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Maps of the Interval [0, 1) with Every Point Chain Recurrent *

区间[0,1)上每个点都为链回归点的映射

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Abstract: Let $X=[0,1)$, $f:X\to X$ be a continuous map. It is showed that if f is pointwise chain recurrent (that is, every point of X is chain recurrent under f), then f is identity if $Fix(f)$ is connected; f is turbulent if $Fix(f)$ is disconnected.

Key words: interval maps, pointwise chain recurrent, turbulent

摘要: 设 $X=[0,1)$, $f:X\to X$ 是连续自映射. 指出: 如果 f 是逐点链回归的(也就是说, X 中的每一点在 f 下是链回归的), 那么, 若 $Fix(f)$ 是连通的, 则 f 是恒等映射; 若 $Fix(f)$ 是不连通的, 则 f 含湍流.

关键词: 区间映射 逐点链回归 湍流

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1 Introduction

Firstly, some notations and definitions are established. Let (X,d) be a metric space and $g:X\to X$ be a continuous map. If $g^n(x)=x\neq g^k(x)$, $k=1,2,\cdots,n-1$, for $x\in X$ and positive integer n, then the point x is called a periodic point of period n, where $g^0=id$, $g^i=g\circ g^{i-1}(i\geqslant 1)$. In particular, if $g(x)=x$, then x is called a fixed point of g, the set of all fixed points of g is denoted by $Fix(g)$. For $x,y\in X$ and $\varepsilon>0$, an ε-chain from x to y is a finite sequence $x=x_0,x_1,\cdots,x_{n-1},x_n=y$ with $d(g(x_i),x_{i+1})<\varepsilon$ for $0\leqslant i\leqslant n-1$. We say that x chains to y under g, if for each $\varepsilon>0$, there is an ε-chain from x to y. A point x is said to be chain recurrent if x chains to itself. The map g is said to be pointwise chain recurrent if every point of X is chain recurrent under g. The following facts about chain recurrent are standard observations.

(a) If g is pointwise chain recurrent, then g maps X onto X.

(b) g is pointwise chain recurrent if and only if g^n is pointwise recurrent for every $n>0$.

(c) [Reference 1, Theorem A] If X is connected and $g:X\to X$ is pointwise chain recurrent, then there is no nonempty open set $U\neq X$ such that $g(\overline{U})\subset U$.

Being chain recurrent is an important dynamical property of a system and has been studied intensively in recent years. For more details see References[1～7].

A map $g:X\to X$ is called turbulent if there are closed non-degenerate connected subsets J and K with disjoint interiors such that $g(J)\cap g(K)\supset J\cup K$.

It is obvious that

(1) If g is turbulent then g^n is turbulent for any $n>1$.

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(2)If there exist $p \in Fix(g), y \in X$ such that $y \in ((g(y), p)$ and $p = g^2(y)$, then g is turbulent.

In Reference[2], it was proved that a pointwise chain recurrent map h of the compact interval must satisfy that either h^2 is the identity or h^2 is turbulent. In Reference[5], it was showed that a pointwise chain recurrent map h of the space Y satisfys that either h^{12} is identity or h^{12} is turbulent. In Reference[6], let T be a tree, $f: T \to T$ be a continuous map. It was shown that if f is pointwise chain recurrent, then either f^{a_n} is identity or f^{a_n} is turbulent if $Fix(f) \cap End(T) = \phi$; either $f^{a_{n-1}}$ is identity or $f^{a_{n-1}}$ is turbulent if $Fix(f) \cap End(T) \neq \phi$, where n denotes the number of the endpoints of T and, a_n denotes the minimal common multiple of $2, 3, \cdots, n$.

In this paper, we prove the following theorem.

Main theorem Let $X = [0,1]$ and $f: X \to X$ be a continuous map. If f is pointwise chain recurrent, then

(1) f is identity if $Fix(f)$ is connected;

(2) f is turbulent if $Fix(f)$ is disconnected.

