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具有连续时滞和功能性反应的非自治竞争系统的持续性和全局吸引性*

Persistence and Global Attractivity for Nonautonomous Competition System with Continuous Time Delay and Functional Responce

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摘要　讨论系数满足一定的条件时,具有连续时滞和II类功能性反应的非自治扩散竞争系统的持续生存性,并给出系统周期解全局吸引的充分条件.

关键词　非自治竞争系统　持续性　全局吸引性

中图法分类号　O175

Abstract　Persistence of a nonautonomous diffusion competition system with continuous time delay and II type functional response is studied under some conditions. Furthermore, sufficient conditions are established for global attractivity of a periodic solution of the system.

Key words　nonautonomous competition system, persistence, global attractivity

众所周知,在种群动力学的研究中,种群的持续生存和灭绝是人们非常关注的课题.近年来,对于扩散对种群的持续生存的影响已有不少的结果[1~4],人们通过扩散可以挽救绝灭的种群,从而使其保持持续生存.但是以往的文献中只有少数工作涉及到时滞问题,而且涉及到的时滞也是局限在离散型时滞.本文在上述工作的基础上,讨论具有连续时滞和功能性反应的非自治竞争模型:

$$\begin{cases}\dot{x}_0 = x_0\Big[a_0(t) - b_0(t)x_0 - c_0(t)\int_{-\mathrm{f}}^{0}k_0(s)\cdot x_0(t+s)\,\mathrm{d}s - \dfrac{\mathrm{U}_0(t)x_1}{1+\mathrm{T}(t)x_1}\Big],\\ \dot{x}_1 = x_1\Big[a_1(t) - b_1(t)x_1 - c_1(t)\int_{-\mathrm{f}}^{0}k_1(s)x_1(t+s)\,\mathrm{d}s - \dfrac{\mathrm{U}_1(t)x_0}{1+\mathrm{T}(t)x_1}\Big] + \sum_{i=2}^{n}D_{i1}(t)(x_i - x_1),\\ \dot{x}_j = x_j\Big[a_j(t) - b_j(t)x_j - c_j(t)\int_{-\mathrm{f}}^{0}k_j(s)x_j(t+s)\,\mathrm{d}s\Big] + \sum_{i=1}^{n}D_{ij}(t)(x_i - x_j),\ (j = 2,3,\cdots,n)\end{cases} \tag{1}$$

其中,$a_i(t)$,$b_i(t)$,$c_i(t)$,$\mathrm{U}_i(t)$,$D_{ij}(t)$ $(i = 0,1,\cdots,n, j = 1,2,\cdots,n)$,$\mathrm{T}(t)$是连续的严格正的函数. x_0和x_1分别表示2个竞争种群X_0和X在斑块1中的密度,x_j $(j = 2,3,\cdots,n)$是种群X在斑块j中的密度. $D_{ij}(t)$ $(i,j = 1,2,\cdots,n)$表示种群X在斑块之间的扩散系数. $k_i(s)$是分段连续函数,并且$\int_{-\mathrm{f}}^{0}k_i(s)\,\mathrm{d}s = 1$, $(i = 0,1,2,\cdots,n)$.

记$f^L = \inf_{t\geq 0}f(t)$, $f^M = \sup_{t\geq 0}f(t)$. 本文假定方程(1)的系数满足下列条件:

$$\min\{a_i^L, b_i^L, c_i^L, \mathrm{T}^L, \mathrm{U}_i^L, D_{ij}^L\} > 0,$$
$$\max\{a_i^M, b_i^M, c_i^M, \mathrm{T}^M, \mathrm{U}_i^M, D_{ij}^M\} < +\infty, (i,j = 0,1,\cdots,n). \tag{2}$$

本文中,采用如下记号和概念.

记$x = (x_0, x_1, \cdots, x_n) \in R_+^{n+1} = \{x \in R^{n+1}: x_i \geq 0, i = 0,1,\cdots,n\}$. 如果$x \in \mathrm{Int}\,R_+^{n+1}$,则记$x > 0$. 基于生态学意义,本文只在$\mathrm{Int}\,R_+^{n+1}$上对系统(1)进行研究.

