned}
&\sum_{i=1}^{4}|b_{ik} - b_{im}| < \frac{TX}{14LF_1}, F_1 = \max\{1, M_3\};\\
&\sum_{i=1}^{4}|D_{ik} - D_{im}| < \frac{TX}{14LF_2}, F_2 = \max\{1 + \frac{M_4}{H_1}, 1 + \frac{M_1}{H_4}\};\\
&\sum_{i=1}^{4}|d_{ik} - d_{im}| < \frac{TX}{14LF_3}, F_3 = \max\{\frac{M_3^2\overline{a_{13}}}{E_1}, \frac{M_3^2\overline{a_{23}}M_2^2}{E_2}\};\\
&\sum_{i=1}^{4}|a_{1ik} - a_{1im}| < \frac{TX}{14LF_4}, F_4 = \max\{M_1, M_2, \frac{M_3^2(\overline{m_1} + \overline{d_1})}{E_1}\};\\
&\sum_{i=1}^{4}|a_{2ik} - a_{2im}| < \frac{TX}{14LF_5}, F_5 = \max\{M_1, M_2, \frac{\overline{m_2}M_3^4}{E_2} + \frac{M_3^2M_2^2\overline{d_2}}{E_2}\};\\
&\sum_{i=1}^{4}|a_{3ik} - a_{3im}| < \frac{TX}{14LF_6}, F_6 = \max\{M_3q, M_3p\};\\
&|a_{44k} - a_{44m}| < \frac{TX}{14LM_4}, \exp\{-T(U_1 + f_k)/L\} < \frac{Xl}{16ML},
\end{aligned}
\right\}
\qquad (9)
$$

其中，p 与 q 均为常数，且满足

$$0 < p < \min\left\{\frac{H_1}{\overline{m_1}M_3 + \overline{d_1}M_1}, \frac{1}{\overline{d_1}}\right\},$$

$$0 < q < \min\left\{\frac{H_2H_3}{\overline{m_2}M_3^2 + \overline{d_2}M_2^2}, \frac{1}{2\overline{m_{2m}d_{2m}}}\right\},$$

$$E_1 = (\underline{m_1}H_3 + \underline{d_1}H_1)^2,$$
$$E_2 = (\underline{m_2}H_3^2 + \underline{d_2}H_2^2)^2,$$
$$E_3 = \underline{m_1}H_3 + \underline{d_1}H_1,$$
$$E_4 = \underline{m_2}H_3^2 + \underline{d_2}H_2^2.$$

沿着系统(1)的正解计算 $V(t)$ 的上右 Dini 导数，可得

$$D^+V(t) = \sum_{i=1,2,4} \mathrm{sign}(x_{im} - x_{ik})\left(\frac{\dot{x}_{im}}{x_{im}} - \frac{\dot{x}_{ik}}{x_{ik}}\right) + \mathrm{sign}(x_{3m} - x_{3k})\left(\frac{\dot{x}_{3m}}{x_{3m}} - \frac{\dot{x}_{3k}}{x_{3k}}\right) \leqslant -T\sum_{i=1}^{4}|x_{im} - x_{ik}| + F_1\sum_{i=1}^{4}|b_{im} - b_{ik}| + F_2\sum_{i=1}^{2}|D_{im} - D_{ik}| + F_3\sum_{i=1}^{2}|d_{im} - d_{ik}| + F_4\sum_{i=1}^{3}|a_{1im} - a_{1ik}| + F_5\sum_{i=1}^{3}|a_{2im} - a_{2ik}| + F_6\sum_{i=1}^{3}|a_{3im} - a_{3ik}| + |a_{44m} - a_{44k}|.$$

由(8)及(9)式可得 $D^+V(t) \leqslant -\frac{T}{L}V(t) + \frac{Tl}{2L}X$，取 $N_0 \geqslant N$，使 $t \in I, R \geqslant N_0$ 时，$t + f_k \geqslant f > 0, (i = 1, 2)$，在 $[-f_k, t]$ 上应用比较定理，得到 $V(t) \leqslant V(-f_k)\exp\left\{-\frac{T(t + f_k)}{L}\right\} + \frac{lX}{2}$. 再由(8)式和 S 的不变性，可得 $\sum_{i=1}^{4}|x_{im} - x_{ik}| \leqslant \frac{8ML}{l}\exp\{U + f_k\} + \frac{X}{2}$. 再由(9)式，得到 $\sum_{i=1}^{4}|x_{im} - x_{ik}| < X, m \geqslant k \geqslant N, t \in R$.

