DOI: 10.13656/j.cnki.gxkx.2002.04.008

广西科学 Guangxi Sciences 2002, 9 (4): 265~ 267

具有食饵补充且捕食者具有阶段结构的捕食系统的持久性与周期解

Persistence and Periodic Solution of a Prey-predator System with Stage-structure for Predator and Prey Supplement

姚志健　　罗桂烈

Yao Zhijian　Luo Guilie

(广西师范大学数学与计算机科学学院　桂林市育才路 3号　541004)

(Dept. of Math. and Comp Sci., Guangxi Normal Univ., 3 Yucailu, Guilin, Guangxi, 541004, China)

摘要　研究具有食饵补充且捕食者具有阶段结构的捕食系统,得到了系统持续生存和周期解存在及全局吸引的充分条件.

关键词　捕食系统　具有食饵补充且捕食者具有阶段结构的捕食系统　持续生存　周期解

中图法分类号　O175.12

Abstract　A predator-prey system with stage-structure for predator and prey supplement is studied. The sufficient conditions are obtained for guaranteeing the permanence of the system and the existence of a positive periodic solution which is globally attractive.

Key words　prey-predator system, prey-predator system with stage-structure for predator and prey supplement, permanence, periodic solution

捕食系统是自然界中重要的生态系统,具有食饵补充的捕食系统也是常见的系统,如在农村地区,由于农药,化肥对某一地区生态环境的影响,使得这一地区的某些昆虫迁向其它地区,从而成为其它地区某些鸟类的食饵,即对其补充了食饵.本文主要研究具有食饵补充且捕食者具有阶段结构的捕食模型的一些性质.假定成年捕食者有捕食能力,幼年捕食者没有捕食能力.

研究下列模型:

$$\begin{cases} x'(t) = \theta(t)x(t) - U(t)x^2(t) - a(t)x(t)y_2(t) + h(t), \\ y'_1(t) = b(t)y_2(t) + k(t)a(t)x(t)y_2(t) - D(t)y_1(t) - r_1(t)y_1(t) - r_2(t)y_1^2(t), \\ y'_2(t) = D(t)y_1(t) - r_3(t)y_2(t) - r_4(t)y_2^2(t), \end{cases} \tag{1}$$

其中 $x(0) > 0, y_i(0) > 0, (i = 1, 2)$. $x(t)$ 表示食饵种群 x 在时刻 t 的规模, $y_1(t), y_2(t)$ 分别表示幼年,成年捕食者在时刻 t 的规模, $\theta(t)$ 表示种群 x 的内禀增长率, $D(t)$ 表示幼年捕食者转化为成年的转化率, y_1, y_2 的死亡率是 logistic型,即分别为 $r_1(t)y_1(t) + r_2(t)y_1^2(t)$ 和 $r_3(t)y_2(t) + r_4(t)y_2^2(t)$. $k(t)a(t)x(t)\cdot y_2(t)$ 表示在时刻 t 由于成年捕食者捕食食饵而增加的生育能力, $h(t)$ 表示食饵的补充率.对于 $[0, +\infty)$ 上的连续有界的正函数 $f(t)$,记 $\underline{f} = \inf f(t)$, $\overline{f} = \sup f(t)$.设 $\theta(t), U(t), a(t), b(t), h(t), D(t), r_i(t), (i = 1, 2, 3, 4)$ 都是连续的严格正的有界函数,且 $\min\{\underline{\theta}, \underline{U}, \underline{a}, \underline{b}, \underline{h}, \underline{k}, \underline{D}, \underline{r_i}\} > 0$, $\max\{\overline{\theta}, \overline{U}, \overline{a}, \overline{b}, \overline{h}, \overline{k}, \overline{D}, \overline{r_i}\} < +\infty$, $i = 1, 2, 3, 4$.

记 $R_+^3 = \{(x, y_1, y_2) | x > 0, y_i > 0, i = 1, 2\}$.

1　持续生存

定义 1　若存在紧集 $E \subset R_+^3$ 使得系统(1)的任何解最终停留在 E 内,则称系统(1)是持续生存的.(其中紧集 E 距坐标超平面有一严格正距离.)

引理 1　R_+^3 是系统(1)的正不变集.

2002-05-08收稿.