$C^+ = C([-\mathrm{f},0], R_+^{n+1})$表示定义在$[-\mathrm{f},0]$上具有范数$\|\mathrm{h}\| = \sup_{s\in[-\mathrm{f},0]}|\mathrm{h}(s)|$ $(\mathrm{h}\in C^+)$的非负连续

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函数构成的 Banach空间 .因此 ,如果选 C^+ 为系统 (1) 的初始函数空间 ,易知对 $\forall\ h = (h_0, h, \cdots, h_n) \in C^+$ 且 $h(0) > 0$,则系统 (1) 在 $[-f, +\infty)$ 上存在唯一解 $x(t, h)$,且对于 $t \in [0, +\infty)$,有 $x(t, h) > 0$,称此解为系统 (1) 的正解 . 因此 ,在下面的研究中 ,总是假定

$$h \in C^+,\ h(0) > 0. \tag{3}$$

定义 1 称系统 (1) 是一致持久的 ,如果存在正常数 m 和 M 使得对任意 $h \in C^+$ 系统 (1) 的正解 $x(t, 0, h) = (x_0(t, 0, h), x_1(t, 0, h), \cdots, x_n(t, 0, h))$ (初始时刻 $t_0 = 0$) 都有

$$m \leqslant \liminf_{t\to\infty} x_i(t, 0, h) \leqslant \limsup_{t\to\infty} x_i(t, 0, h) \leqslant M,$$

$(i = 0, 1, 2, \cdots, n)$.

1 一致持久性

引理 1 R_+^{n+1} 是系统 (1) 的正向不变集 .

证明 对于 $\forall\ t \in [0, +\infty)$,由系统 (1) 有

$$x_0(t) = x_0(0)\exp\{\int_0^t [a_0(u) - b_0(u)x_0(u) - c_0(u)\int_{-f}^0 k_0(s)x_0(u+s)\mathrm{d}s - \frac{U_0(u)x_1(u)}{1+T(u)x_1(u)}]\mathrm{d}u\} \geqslant 0,$$

$$\dot{x}_i(t)|_{x_i=0} = \sum_{j=1}^n D_{ji}(t)x_j(t) \geqslant 0, i \neq j; i, j = 1, 2, \cdots, n.$$ 当 $x_j(t) > 0$ 时 ,

定理 1 假设系统 (1) 满足

$$\min_{j=2,3,\cdots,n}\{\frac{a_j^L}{c_j^M}, \frac{a_0^L}{U_0^M}\} > \max_{i=1,2,\cdots,n}\{\frac{a_i^M}{b_i^L}\}, \min\{\frac{a_1^L}{U_1^M + c_1^M}, \frac{a_0^L}{U_0^M + c_0^M}\} > \max_{i=0,1,\cdots,n}\{\frac{a_i^M}{b_i^L}\}, \tag{4}$$

则系统 (1) 是一致持久的 .

证明 假设 $x(t) = (x_0(t), x_1(t), \cdots, x_n(t))$ 表示满足初始条件 (3) 的任意正解 . 要证明系统 (1) 的一致持久性只需要证明存在集合

$$D = \{(x_0, x_1, \cdots, x_n) | m \leqslant \liminf_{t\to\infty} x_i(t) \leqslant \limsup_{t\to\infty} x_i(t) \leqslant M, i = 0, 1, \cdots, n\},$$

其中 $m = \min(m_1, m_2)$, $M = \max(M_1, M_2)$,使得存在 $T > 0$,当 $t > T$ 时 ,$x(t) \in D$. 这里 $m_i, M_i (i = 1, 2)$ 满足

$$0 < m_i < m_i^*, M_i > M_i^*, (i = 1, 2). \tag{5}$$

$$M_1^* = \max_{i=1,2,\cdots,n}\{\frac{a_i^M}{b_i^L}\}, M_2^* = \frac{a_0^M}{b_0^L},$$

$$m_1^* = \min\{\frac{a_1^L - (U_1^M + c_1^L)M}{b_1^M}, \frac{a_j^L - c_j^M M_1}{b_j^M}\}, (j = 2, 3, \cdots, n) \tag{6}$$

$$m_2^* = \frac{a_0^L - (U_0^M + c_0^M)M}{b_0^M}.$$

在条件 (4) 成立时 ,由于能选择 M_i 充分靠近 M_i^*,所以可得 $m_i^* > 0 (i = 1, 2)$.

