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Almost Periodic Solutions of Delayed Differential Equations Appeared in Power System*

电力系统中时滞微分方程的概周期解

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Abstract Almost periodic oscillations appeared in power system are considered. A sufficient condition on the existence and uniqueness of almost periodic solutions of delayed differential equations appeared in power system is obtained.

Key words power system, delay, almost periodic solution, existence and uniqueness

摘要: 研究电力系统中时滞微分方程的概周期解 ,得到保证系统存在唯一概周期振动的一组充分条件 .

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In reference [1], Wang Lian has investigated the following nonlinear differential equation appeared in high-voltage electricity network

$$x'' + RF'(x)x' + \frac{1}{L}F(x) = K\cos kt, F(x) = Tx + Ux^3. \quad (1)$$

Since there are not only periodic oscillations but also almost periodic oscillations appeared in high-voltage electricity network, we have extended the equatiion (1) to an almost periodic system and discussed the existence and uniqueness of almost periodic solutions[2]. However, we still neglect the effect for the system of time delays. It is not to be ignored delaying effect for the system from the research result[3,4]. In this paper, we consider the following delayed differential equation

$$x'' + RF'(x)x' + \frac{1}{L}F(x(t - f)) = Ke(t),$$
$$F(x) = Tx + Ux^3, \quad (2)$$

where L, R, T, U, K and f are positive constants; $e(t)$ is a continuous almost periodic function. let $E(t) = \int_0^t e(s)ds$, and suppose that $E(t)$ is a continuously bounded function, therefore, $E(t)$ is also an almost periodic function. Let $M = \sup_{\in R}|E(t)|$, then equation (2) equals to the following system

$$\begin{cases} x'(t) = y(t) - RTx(t) - RUx^3(t) + \\ \quad KE(t) + \frac{1}{L}\int_{t-f}^{t}[Tx(s) + Ux^3(s)]ds, \\ y'(t) = -\frac{1}{L}[Tx(t) + Ux^3(t)]. \end{cases} \quad (3)$$

It is known that the periodic system is a special case of almost periodic system. For functional differential periodic equation with finite delay, it is implied the existence of a periodic solution by the uniformly ultimate boundedness of the solutions[5]. However, even if for ordinary differential almost periodic system, it does not imply the existence of an almost periodic solution from the ultimate boundedness of the solutions. Theoretically, one can investigate the existence of a almost periodic solution by Liapunov funcional[6]. However, there exists no general rule to guide how a proper Liapunov functional can be constructed for a given nonlinear delayed system. In this paper, we investigate the existence and uniqueness of almost periodic solutions for system (3) by constructing of a suitable Liapunov functional.

By using Liapunov functional to investigate the existence and uniqueness of almost periodic solutions, there is a following statement[6]:

Consider an almost periodic differential equation with time delays as follws

$$x' = f(t, x_t), \quad (4)$$

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where $x_t = x(t+\theta), \theta \in [-r, 0]$, let $C = C([-r, 0], R^n)$ be a Banach space of all continuous mappings of $[-r, 0] \to R^n$, $\|h\| = \sup_{\theta \in [-r,0]} |h(\theta)|$ represents the norm of $h(\theta)$. $C_H = \{h: h \in C, \|h\| < H, H \in R^+\}$. $x(t) = x(t, e, h)$ denotes the solution through (e, h) of system (4). The associated product system of (4) as follows

$$x' = f(t, x_t), y' = f(t, y_t). \quad (5)$$

The derivative of Liapunov functional $V(t, h, j)$ along the solutions of system (5) will be denoted by $V'_{(5)}(t, h, j)$ and is defined to be

$$V'_{(5)}(t, h, j) = \overline{\lim}_{h \to 0^+} \frac{1}{h}\{V(t+h, x_{t+h}(t, h), y_{t+h}(t, j)) - V(t, h, j)\}.$$

Theorem A Suppose that there exists a continuous Liapunov functional $V(t, h, j)$ for $t \geq 0$, $h \in C_H$ and $j \in C_H$ whick satisfies the following conditions:

(i) $a(|h - j|) \leq V(t, h, j) \leq b(|h - j|)$, where $a(r)$ is continuous, increasing, positive definite and $b(r)$ is continuous, increasing and such that $b(r) \to 0$ as $r \to 0$.

(ii) $|V(t, h_1, j_1) - V(t, h_2, j_2)| \leq k\{|h_1 - h_2| + |j_1 - j_2|\}$, where k is a positive constant.

(iii) $V'_{(5)}(t, h, j) \leq -cV(t, h, j)$, where c is a positive constant.

Moreover, we assume that there exists a solution $x(t)$ of system (4) such that $|x(t)| \leq H_1$, where $H_1 \leq H$. Then in the region C_H there exists a unique uniformly asymptotically stable almost periodic solution of (4) which is bounded by H_1.