证明

$x'|_{x(t)=0} = \lambda(t) > 0,$

$y'_1|_{y_1(t)=0} = b(t)y_2(t) + k(t)a(t)x(t)y_2(t) > 0,$

$x(t) > 0, y_2(t) > 0,$

$y'_2|_{y_2(t)=0} = D(t)y_1(t) > 0, y_1(t) > 0.$

所以, R_+^3 是系统(1)的正不变集.

引理 **2** 系统(1)的解关于 R_+^3 是最终有界的.

证明 作函数 $P(t) = hx(t) + h_1y_1(t) + h_2y_2(t)$,其中 h, h_1, h_2 为待定的正常数. 函数 $P(t)$ 沿系统(1)的导数为:

$P'(t) = hx'(t) + h_1y'_1(t) + h_2y'_2(t) = h\theta(t)x(t) - h\mu(t)x^2(t) - ha(t)x(t)y_2(t) + h\lambda(t) + h_1b(t)y_2(t) + h_1k(t)a(t)x(t)y_2(t) - h_1D(t)y_1(t) - h_1r_1(t)y_1(t) - h_1r_2(t)y_1^2(t) + h_2D(t)y_1(t) - h_2r_3(t)y_2(t) - h_2r_4(t)y_2^2(t),$

$P' + cP = [h\theta(t) + ch - h\mu(t)x(t)]x(t) + [h_2D(t) + ch_1 - h_1D(t) - h_1r_1(t) - h_1r_2(t)y_1(t)]\cdot y_1(t) + [ch_2 - h_2r_3(t) + h_1b(t) - h_2r_4(t)y_2]y_2(t) + [h_1k(t)a(t) - ha(t)]x(t)y_2(t) + h\lambda(t).$

设 $k_1(t) = h\theta(t) + ch, k_2(t) = h_2D(t) - h_1D(t) - h_1r_1(t) + ch_1, K_3(t) = ch_2 - h_2r_3(t) + h_1b(t)$, 则 $P' + cP = [k_1(t) - h\mu(t)x(t)]x(t) + [k_2(t) - h_1r_2(t)y_1(t)]y_1(t) + [k_3(t) - h_2r_4(t)y_2(t)]y_2(t) + [h_1k(t)a(t) - ha(t)]x(t)y_2(t) + h\lambda(t).$

为使 $h_1k(t)a(t) - ha(t) \leqslant 0$,可选取 $h_1 = h_2 = 1, h = \overline{k}$.

可选取某正整数 c,使得 $k_i(t) > 0, (i = 1, 2, 3)$.

从而 $P' + cP \leqslant x(t)[\overline{k_1} - \overline{k}\underline{\mu}x(t)] + y_1(t)[\overline{k_2} - \underline{r_2}y_1(t)] + y_2(t)[\overline{k_3} - \underline{r_4}y_2(t)] + \overline{k}\,\overline{\lambda}.$

由于 $x(t)[\overline{k_1} - \overline{k}\underline{\mu}x(t)], y_1(t)[\overline{k_2} - \underline{r_2}y_1(t)], y_2(t)[\overline{k_3} - \underline{r_4}y_2(t)]$ 的二次项的系数均为负,所以可找到某一个正常数 $A > 0$,使得 $P' + cP \leqslant A$. 从而 $P = P(t) \leqslant \frac{A}{c} + \frac{B}{e^{ct}}$, B 是某个适当的常数. $\lim\limits_{t\to+\infty} P(t) \leqslant \frac{A}{c}$,可取一个常数 $L > \frac{A}{c}$,使得当 t 大于某个 T 时,有 $P(t) \leqslant L$.

令 $E_1 = \{(x, y_1, y_2) \in R_+^3 \mid P(t) \leqslant L\}$, 则 E_1 是系统(1)的最终有界区域.

定理 **1** 若系统(1)的系数满足下列条件:

$(H_1)\ \underline{D} - \overline{r_3} > 0,$

$(H_2)\ \underline{b} + \underline{k}\,\underline{a}l_1 - \overline{D} - \overline{r_1} > 0,$

其中

$0 < l_1 < x^*,$

$$x^* = \frac{\underline{\theta} - \overline{a}L + \sqrt{(\underline{\theta} - \overline{a}L)^2 + 4\overline{\mu}\underline{\lambda}}}{2\overline{\mu}} > 0,$$

令 $l_2 = \min\{\frac{\underline{D} - \overline{r_3}}{\overline{r_4}}, \frac{\underline{b} + \underline{k}\,\underline{a}l_1 - \overline{D} - \overline{r_1}}{\overline{r_2}}\}$,

则系统(1)是持续生存的.