首先 ,从系统 (1) 能得到

$$\dot{x}_0|_{x_0=M_2} \leqslant x_0(a_0^M - b_0^L x_0)|_{x_0=M_2} < 0,$$

$$\dot{x}_1\Big|_{\substack{x_1=M_1\\x_1\geqslant x_i}} \leqslant x_1(a_1^M - b_1^L x_1) + \sum_{i=2}^n D_{i1}(t)(x_i - x_1)\Big|_{\substack{x_1=M_1\\x_1\geqslant x_i}} < 0,$$

$$\dot{x}_j\Big|_{\substack{x_j=M_1\\x_j\geqslant x_i}} \leqslant x_j(a_j^M - b_j^L x_j) + \sum_{i=1}^n D_{ij}(t)(x_i - x_j)\Big|_{\substack{x_j=M_1\\x_j\geqslant x_i}} < 0, (j = 2, 3, \cdots, n),$$

令 $h(t) = \max\{x_1(t), x_2(t), \cdots, x_n(t)\}$,得到

$h(0) \leqslant M_1 \Rightarrow h(t) \leqslant M_1$, $(t \geqslant 0)$, $x_0(0) \leqslant M_2 \Rightarrow x_0(t) \leqslant M_2$, $(t \geqslant 0)$,

显然 ,可选取 $\lambda = \min_{i=1,2,\cdots,n} M_1(b_i^L M_1 - a_i^M) > 0$,使得

(i) 如果 $x_1 \geqslant M_1, x_1 \geqslant x_j (j = 2, 3, \cdots, n)$,有

$$\dot{x}_i(t)|_{x_i=0} = \sum_{j=1}^n D_{ji}(t)x_j(t) \geqslant 0, \tag{7}$$

(ii) 如果 $x_j \geqslant M_1, x_j \geqslant x_i (j = 2, \cdots, n, i = 1, 2, \cdots, n, i \neq j)$,有

$$\dot{x}_j \leqslant x_j(a_j^M - b_j^L x_j) \leqslant M_1(a_j^M - b_j^L M_1) \leqslant -\lambda < 0, \tag{8}$$

这意味着 $h(t)$ 以速度 λ 单调递减 ,因此如果 $h(0) > M_1$,则存在 $T_1 > 0$,当 $t \geqslant T_1$ 时 ,有 $h(t) \leqslant M_1$. 这说明当 $t \geqslant T_1$ 时 ,有

$$x_i(t) \leqslant M_1, (i = 1, 2, \cdots, n), \tag{9}$$

如果 $x_0 > M_2$,选取 $U = M_2(b_0^L M_2 - a_0^M)$ 有

$$\dot{x}_0 \leqslant x_0(t)(a_0^M - b_0^L x_0(t)) \leqslant M_2(a_0^M - b_0^L M_2) = -U < 0, \tag{10}$$

这意味着 $x_0(t)$ 以速度 U 严格单调递减 ,所以存在 $T_2 > T_1$,当 $t \geqslant T_2$ 时 ,$x_0(t) \leqslant M_2$.

其次 ,令 $T_3 > T_2$,当 $t \geqslant T_3$ 时 ,由系统 (1) 得

$$\dot{x}_0|_{x_0=m_2} \geqslant x_0(a_0^L - (c_0^M + U_0^M)M - b_0^M x_0)|_{x_0=m_2} > 0,$$

$$\dot{x}_1\Big|_{\substack{x_2=M_1\\x_1\leqslant x_i}} \geqslant x_1(a_1^L - (c_1^M + U_1^M)M - b_1^M x_1) + \sum_{i=2}^n D_{i1}(t)(x_i - x_1)\Big|_{\substack{x_2=M_1\\x_1\leqslant x_i}} > 0,$$

$$\dot{x}_j\Big|_{\substack{x_j=M_1\\x_j\leqslant x_i}} \geqslant x_j(a_j^L - b_j^M x_j - c_j^M M_1) + \sum_{i=1}^n D_{ij}(t)(x_i - x_j)\Big|_{\substack{x_j=M_1\\x_j\leqslant x_i}} > 0, (j = 2, 3, \cdots, n).$$