2 Proof of main results

In this section, $f: X \to X$ be defined a continuous map, where $X = [0,1)$. Some lemmas are established before the proving of the main results.

Lemma 2.1 Let $X = [0,1), f: X \to X$ be a pointwise chain recurrent map, then $Fix(f) \neq \phi$.

Proof Suppose that $Fix(f) = \phi$. Therefore $f(x) > x$ for each $x \in X$ since f is a continuous map and $f(0) \geqslant 0$. Let $m = \min\{f(x): x \in [0, f(0)]\}$. It is obvious that $f(x) > m$ for each $x \in [f(0), 1)$. Therefore $f(\overline{X}) = f(X) \subset [m, 1) \subseteq X$, which contradicts with the fact that f is a pointwise chain recurrent map.

Lemma 2.2 Let $X = [0,1), f: X \to X$ be a continuous map and $Fix(f) \subseteq [0, a]$ for $a < 1$, then f can not be pointwise chain recurrent.

Proof Firstly, in terms of the continuity of f, we have $f(x) > x$ for each $x > a$ or $f(x) < x$ for each $x > a$.

Case 1 $f(x) > x$ for each $x > a$.

Given a point $b \in (a, 1))$, we take $m = \min\{f(x): x \in [b, f(b)]\}$. Then $m > b$ and $f([b, 1)) \subset [m, 1) \subset (b, 1)$. Therefore, f can not be pointwise chain recurrent.

Case 2 $f(x) < x$ for each $x > a$.

Let $M_1 = \max\{f(x): x \in [0, a]\}$ and $M = \max\{M_1, a\}$. Choose a point $M_2 \in (M, 1)$ and let $M_3 = \max\{f(x): x \in [0, M_2]\}$. Then $M_3 < M_2$ and $f[0, M_2] = [0, M_3] \subset [0, M_2)$. Therefore, f can not be pointwise chain recurrent.

The proof is completed.

Corollary 2.1 Let $X = [0,1)$ and $f: X \to X$ be a continuous map. If f is pointwise chain recurrent, then there exist points $\{e_n\} \subset Fix(f)$ with $\lim\limits_{n \to \infty} e_n = 1$.

Lemma 2.3 Let $X = [0,1), f: X \to X$ be a continuous map and $Fix(f) = [a, 1)$ for $0 < a < 1$, then f can not be pointwise chain recurrent.

Proof Firstly, in terms of the continuity of f, we have $f(x) > x$ for each $x < a$ or $f(x) < x$ for each $x < a$.

Case 1 $f(x) > x$ for each $x < a$.

Let $m = \min\{f(x): x \in X\}$. It is obvious that $m > 0$. Then $f(X) = [m, 1) \subset (\frac{m}{2}, 1)$. Therefore, f can not be pointwise chain recurrent.

Case 2 $f(x) < x$ for each $x < a$.

Given any point $b \in (0, a), f(b) < b$. It follows that $Fix(f) \cap [0, b) \neq \phi$ since $f(0) \geqslant 0$, which contradicts with $Fix(f) = [a, 1)$.

The proof is completed.

By Lemmas 2.2 to 2.3, we have Theorem 2.1.

Theorem 2.1 Let $X = [0,1)$ and $f: X \to X$ be a continuous map. Then f is identity if f is pointwise chain recurrent and $Fix(f)$ is connected.

Lemma 2.4 Let $X = [0,1)$ and f be a pointwise chain recurrent continuous map, if there exists $e_1 < e_2 < e_3 \in Fix(f)$ satisfying

(1) $(e_1, e_2) - Fix(f) \neq \phi$ and $(e_2, e_3) - Fix(f) \neq \phi$;

(2) $f(x) \geqslant x$ for each $x \in [e_1, e_2]$;

(3) $f(x) \leqslant x$ for each $x \in [e_2, e_3]$;

then f is turbulent.