证明 定义 $v_1(t) = x(t)$. 沿系统(1)的正解计算 $v_1(t)$ 的上右导数:

$v'_1(t) = x'(t) = \theta(t)x(t) - \mu(t)x^2(t) - a(t)x(t)\cdot y_2(t) + \lambda(t) \geqslant \underline{\theta}x(t) - \overline{\mu}x^2(t) - \overline{a}Lx(t) + \underline{\lambda} = -\overline{\mu}x^2(t) + (\underline{\theta} - \overline{a}L)x(t) + \underline{\lambda},$

(i) 当 $x(t) \geqslant l_1$ 时, $x'(t)|_{x(t)=l_1} \geqslant -\overline{\mu}l_1^2 + (\underline{\theta} - \overline{a}L)l_1 + \underline{\lambda} > 0$. 所以, $x(t) \geqslant l_1, (t \geqslant 0)$.

(ii) 当 $0 < x(t) \leqslant l_1$ 时, $x'(t) \geqslant q > 0$. $q = \min\{-\overline{\mu}l_1^2 + (\underline{\theta} - \overline{a}L)l_1 + \underline{\lambda}, \underline{\lambda}\}$.

所以存在 $T_1 > 0$,当 $t \geqslant T_1$ 时, $x(t) \geqslant l_1$.

定义 $v_2(t) = \min\{y_1(t), y_2(t)\}$. 沿系统(1)的正解计算 $v_2(t)$ 的上右导数:

(i) 若 $y_1(t) > y_2(t)$ 或 $y_1(t) = y_2(t)$ 且 $y'_1(t) \geqslant y'_2(t)$,则

$D^+v_2(t) = y'_2 = D(t)y_1(t) - r_3(t)y_2(t) - r_4(t)y_2^2(t) \geqslant \underline{D}y_2(t) - \overline{r_3}y_2(t) - \overline{r_4}y_2^2(t) = y_2(t)[\underline{D} - \overline{r_3} - \overline{r_4}y_2(t)],$

(ii) 若 $y_1(t) < y_2(t)$,或 $y_1(t) = y_2(t)$ 且 $y'_1(t) \leqslant y'_2(t)$,则

$D^+v_2(t) = y'_1 = b(t)y_2(t) + k(t)a(t)x(t)y_2(t) - D(t)y_1(t) - r_1(t)y_1(t) - r_2(t)y_1^2(t) \geqslant \underline{b}y_1(t) + \underline{k}\,\underline{a}l_1y_1(t) - \overline{D}y_1(t) - \overline{r_1}y_1(t) - \overline{r_2}y_1^2(t) = y_1(t)[\underline{b} + \underline{k}\,\underline{a}l_1 - \overline{D} - \overline{r_1} - \overline{r_2}y_1(t)],$

当 $v_2(0) \geqslant l_2$ 时,

$v'_2(t)|_{v_2(t)=l_2} \geqslant l_2(\underline{D} - \overline{r_3} - \overline{r_4}l_2) \geqslant 0,$

或 $v'_2(t)|_{v_2(t)=l_2} \geqslant l_2(\underline{k}\,\underline{a}l_1 + \underline{b} - \overline{D} - \overline{r_1} - \overline{r_2}l_2) \geqslant 0,$

所以 $v_2(t) \geqslant l_2. (t \geqslant 0)$.

当 $v_2(o) < l_2$ 时,则存在 $T_2 > 0$,当 $t \geqslant T_2$ 时, $v_2(t) \geqslant l_2$. (用反证法)

若不成立,则 $v_2(t) < l_2$,从而存在某个 $\varepsilon > 0$,使 $v_2(t) - l_2 \leqslant -\varepsilon$.