令 $f(t) = \min\{x_1(t), x_2(t), \cdots, x_n(t)\}$,用上述类似方法可以证明 ,如果 $0 < f(t) < m_1$,则存在 $W > 0$, $f(t)$ 以速度 W 单调递增 ,所以 ,如果 $0 < f(0) <$

m_1,则存在 $T_4 > T_3$,当 $t \geqslant T_4$ 时,有 $f(t) \geqslant m_1$,如果 $0 < x_0(0) < m_2$,则

$$\dot{x}_0(t) \geqslant x_0(0)[a_0^L - (c_0^M + U_0^M)M_1 - b_0^M M_2] > 0.$$

所以,如果 $x_0(0) \leqslant m_2$,则存在 $T > T_4$,当 $t \geqslant T$ 时,有

$$x_0(t) \geqslant m_2, x_i(t) \geqslant m_1, (i = 1, 2, \cdots, n).$$

最后,当 $t \geqslant T$ 时

$$x(t) = (x_0(t), x_1(t), \cdots, x_n(t)) \in D,$$

所以系统(1)是一致持久的.证毕.

2 周期解的存在性及全局吸引性

假设系统(1)除满足条件(4)外,所有系数都是以 T 为周期的周期函数.由定理 1和文献[5]的定理 2有

定理 2 如果定理 1的条件成立,则周期系统(1)至少存在一个严格正的 T 周期解.

定理 3 假设定理 1的条件成立,且还满足

$$\frac{U_1^M}{1+T_1^L m_1^*} + c_0^M < b_0^L,$$

$$\sum_{i=2}^n \frac{D_i^M}{m_1^*} + \frac{U_0^M + U_1^M T_1^M M_2}{(1+T_1^L m_1^*)^2} + c_1^M < b_1^L, \qquad (11)$$

$$\frac{D_{j1}^M}{m_1^*} + \sum_{i=2}^n \frac{D_{ij}^M}{m_1^*} + c_j^M < b_j^L, (j = 2, 3, \cdots, n),$$

则系统(1)有唯一的正周期解,且此周期解是全局吸引的.

证明 由定理 2知系统(1)存在正周期解,设此周期解为 $Z(t) = (x_0(t), x_1(t), \cdots, x_n(t))$. 再设 $Z^*(t) = (u_0(t), u_1(t), \cdots, u_n(t))$ 是系统(1)的任一正解,且 $u_i(0) > 0, (i = 0, 1, \cdots, n)$. 则存在 $T > 0$,使当 $t \geqslant T$ 时,有 $Z(t) \in D, Z^*(t) \in D$. 令

$$X_i(t) = \ln x_i(t), (i = 0, 1, \cdots, n),$$

$$U_i(t) = \ln u_i(t), (i = 0, 1, \cdots, n).$$

定义 Lyapunov 函数

$$V(t) = \sum_{i=0}^n \{|X_i(t) - U_i(t)| + c_i^M \int_{-f}^0 k_i(s)(\int_{t+s}^t |x_i(\theta) - u_i(\theta)| d\theta)ds\}.$$

沿系统(1)计算 $V(t)$ 的上右导数.

$$D^+ V(t) =$$

$$\frac{X_0(t) - U_0(t)}{|X_0(t) - U_0(t)|}[-b_0(t)(x_0(t) - u_0(t)) - c_0(t)\int_{-f}^0 k_0(s)(x_0(t+s) - u_0(t+s))ds - U_0(t)(\frac{x_1(t)}{1+T(t)x_1(t)} - \frac{u_1(t)}{1+T(t)u_1(t)})] +$$

$$\frac{X_1(t) - U_1(t)}{|X_1(t) - U_1(t)|}[-b_1(t)(x_1(t) - u_1(t)) - c_1(t)\int_{-f}^0 k_1(s)x_1(t+s)dx + \sum_{i=2}^n D_{ij}(t)(\frac{x_i(t)}{x_1(t)} - \frac{u_i(t)}{u_1(t)}) - U_1(t)(\frac{x_0(t)}{1+T(t)x_1(t)} - \frac{u_0(t)}{1+T(t)u_1(t)})] +$$