In order to use the result of Theorem A, we have

Theorem 1 For system (3), suppose that positive constants T, U, R, L, K, f and M satisfy the following conditions

(I) $R^2T^2 - KMRT - \frac{fRT}{2L} - \frac{fR(T+U)T^2}{2L} > 0$;

(II) $2R^2TU - \frac{fRTU(T+U)}{L} \geq 0$;

(III) $R^2U^2 - KMRU - \frac{fRU}{2L} - \frac{fR(T+U)U^2}{2L} \geq 0$.

Then there exists a bounded solution of system (3).

Proof Let h be any given initial function, $(x, y) = (x(t), y(t))$ is a solution of system (3), consider Liapunov functional

$$W(t) = \frac{RL}{2}y^2 + \frac{RT}{2}x^2 + \frac{RU}{4}x^4 + \frac{R(T+U)}{2L}\int_{-f}^{0}\int_{t+s}^{t}[Tx(\theta) + Ux^3(\theta)]^2 d\theta ds, \quad (6)$$

then $W(t) \to +\infty$ ($|x| \to \infty$) and $W(t) \to +\infty$ ($|y| \to \infty$), meanwhile we have

$$W'_{(3)}(t) = -Ry(Tx + Ux^3) + R(Tx + Ux^3)\{y - RTx - RUx^3 + KE(t) + \frac{1}{L}\int_{t-f}^{t}[Tx(s) + Ux^3(s)]ds\} + \frac{R(T+U)}{2L}\int_{-f}^{0}[Tx(t) + Ux^3(t)]^2 ds - \frac{R(T+U)}{2L}\int_{-f}^{0}[Tx(t+s) + Ux^3(t+s)]^2 ds = -(RTx + RUx^3)^2 + (RTx + RUx^3)KE(t) + \frac{1}{L}(RTx + RUx^3)\int_{t-f}^{t}[Tx(s) + Ux^3(s)]ds + \frac{fR(T+U)}{2L}(Tx + Ux^3)^2 - \frac{R(T+U)}{2L}\int_{t-f}^{t}[Tx(s) + Ux^3(s)]^2 ds, \quad (7)$$

notice that

$$\frac{1}{L}(RTx(t) + RUx^3(t))\int_{t-f}^{t}[Tx(s) + Ux^3(s)]ds = \frac{RT}{L}\int_{t-f}^{t}x(t)[Tx(s) + Ux^3(s)]ds + \frac{RU}{L}\int_{t-f}^{t}x^3(t)[Tx(s) + Ux^3(s)]ds \leq \frac{RT}{2L}\int_{t-f}^{t}\{x^2(t) + [Tx(s) + Ux^3(s)]^2\}ds + \frac{RU}{2L}\int_{t-f}^{t}\{x^6(t) + [Tx(s) + Ux^3(s)]^2\}ds = \frac{fRT}{2L}x^2 + \frac{fRU}{2L}x^6 + \frac{R(T+U)}{2L}\int_{t-f}^{t}[Tx(s) + Ux^3(s)]^2 ds. \quad (8)$$

Since T, U, R are positive constants, therefore, when $|x| \geq 1$, then $RTx + RUx^3 \leq RTx^2 + RUx^3$. Namely, when $|x| \geq 1$, for any y we have

$$W'_{(3)}(t) = -(RTx + RUx^3)^2 + (RTx^2 + RUx^6)KM + \frac{fRT}{2L}x^2 + \frac{fRU}{2L}x^6 + \frac{fR(T+U)}{2L}(Tx + Ux^3)^2 = -[R^2T^2 - KMRT - \frac{fRT}{2L} - \frac{fR(T+U)T^2}{2L}]x^2 - [2R^2TU - \frac{fRTU(T+U)}{L}]x^4 - [R^2U^2 - KMRU - \frac{fRU}{2L} - \frac{fR(T+U)U^2}{2L}]x^6 < 0. \quad (9)$$

This means (x, y) is bounded.

Now we consider the associated product system (3) as the following system (10):

$$\begin{cases} x'(t) = y(t) - RTx(t) - RUx^3(t) + KE(t) + \frac{1}{L}\int_{t-f}^{t}[Tx(s) + Ux^3(s)]ds, \\ y'(t) = -\frac{1}{L}[Tx(t) + Ux^3(t)], \\ u'(t) = v(t) - RTu(t) - RUu^3(t) + KE(t) + \frac{1}{L}\int_{t-f}^{t}[Tu(s) + Uu^3(s)]ds, \\ v'(t) = -\frac{1}{L}[Tu(t) + Uu^3(t)]. \end{cases} \quad (10)$$

For the solution $(x, y, u, v) = (x(t), y(t), u(t), v(t))$ of system (10), Let $X = X(t) = x - u$, $Y = Y(t) = y - v$, and

$$f(t) = \begin{cases} x^2 + xu + u^2, x \neq u, \\ 3x^2, \quad x = u. \end{cases} \qquad (11)$$

we know that $f(t) \geqslant 0$ and $x^3 - u^3 = f(t)X$. Let again $A = 2RT + 2RUf(t) - \frac{1}{L}(\frac{5fT}{2} + T + Uf(t) + \frac{3fU}{2}f^2(t))$; $B = 2 - RT - RUf(t) - \frac{1}{L}(T + 2bT + Uf(t) + 2bUf(t))$; $C = 1 - \frac{fT}{2L}$. where b is some positive constant such that $B^2 < 4AC$. Now again assume that

(Ⅳ) $A > 0$ and $B^2 < 4AC$.