这表明 $(x_1(t + f_k), x_2(t + f_k), x_3(t + f_k), x_4(t + f_k))$ 在 $[U, +\infty)$ 的一切紧子集上一致收敛，记极限函数为 $(u(t), v(t), Y(t), Z(t))$. 由于 U 的任意性．实际上，$(u(t), v(t), Y(t), Z(t))$ 在 $(-\infty, +\infty)$ 上有定义，由 $t \geqslant 0$ 时，$(x_1(t), x_2(t), x_3(t), x_4(t))$ 的值域含于 S，知 $\mathrm{Rang}(u(t), v(t), Y(t), Z(t)) \subset S$.

仿文献[8]第 352～353 页，可以证明 $(u(t), v(t), Y(t), Z(t))$ 是可微的，并且满足系统(1).

为证明 $(u(t), v(t), Y(t), Z(t))$ 是概周期函数，由各系数的概周期性，显然可以知道 (f_1, f_2, f_3, f_4) 关于 $(x_1, x_2, x_3, x_4) \in R$ 是 t 的一致概周期的．于是任给定数列 $\{f_k\}$，存在子序列 $\{f_k\}$ 及 $(f_1(t + f_k), f_2(t + f_k), f_3(t + f_3), f_4(t + f_k))$ 在 $R \times D$ 一致收敛，这里 D 是 S 的任何紧子集（文献[9]定理 2.2）. 必要时，选取子序列，可以认为 f_k 有极限（有限或者无限），若可以证明解 $(u(t + f_k), v(t + f_k), Y(t + f_k), Z(t + f_k))$ 在 R 上一致收敛，则 $(u(t), v(t), Y(t), Z(t))$ 为概周期函数，而由文献[9]第 18 页中定理 2.8. 可以证明 $\mathrm{mod}(u(t), v(t), Y(t), Z(t)) \subset \mathrm{mod}(f_1, f_2, f_3, f_4)$.

现在来证明 $(u(t + f_k), v(t + f_k), Y(t + f_k), Z(t + f_k))$ 在 R 上一致收敛．

若 $\lim_{k\to+\infty} f_k = f$（有限数），注意到 $\mathrm{Rang}(u(t), v(t), Y(t), Z(t)) \subset S$，则

$$|u(t + f_k) - u(t + f)| = |\dot{u}(t + \theta(f_k - f))(f_k - f)| = |f_1(t + \theta(f_k - f))||f_k - f| \leqslant k|f_k - f|,$$

其中 k 为常数，$\theta \in [0, 1]$，所以，$u(t + f_k) \rightrightarrows u(t + f)$. 同理可证 $v(t + f_k) \rightrightarrows v(t + f)$，$Y(t + f_k) \rightrightarrows Y(t + f)$，$Z(t + f_k) \rightrightarrows Z(t + f)$.

若 $\lim_{k\to+\infty} f_k = +\infty$，由 $(u(t), v(t), Y(t), Z(t))$ 是系统(1)的解，且 $t \in R$ 有定义，令

$$V(t) = |\ln u(t + f_k) - \ln u(t + f_m)| + |\ln v(t + f_k) - \ln v(t + f_m)| + |\ln Y(t + f_k) - \ln Y(t + f_m)| + |\ln Z(t + f_k) - \ln Z(t + f_m)|, m \geqslant k \geqslant K, s \in R.$$

则由(8)式，对 $S \in R$ 成立．以 $\exp\{-\frac{Tf_k}{L}\} < \frac{Xl}{16ML}$ 代替 $\exp\{-T(U + f_k)/L\} < \frac{Xl}{16ML}$，利用已知各条件及(3)～(9)式，仍然可以利用微分不等式原理，得到

$$|u(t + f_k) - u(t + f_m)| + |Y(t + f_k) - Y(t + f_m)| + |Z(t + f_k) - Z(t + f_m)| + |v(t + f_k) - v(t + f_m)| \leqslant X, m \geqslant k \gg 1, t \in R.$$

这表明 $(u(t + f_k), v(t + f_k), Y(t + f_k), Z(t + f_k))$ 在 R 上一致收敛．

若 $\lim_{k\to+\infty} f_k = -\infty$，可类似证明．

如前所述，就可以证明了 $(u(t), v(t), Y(t), Z(t))$ 是概周期函数，且 $\mathrm{mod}(u(t), v(t), Y(t), Z(t)) \subset \mathrm{mod}(f_1, f_2, f_3, f_4)$. 由定理 5 已知条件，由定理 3 可以知道 $(u(t), v(t), Y(t), Z(t))$ 对位于 R_+^4 的其它解而言是全局吸引的．而由此，与定理 4 的证明类似，有可以得出正概周期解的唯一性．证毕．