$$\sum_{j=2}^n \frac{X_j(t) - U_j(t)}{|X_j(t) - U_j(t)|}[-b_j(t)(x_j(t) - u_j(t)) - c_j(t)\int_{-f}^0 k_j(s)(x_j(t+s) - u_j(t+s))ds + \sum_{i=1}^n D_{ij}(t)(\frac{x_i(t)}{x_j(t)} - \frac{u_i(t)}{u_j(t)})] +$$

$$D^+\{\sum_{i=0}^n [c_i^M \int_{-f}^0 k_i(s)(\int_{t+s}^t |x_i(\theta) - u_i(\theta)| d\theta)ds]\} \leqslant$$

$$-(b_0^L - \frac{U_1^M}{1+T_1^L m_1^*} - c_0^M)|x_0(t) - u_0(t)| - (b_1^L - \sum_{i=2}^n \frac{D_{1i}^M}{m_1^*} - \frac{U_0^M + U_1^M T_1^M M_2}{(1+T_1^L m_1^*)^2} - c_1^M)|x_1(t) - u_1(t)| - \sum_{j=2}^n (b_j^L - c_j^M - \frac{D_{j1}^M}{m_1^*} - \sum_{i=2}^n \frac{D_{ij}^M}{m_1^*})|x_j(t) - u_j(t)|,$$

由条件(11),令

$$V = \min\{b_0^L - \frac{U_1^M}{1+T_1^L m_1^*} - c_0^M, b_1^L - \sum_{i=2}^n \frac{D_{1i}^M}{m_1^*} - \frac{U_0^M + U_1^M T_1^M M_2}{(1+T_1^L m_1^*)^2} - c_1^M, b_j^L - c_j^M - \frac{D_{j1}^M}{m_1^*} - \sum_{i=2}^n \frac{D_{ij}^M}{m_1^*}\} > 0,$$

$(j = 2, 3, \cdots, n)$,

从而可得

$$D^+ V(t) \leqslant -V(\sum_{i=0}^n |x_i(t) - u_i(t)|),$$

因此有

$$\lim_{t \to \infty} \sum_{i=0}^n |x_i(t) - u_i(t)| = 0,$$

从而

$$\lim_{t \to \infty} |x_i(t) - u_i(t)| = 0, (i = 0, 1, \cdots, n),$$

因此,根据文献[6]知道系统(1)的 T 周期解 $Z(t)$ 是全局吸引的.

下证 T 周期解是唯一的.设

$$\bar{Z}(t) = (\bar{x}_0(t), \bar{x}_1(t), \cdots, \bar{x}_n(t))$$

是系统(1)的另一周期解,由上述证明可知:

$$\sum_{i=0}^n |\bar{x}_i(t) - x_i(t)| \to 0, (t \to +\infty).$$

从而 $\bar{x}_i(t) \equiv x_i(t) (i = 0, 1, \cdots, n)$.

证毕.

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(上接第 178页 Continue from page 178)

证毕.

定理 2 设 G是一个图, g和 f是定义在 $V(G)$上的 2个整数值函数,且对任意的 $x, y \in V(G)$且 $x \neq y$,有 $f(x)(d_G(y)-2) \geq d_G(x)g(y)$,则图 G是分数 (g, f)-2-消去图.

证明 设 $e_i = u_i v_i, i = 1, 2$为图 G的任何两条边,令 $G' = G - e_1 - e_2$,显然对任意的 $x \in V(G)$,有 $d_G(x) \geq d_{G'}(x) \geq d_G(x) - 2$.要证明定理成立,只需证明 G'有一个分数 (g, f)-因子.由引理 3知,只要证明对任意的 $x, y \in V(G)$且 $x \neq y$,有

$$f(x) d_{G'}(y) \geq d_{G'}(x) g(y) \tag{3}$$

即可.

对任意的 $x, y \in V(G)$且 $x \neq y$,有:

$$f(x) d_{G'}(y) \geq f(x)(d_G(y) - 2) \geq d_G(x) g(y) \geq d_{G'}(x) g(y).$$

所以(3)式成立,由引理 3知, G'有一个分数 (g, f)-因子.于是,图 G是分数 (g, f)-2-消去图.

证毕.

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