Theorem 2 Suppose that the conditions (Ⅰ) ~ (Ⅳ) are satisfied, then there exists a unique uniformly asymptotically stable almost periodic solution of system (3).

Proof From theorem 1, there exists a bounded solution of system (3) under assumptions, so that $f(t)$ is also bounded. For system (10), consider Liapunov functional

$$V(t) = X^2 - XY + (\frac{1}{2} + b)Y^2 + \frac{3T}{2L}\int_{-f}^{0}\int_{t+s}^{t} X^2(\theta)\,d\theta\,ds + \frac{3U}{2L}\int_{-f}^{0}\int_{t+s}^{t} f^2(\theta)X^2(\theta)\,d\theta\,ds, \qquad (12)$$

then $V(t)$ satisfies conditions (i), (ii) of Theorem A, and

$$V'_{(10)}(t) = 2X\{Y - RTX - RUf(t)X + \frac{1}{L}\int_{t-f}^{t}[TX(s) + Uf(s)X(s)]ds\} - Y\{Y - RTX - RUf(t)X + \frac{1}{L}\int_{t-f}^{t}[TX(s) + Uf(s)X(s)]ds\} + \frac{1}{L}X(TX + Uf(t)X) - (\frac{1}{L} + \frac{2b}{L})(TX + Uf(t)X)Y + \frac{3T}{2L}\int_{-f}^{0}X^2(t)ds - \frac{3T}{2L}\int_{-f}^{0}X^2(t+s)ds + \frac{3U}{2L}\int_{-f}^{0}f^2(t)X^2(t)ds - \frac{3U}{2L}\int_{-f}^{0}f^2(t+s)X^2(t+s)ds$$

$$= 2XY - 2RTX^2 - 2RUf(t)X^2 + \frac{2}{L}X(t)\int_{t-f}^{t}[TX(s) + Uf(s)X(s)]ds - Y^2 - RTXY - RUf(t)XY - \frac{1}{L}Y(t)\int_{t-f}^{t}[TX(s) + Uf(s)X(s)]ds + (\frac{T}{L} + \frac{U}{L}f(t))X^2 - (\frac{1}{L} + \frac{2b}{L})(T + Uf(t))XY + \frac{3fT}{2L}X^2 - \frac{3T}{2L}\int_{t-f}^{t}X^2(s)ds + \frac{3fU}{2L}f^2(t)X^2 - \frac{3U}{2L}\int_{t-f}^{t}f^2(s)X^2(s)\cdot ds$$

$$\leqslant 2XY - 2RTX^2 - 2RUf(t)X^2 + \frac{fT}{L}X^2 - Y^2 - RTXY - RUf(t)XY + \frac{fT}{2L}Y^2 + (\frac{T}{L} + \frac{U}{L}f(t))X^2 - \frac{1}{L}(T + 2bT + Uf(t) + 2bUf(t))XY + \frac{3fT}{2L}X^2 + \frac{3fU}{2L}f^2(t)X^2$$

$$= -[2RT + 2RUf(t) - \frac{1}{L}(\frac{5fT}{2} + T + Uf(t) + \frac{3fU}{2}f^2(t))]X^2 + [2 - RT - RUf(t) - \frac{1}{L}(T + 2bT + Uf(t) + 2bUf(t))]XY - (1 - \frac{fT}{2L})Y^2 = -AX^2 + BXY - CY^2 = -C(Y - \frac{B}{2C}X)^2 - A(1 - \frac{B^2}{4AC})X^2. \qquad (13)$$

We also have

$$V'_{(10)}(t) = -AX^2 + BXY - CY^2 = -A(X - \frac{B}{2A}Y)^2 - C(1 - \frac{B^2}{4AC})Y^2. \qquad (14)$$

Combine (13) and (14), we can obtain

$$V'_{(10)}(t) = -AX^2 + BXY - CY^2 \leqslant -\frac{1}{2}A(1 - \frac{B^2}{4AC})X^2 - \frac{1}{2}C(1 - \frac{B^2}{4AC})Y^2. \qquad (15)$$

From condition (Ⅳ), there exists a positive constant c such that

$$V'_{(10)}(t) \leqslant -c(X^2 + Y^2). \qquad (16)$$

This means condition (iii) of Theorem A is also satisfied. Therefore, there exists a unique uniformly asymptotically stable almost periodic solution of system (3), namely, there is a almost periodic oscillation of equation (2).

Remark The value of B can be positive or negative, so the value of b may be in an interval of R^+. It is easily to verify that the conditions (Ⅰ) ~ (Ⅲ) are indepedent and easily to give some examples satisfying conditions (Ⅰ) ~ (Ⅲ) of the parameters L, R, T, U, K and f.

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