定理 6 系统(1)满足条件(2)～(5)和(6)，概周期解 $(u_1(t), u_2(t), u_3(t), u_4(t))$ 对于 $t \geqslant 0$ 关于 S 是在壳扰动下是稳定的．

证明 对于系统(1)的任意壳方程．

$$\begin{cases}
\dot{x}_1(t) = x_1(t)(B_1(t) - A_{11}(t)x_1(t) - A_{12}(t)x_2(t) - A_{13}(t)x_3(t)/(M_1(t)x_3(t) + G_1(t)x_1(t))) + F_1(t)(x_4(t) - x_1(t)), \\
\dot{x}_2(t) = x_2(t)(B_2(t) - A_{21}(t)x_1(t) - A_{22}(t)x_2(t) - A_{23}(t)x_3^2(t)/(M_2(t)x_3^2(t) + G_2(t)x_2^2(t))), \\
\dot{x}_3(t) = x_3(t)(-B_3(t) + A_{31}(t)x_1(t - f_1)/(M_1(t)x_3(t - f_1) + G_1(t)x_1(t - f_1)) + (A_{32}(t)x_2(t - f_2)x_3(t - f_2)/(M_2(t)x_3^2(t - f_2) + G_2(t)x_2^2(t - f_2)))), \\
\dot{x}_4(t) = x_4(t)(B_4(t) - A_{44}x_4(t)) + F_2(t)(x_1(t) - x_4(t)).
\end{cases}$$

其中,$B_i(t)\in H(b_i(t))$, $A_{ij}(t)\in H(a_{ij}(t))$, $M_i(t)\in H(m_i(t))$, $G_i(t)\in H(d_i(t))$, $F_i(t)\in H(D_i(t))$, (H 表示壳),显然,$B_i(t), A_{ij}(t), M_i(t), G_i(t), F_i(t)$ 分别与 $b_i(t), a_{ij}(t), m_i(t), d_i(t), D_i(t)$满足相同的条件,因而(4)式存在对于 $t\geqslant 0$位于 S中的解. 以$(x_1(t), x_2(t), x_3(t), x_4(t))$表示这样的解.

令

$$W(t)=\sum_{i=1,2,4}|\ln u_i(t+f)-\ln x_i(t)|+|u_3(t+f)-x_3(t)|,$$

所以,$l\sum_{i=1}^{4}|\ln u_i(t+f)-\ln x_i(t)|\leqslant W(t)\leqslant L\sum_{i=1}^{4}|\ln u_i(t+f)-\ln x_i(t)|$.

任意 X,取 W>0,使 $(\frac{L}{l}+\frac{4L}{THl})W<X\sum_{i=1}^{4}|\ln u_i(t+f)-\ln x_i(t)|<W$,

$\sup\{x_1|(b_1(t+f)-B_1(t))-(D_1(t+f)-F_1(t))+(D_2(t+f)-F_2(t))-(a_{11}(t+f)-A_{11}(t))x_1-(a_{12}(t+f)-A_{12}(t))x_2-a_{13}(t+f)x_3(t)/(m_1(t+f)x_3(t)+x_1(t)W(t+f))-A_{13}(t)x_3(t)/(M_1(t)\cdot x_3(t)+x_1(t)G_1(t))|+x_2|(b_2(t)-B_2(t))-(a_{21}(t+f)-A_{21}(t))x_1-(a_{22}(t+f)-A_{22}(t))x_2-(a_{23}(t+f)x_3^2/(m_2(t+f)x_3^2+d_1(t+f)x_2^2)-A_{23}(t+f)x_3^2(t)/(M_2(t+f)x_3^2+G_2(t+f)x_2^2))|+x_3|(b_3(t)-B_3(t))+(a_{31}(t+f)x_1(t+f_1)/(m_1(t+f)x_1(t+f)+d_1(t+f)x_1(t-f_2))-A_{31}(t)x_1(t+f_1)/(M_1(t+f_1)x_1(t+f_1)+G_1(t+f_1)x_1(t-f_1)))+(A_{32}(t+f)x_2(t+f_2)x_3(t+f_2)/(m_2(t+f)x_3^2(t-f_2)+d_2(t+f)x_2^2(t-f_2))-A_{32}(t)x_2(t+f_2)x_3(t+f_2)/(M_2(t)x_3^2(t-f_2)+G_2(t)x_2^2(t-f_2)))|+x_4|(b_4(t+f)-B_4(t))-(a_{44}(t+f)-A_{44}(t))x_4+(D_1(t+f)-F_1(t))-(D_2(t+f)-F_2(t))|;t\geqslant 0,(x_1,x_2,x_3,x_4)\in S\}<W,$