Proof **Case 1** $f(x) \neq e_3$ for all $x \in (e_1, e_2)$ and $f(x) \neq e_1$ for all $x \in (e_2, e_3)$. Let $M_1 = \max\{f(x): x \in [e_1, e_2]\}$ and $m_2 = \min\{f(x): x \in [e_2, e_3]\}$, then $e_1 < m_2 \leqslant M_1 < e_3$. Let $U = (\frac{e_1 + m_2}{2}, \frac{M_1 + e_3}{2})$, then $U \neq \phi$ and $f(\overline{U}) \subset U$, which contradicts with the fact that f is pointwise chain recurrent.

Case 2 $f(x) = e_3$ for some $x \in (e_1, e_2)$ or $f(x) = e_1$ for some $x \in (e_2, e_3)$.

Without loss of generality, we assume that there exists $y \in (e_1, e_2)$ such that with $f(y) = e_3$ and $f^{-1}(e_3) \cap (y, e_2) = \phi$.

If $m_2 > y$. Let $M_2 = \max\{f(x) : x \in [\frac{y + m_2}{2}, e_2]$ and $\delta = \min\{\frac{m_2 - y}{3}, \frac{e_3 - M_2}{2}\}$, then, in terms of the continuity of f, $f([y+\delta, e_3 - \delta]) \subset (y+\delta, e_3 - \delta)$. That is a contradiction. Else, $f(y_0) = y$ for some $y_0 \in (e_2, e_3)$, then $y_0 \in (f(y_0), e_3)$ and $f^2(y_0) = e_3$. It follows that f is turbulent.

The proof is completed.

Theorem 2.2 Let $X = [0,1)$ and f be a pointwise chain recurrent continuous map, if there exists $e_0 < e_1 < e_2, \cdots \in Fix(f)$ such that $(e_{n-1}, e_n) \not\subset Fix(f)$ for each $n \in N$ and $\lim_{n\to\infty} e_n = 1$, then f is turbulent.

Proof Without loss of generality, we assume that $(e_i, e_{i+1}) \cap Fix(f) = \phi$.

Case 1 There exists positive integer i_0 such that $f(x) \leqslant x$ for all $x \in [e_{i_0}, 1)$.

Let $M = \max\{f(x) : x \leqslant e_{i_0}$, then $M < 1$. There exists $e_{i_1} > M$ since $\lim_{n\to\infty} e_n = 1$. It follows that $f([0,b]) \subset [0,b)$ for each $b \in (e_{i_1}, e_{i_1+1})$, which contradicts with the fact that f is pointwise chain recurrent.

Case 2 There exists positive integer i_0 such that $x \leqslant f(x)$ for all $x \in [e_{i_0}, 1)$, then $f([b,1)) \subset (b,1)$ for each $b \in (e_{i_0}, e_{i_0+1})$, which contradicts with the fact that f is pointwise chain recurrent.

Case 3 There exists $n \in N$ satisfying $f(x) > x$ for all $x \in (e_n, e_{n+1})$ and $f(x) < x$ for all $x \in (e_{n+1}, e_{n+2})$. By Lemma 2.4, it is obvious that f is turbulent.

The proof is completed.

Lemma 2.5 Let $X = [0,1)$ and f be a pointwise chain recurrent continuous map. If there exists $a < 1$ such that $[a,1)$ is a connected component of $Fix(f)$ and f is not identity, then there exists $b < a$ such that $f(x) \leqslant x$ for each $x \in (b,a)$.

The proof of Lemma 2.5 is easy, and omitted.

Theorem 2.3 Let $X = [0,1)$ and f be a pointwise chain recurrent continuous map. If there exists $a \neq 0$ such that $\{0\} \cup [a,1) \subset Fix(f)$ and f is not an identity map, then f is turbulent.

Proof Without loss of generality, we assume that $0 = \max\{x : [0,x] \subset Fix(f)\}$ and $a = \min\{x : [x,1) \subset Fix(f)\}$.