$D^+v_2(t) \geqslant y_2(t)[\underline{D} - \overline{r_3} - \overline{r_4}(l_2 - \varepsilon)] = y_2(t)(\underline{D} - \overline{r_3} - \overline{r_4}l_2 + \overline{r_4}\varepsilon) \geqslant \overline{r_4}\varepsilon y_2(t) = \overline{r_4}\varepsilon v_2(t)$

或者 $D^+v_2(t) \geqslant y_1(t)[\underline{b} + \underline{k}\,\underline{a}l_1 - \overline{D} - \overline{r_1} - \overline{r_2}(l_2 - \varepsilon)] = y_1(\underline{b} + \underline{k}\,\underline{a}l_1 - \overline{D} - \overline{r_1} - \overline{r_2}l_2 + \overline{r_2}\varepsilon) \geqslant \overline{r_2}\varepsilon y_1(t) = \overline{r_2}\varepsilon v_2(t),$

取 $r = \min\{\overline{r_2}, \overline{r_4}\}$,则 $D^+v_2(t) \geqslant r\varepsilon v_2(t), v_2(t) \geqslant v_2(o)e^{r\varepsilon} \to +\infty. (t \to +\infty)$. 矛盾.

故 $v_2(t) \geqslant l_2. (t \geqslant T_2)$.

令 $E = \{(x, y_1, y_2) \in R_+^3 \mid P(t) \leqslant L, x \geqslant l_1, y_i \geqslant l_2, i = 1, 2\}$. 则紧集 E 是系统(1)的最终有界区域. 所以系统(1)是持续生存的.

2 周期解

定理 2 设系统(1)的所有系数是连续的正的 k 周期函数,且满足定理 1的条件.则系统(1)存在正的 k周期解.

证明 设 $Z(t)=(x(t),y_1(t),y_2(t))$ 是系统(1)的初值为 $Z_0=(x^0,y_1^0,y_2^0)$ 的正解.定义 Poincare 映射 $Q: R_+^3\to R_+^3$ $Q(Z_0)=Z(k,Z_0)=(x(k;x^0,y_1^0,y_2^0),y_1(k;x^0,y_1^0,y_2^0),y_2(k;x^0,y_1^0,y_2^0))$ 因为紧集 E是系统(1)的最终有界区域,由 Brouwer不动点定理知:映射 Q在 E中存在不动点.设此不动点为 $(\bar x,\bar y_1,\bar y_2)$.则 $\bar x=x(k;\bar x,\bar y_1,\bar y_2)$, $\bar y_1=y_1(k;\bar x,\bar y_1,\bar y_2)$, $\bar y_2=y_2(k;\bar x,\bar y_1,\bar y_2)$.即 $(x(t;\bar x,\bar y_1,\bar y_2),y_1(t;\bar x,\bar y_1,\bar y_2),y_2(t;\bar x,\bar y_1,\bar y_2))$ 是系统(1)的正的 k周期解.定理得证.

定理 3 设系统(1)的所有系数是连续的正的 k 周期函数,满足定理 1的条件,且满足下列不等式:

(H3) $\bar\theta+(\bar k+1)\bar aL<2\underline{U}l_1$,

(H4) $\bar aL<(\underline{r_3}+2\underline{r_4}l_2-\bar b-\bar aL)\bar k$,

则系统(1)存在唯一的正 k周期解,且此周期解是全局吸引的.

证明 由定理 2知系统(1)存在正 k周期解,设此周期解为 $Z^*(t)=(x^*(t),y_1^*(t),y_2^*(t))$.再设 $Z(t)=(x(t),y_1(t),y_2(t))$是系统(1)的任一正解.则存在 $T>0$,使当 $t\geqslant T$时,有 $Z^*(t)\in E$, $Z(t)\in E$.