则对于 $t\geqslant 0$, (x_1,x_2,x_3,x_4)有

$$|(b_1(t+f)-B_1(t))-(D_1(t+f)-F_1(t))+(D_2(t+f)-F_2(t))-(a_{11}(t+f)-A_{11}(t))x_1-(a_{12}(t+f)-A_{12}(t))x_2-(a_{13}(t+f)x_3(t)/(m_1(t+f)x_3(t)+x_1(t)W(t+f))-A_{13}(t)x_3(t)/(M_1(t)x_3(t)+x_1(t)G_1(t)))|<\frac{W}{H},$$

$$|(b_2(t)-B_2(t))-(a_{21}(t+f)-A_{21}(t))x_1-(a_{22}(t+f)-A_{22}(t))x_2-a_{23}(t+f)x_3^2/(m_2(t+f)x_3^2+d_2(t+f)x_2^2)-A_{23}(t+f)x_3^2/(M_2(t+f)x_3^2+G_2(t+f)x_2^2))|<\frac{W}{H},$$

$$|(b_3(t)-B_3(t))+(a_{31}(t+f)x_1(t+f_1)/(m_1(t+f)x_1(t+f)+d_1(t+f)x_1(t-f_2))-A_{31}(t)x_1(t+f_1)/(M_1(t+f_1)x_1(t+f_1)+G_1(t+f_1)x_1(t-f_1)))+(a_{32}(t+f)x_2(t+f_2)x_3(t+f_2)/(m_2(t+f)x_3^2(t-f_2)+d_2(t+f)x_2^2(t-f_2))-A_{32}(t)x_2(t+f_2)x_3(t+f_2)/(M_2(t)x_3^2(t-f_2)+G_2(t)x_2^2(t-f_2)))|<\frac{W}{H},$$

$$|(b_4(t+f)-B_4(t))-(a_{44}(t+f)-A_{44}(t))x_4+(D_1(t+f)-F_1(t))-(D_2(t+f)-F_2(t))|<\frac{W}{H},$$

类似于定理 5,可以得到,

$$D^+W(t)\leqslant-\frac{T}{L}W(t)+\frac{4W}{H}.$$

在$[0,t]$上积分得到 $W(t)\leqslant LW+\frac{4LW}{TH}$. 所以,

$$\sum_{i=1}^{4}|u_i(t+f)-x_i(t)|\leqslant\frac{W(t)}{l}\leqslant(\frac{L}{l}+\frac{4L}{THL})W<X$$

即$(u_1(t),u_2(t),u_3(t),u_4(t))$在壳扰动下是稳定的. 定理证毕.

参考文献

1 Song Xinyu, Chen Lansun. Harmless delays and global attractivity for nonautonomous predator-prey system with diffusion. Computers and Mathematics with Application, 2000, 39: 33~42.

2 Hey Z. Stability and delays in a predator-prey system. J Math Anal Appl, 1996, 198: 355~370.

3 Freedman H I, So J, Waltman P. Coexistence in a model of competition in the chemostat incorporating discrete time delays. SIAM J Appl Math, 1989, 49: 859~870.

4 Cosner C DeAnglis D L. Effects of spatial grouping on the functional response of predators. Theor popul Biol, 1999, 56: 65~75.

5 Gutierrez A P. The physiological basis of ratio-dependent predator-prey theory: a methbolic pool model of Nicholson's blowflies as an example. Ecology, 1992, 73: 1552~1563.

6 滕志东,陈兰荪.高维时滞周期的 Kolmogorov型系统的正周期解.应用数学学报, 1999, 22(3): 446~456.

7 Barbalat I. System equations differentielle oscillations. Rev Roumaine Math Pures Appl, 1959, 4: 261~270.

8 Gopalsamy K. Global asymptatic stability in an almost periodic Lotka-Volterra system. J Austral Math Soc Ser, 1986, 27: 346~360.

9 Yoshizawa T. Stability theory and the existence of Periodic Solutions and Almost Periodic Solutions. Berlin: Springer-Verlag. 1975. 205~206.

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