Case 1 There exists $b \in (0,a)$ such that $f(x) < x$ for each $x \in (0,b]$. Let $M = \max\{f(x) : x \in [0, b]\}$. Then $M < b$ and $f([0,b]) = [0,M] \subset [0,b)$, which contradicts with the fact that f is pointwise chain recurrent.

Case 2 There exists $e_1 = 0 < e_2 < e_3 \leqslant a \in Fix(f)$ satisfying the three conditions of Lemma 2.4, it follows that f is turbulent.

Lemma 2.6 Let $X = [0,1)$ and f be a pointwise chain recurrent continuous map. If there exists $0 < a \leq b < c < 1$ such that $Fix(f) = [a,b] \cup [c,1)$, then f is turbulent.

Proof It follows, by Lemma 2.5, that $f(x) < x$ for each $x \in (b,c)$.

Case 1 $f^{-1}(c) \cap [0,c) = \phi$, then $f([0,s]) \subset [0,s)$ for each $s \in (\max_{x\in[0,b]} f(x), c) - Fix(f)$, which contradicts with the fact that f is pointwise chain recurrent.

Case 2 $f^{-1}(c) \cap [0,c) = \phi$. Let $y \in f^{-1}(c) \cap [0,a)$ and $f^{-1}(c) \cap [y,a) = \phi$. If $f^{-1}(y) \cap (b,c) = \phi$, there exist some nonempty intervals $U \subset (y,c)$ such that $f(\overline{U}) \subset U$ in terms of the continuity of f, which contradicts with the fact that f is pointwise chain recurrent. Additionally, there exists $z \in (b,c)$ such that $f(z) = y$, then f is turbulent.

Theorem 2.4 Let $X = [0,1)$ and f be a pointwise chain recurrent continuous map. If there exists $a \neq 0$ such that $[a,1) \subset Fix(f)$ and $0 \notin Fix(f)$, then f is turbulent.

Proof Let $b = \min\{x : f(x) = x\}$. It is obvious that $f(x) > x$ for each $x < b$ since f is a continuous map, and that $[b,a] \cap Fix(f)$ is disconnected in terms of Lemma 2.3. Let $c = \min\{x : [b,c] \cap Fix(f)$ is disconnected $\}$ and $d = \min\{x : [x,1) \cap Fix(f)$ is connected $\}$.

Case 1 $c = d$. It is followed by Lemma 2.6 immediately.

Case 2 $c < d$. It follows, by Lemma 2.5, that $f(x) < x$ for each $x \in (s,d)$ for some $s \in (c,d)$.

(下转第102页 Continue on page 102)

由表5可以看出，用进化策略算法不但能计算低阶多项式，还能计算高阶多项式，速度快，精度高.

4 结论

利用进化策略的群体搜索和全局收敛的特性，提出在整个实数域(或复数域)上进行求根的进化策略算法，能有效的解决了传统算法在求解过程中存在迭代初值选取难的问题，而且能解决系数为复(实)系数的高阶多项式在复数(实数)域上求根的问题，比一般的求多项式根的智能算法还要好.该算法收敛速度快，精度高.

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(上接第97页 Continue from page 97)

Subcase 2.1 $f(x) \geqslant x$ for each $x \in [b,c]$. We can prove it by using the same method as Lemma 2.6.

Subcase 2.2 $f(x) \geqslant x$ for each $x \in [b,c]$. There exist $e_1 = b < e_2 = c < e_3 \in Fix(f)$ satisfying the three conditions of Lemma 2.4. It follows that f is turbulent.

The Main Theorem is obtained by Theorems 2.1 to 2.4 in the following.

Main theorem Let $X = [0,1)$ and $f: X \to X$ be a continuous map. If f is pointwise chain recurrent, then

(1) f is identity if $Fix(f)$ is connected;

(2) f is turbulent if $Fix(f)$ is disconnected.

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