令 $V(t)=|x(t)-x^*(t)|+|y_1(t)-y_1^*(t)|+|y_2(t)-y_2^*(t)|$,

沿系统(1)的解计算 $V(t)$的上右导数:

$$D^+V(t)=\mathrm{sgn}(x-x^*)[x'-(x^*)']+\mathrm{sgn}(y_1-y_1^*)[y_1'-(y_1^*)']+\mathrm{sgn}(y_2-y_2^*)[y_2'-(y_2^*)']=\mathrm{sgn}(x-x^*)[\theta(t)(x-x^*)-U(t)(x^2-x^{*2})-a(t)(xy_2-x^*y_2^*)]+\mathrm{sgn}(y_1-y_1^*)[b(t)(y_2-y_2^*)+k(t)a(t)(xy_2-x^*y_2^*)-D(t)(y_1-y_1^*)-r_1(t)(y_1-y_1^*)-r_2(t)(y_1^2-y_1^{*2})]+\mathrm{sgn}(y_2-y_2^*)[D(t)(y_1-y_1^*)-r_3(t)(y_2-y_2^*)-r_4(t)(y_2^2-y_2^{*2})]\leqslant[\theta(t)-2U(t)l_1+(k(t)+1)a(t)L]|x-x^*|+(-r_1(t)-2l_2r_2(t))|y_1-y_1^*|+[b(t)-r_3(t)-2r_4(t)l_2+(k(t)+1)a(t)\frac{L}{k}]|y_2-y_2^*|\leqslant[\bar\theta-2\underline{U}l_1+(\bar k+1)\bar aL]|x-x^*|+[-r_1(t)-2l_2r_2(t)]|y_1-y_1^*|+[\bar b-\underline{r_3}-2\underline{r_4}l_2+(\bar k+1)\bar a\frac{L}{k}]|y_2-y_2^*|,$$

由(H3),(H4)知 $\bar\theta-2\underline{U}l_1+(\bar k+1)\bar aL<0$, $\bar b-\underline{r_3}-2\underline{r_4}l_2+(\bar k+1)\bar a\frac{L}{k}<0$,

从而存在某个正数 d使得 $D^+V(t)\leqslant -d(|x-x^*|+|y_1-y_1^*|+|y_2-y_2^*|)$,

对上式积分得 $V(t)-V(T)\leqslant -d\int_T^t[|x(s)-x^*(s)|+|y_1(s)-y_1^*(s)|+|y_2(s)-y_2^*(s)|]ds$,

$$V(t)+d\int_T^t[|x(s)-x^*(s)|+|y_1(s)-y_1^*(s)|+|y_2(s)-y_2^*(s)|]ds\leqslant V(T),$$

$$\int_T^t[|x(s)-x^*(s)|+|y_1(s)-y_1^*(s)|+|y_2(s)-y_2^*(s)|]ds\leqslant\frac{V(T)}{d},$$

$$\int_T^{+\infty}[|x(s)-x^*(s)|+|y_1(s)-y_1^*(s)|+|y_2(s)-y_2^*(s)|]ds\leqslant\frac{V(T)}{d}<+\infty.$$

易知 $V(t)=|x(t)-x^*(t)|+|y_1(t)-y_1^*(t)|+|y_2(t)-y_2^*(t)|$ 在 $[0,+\infty)$上一致连续.

所以 $\lim\limits_{t\to+\infty}V(t)=0$.

即 $|x(t)-x^*(t)|+|y_1(t)-y_1^*(t)|+|y_2(t)-y_2^*(t)|\to 0.(t\to+\infty)$

故周期解 $Z^*(t)$是全局吸引的.

下证 k周期解是唯一的.

设 $\bar Z(t)=(\bar x(t),\bar y_1(t),\bar y_2(t))$是系统(1)的另一周期解.由上述证明过程可知: $|\bar x(t)-x^*(t)|+|\bar y_1(t)-y_1^*(t)|+|\bar y_2(t)-y_2^*(t)|\to 0.(t\to+\infty)$.

从而 $\bar x(t)\equiv x^*(t)$, $\bar y_1(t)\equiv y_1^*(t)$, $\bar y_2(t)\equiv y_2^*(t)$.定理 3得证.

参考文献

1 Wang W, Chen L. A predator-prey system with stage-structure for predator. Computers Math Applic, 1997, 33(6): 83~91.

2 He X Z. Stability and delays in a predator-prey system. J Math Anal Appl, 1996, 198(3): 335~370.

3 Yoshizawa T. Stability theory and the existence of periodic solutions and almost periodic solutions, New York: Springer-Verlag, Heidelbery. Berlin, 1975.

4 Barbalat I. Systems d'equations differentielle d'oscillations. Rev Roumaine Math Pures Appl, 1959, 4 261~270.

(责任编辑:黎贞崇)