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一类中立型时滞系统的绝对稳定性*

Robust Stabilization of a Class of Linear Neutral Systems with Nonlinear Perturbations

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摘要：利用 Lyapunov 和 LMI 求解工具，用相对直接的矩阵运算来得到控制器的设计方法，获得一类中立型不确定时滞系统绝对稳定的充分条件.

关键词：时滞系统　稳定性　LMI

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Abstract: Robust stabilization of a class of linear neutral systems with nonlinear perturbation is discussed in this paper. By using Lyapunov functional method and the method of LMI, we have derived some absolute stability criterions.

Key words: neutral system, stabilization, LMI

控制系统的时滞现象普遍存在于实际控制问题中，如飞行控制系统、化工控制系统等都存在滞后现象。许多控制系统不仅状态中存在时滞，而且在状态导数中也存在时滞，这样的一类系统一般称之为中立型时滞系统[1]. 对于中立型时滞系统的稳定性已有很多的结论[2~6]. 但是，讨论的系统中输出大多不含有时滞状态以及干扰输入，这在实际控制系统中不具有一般性，并且所讨论的方法大都归结为代数 Riccati 方程(或不等式)的求解问题，这对系统带有某些约束条件的情形，给设计控制器带来一定困难. 本文利用 Lyapunov 和 LMI 求解工具，用相对直接的矩阵运算来得到控制器的设计方法，进一步获得了一类中立型时滞系统的相关绝对稳定的充分条件.

本文研究的中立型结构时滞系统如下：

$$\dot{x}(t) - M\dot{x}(t-h) = Ax(t) + Bx(t-h) + C\int_{t-\tau}^{t} x(s)\mathrm{d}s + b\phi(y) + e\omega(t),\ y = d^T x(t) + fx(t-h) + g\omega(t), \tag{1}$$

其中 $x(t) \in R^n$ 为状态变量，$\omega(t) \in R^n$ 为外部扰动，$y \in R^n$ 为被调输出. $A,B,C,M \in R^{n\times n}$，$b,d,e,f,g \in R^n$，$h \geqslant 0$，$\tau > 0$ 为常数，$\phi(y) \in K[0,k]$，$K[0,k] = \{\phi(y) \mid \phi(0) = 0, 0 < y\phi(y) \leqslant ky^2 (y \neq 0)\}$. 以 $|\phi(t)|$ 表示 R^n 空间中的范数，$\|\phi\| = \sup\limits_{t\in[-h_0,0]} |\phi(t)|$.

引理[7]　对给定的对称矩阵

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix},$$

其中 S_{ii} 是 $R \times R$ 维实矩阵. 则以下 3 个条件是等价的：

(1) $S < 0$；

(2) $S_{11} < 0, S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$；

(3) $S_{22} < 0, S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

其中 $S < 0$ 表示 S 为负定的.

首先考虑无输出时滞控制系统

$$\dot{x}(t) - M\dot{x}(t-h) = Ax(t) + Bx(t-h) + C\int_{t-\tau}^{t} x(s)\mathrm{d}s + b\phi(y) + e\omega,\ y = d^T x(t) + fx(t-h), \tag{2}$$

的渐近稳定性.

定义

$V(x(t), x(t-h)) = (x(t) - Mx(t-h))^T P(x(t) - Mx(t-h)) + \alpha\int_0^t (y\phi(y) -$

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$$\frac{\phi^2(y)}{k})\mathrm{d}\theta + \int_{t-h}^{t} x^T(\theta)Gx(\theta)\mathrm{d}\theta +$$
$$\int_0^\tau\int_{-s}^{t} x^T(\theta)Rx(\theta)\mathrm{d}\theta\mathrm{d}s.$$

$$\dot{V} = x(t)(A^TP + PA + G + \tau R)x(t) + x(t)(PB - A^TPM)x(t-h) + x^T(t-h)(B^TP - M^TPA)x(t) + x^T(t-h)(-B^TPM - M^TPB - G)x(t-h) + \phi^T(y)b^TPx(t) + x^T(t)Pb\phi(y) - \phi^T(y)b^TPMx(t-h) - x^T(t-h)M^TPb\phi(y) + \phi(y)\alpha d^Tx(t) + \phi(y)\alpha fx(t-h) - \frac{\alpha\phi^2(y)}{k} + \omega^Te^TPx(t) - \omega^Te^TPMx(t-h) + x(t)Pe\omega - x^T(t-h)M^TPe\omega + \int_{-\tau}^{t} x^T(s)CPx(t)\mathrm{d}s - \int_{-\tau}^{t} x^T(s)C^TPMx(t-h)\mathrm{d}s + \int_{-\tau}^{t} x(t)PCx(s)\mathrm{d}s - \int_{-\tau}^{t} x^T(t-h)M^TPCx(s) - \int_{-\tau}^{t} x^T(s)Rx(s)\mathrm{d}s -$$

$$\frac{1}{2}x^T(s)Rx(s) + x^T(s)C^TPx(t) + x^T(t)PCx(s) = -[C^TPx(t) - \frac{1}{2}Rx(s)]^T2R^{-1}[C^TPx(t) - \frac{1}{2}Rx(s)] + 2x^T(t)PCR^{-1}C^TPx(t) -$$

$$\frac{1}{2}x^T(s)Rx(s) - x^T(s)C^TPMx(t-h) - x^T(t-h)M^TPCx(s) = -[C^TPMx(t-h) - \frac{1}{2}Rx(s)]^T2R^{-1}[C^TPMx(t) - \frac{1}{2}Rx(s)] + 2x^T(t-h)M^TPCR^{-1}C^TPMx(t).$$

因此

$$\dot{V} \leqslant x(t)(A^TP + PA + G + \tau R)x(t) + x(t)(PB - A^TPM)x(t-h) + x^T(t-h)(B^TP - M^TPA)x(t) + x^T(t-h)(-B^TPM - M^TPB - G)x(t-h) + \phi^T(y)b^TPx(t) + x^T(t)Pb\phi(y) - \phi^T(y)b^TPMx(t-h) - x^T(t-h)M^TPb\phi(y) + 2\tau x^T(t)PCR^{-1}C^TPx(t) + x^T(t-h)2\tau M^TPCR^{-1}C^TPMx(t-h) + \phi(y)\alpha d^Tx(t) + \phi(Y)\alpha fx(t-h) - \frac{\alpha}{k}\phi^2(y) + \omega^Te^TPx(t) - \omega^Te^TPMx(t-h) + x^T(t)Pe\omega - x^T(t-h)M^TPe\omega =$$

$$\begin{bmatrix} x(t) \\ x(t-h) \\ \phi(y) \\ \omega \end{bmatrix}^T H \begin{bmatrix} x(t) \\ x(t-h) \\ \phi(y) \\ \omega \end{bmatrix},$$

$$H = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & 0 \\ a_{41} & a_{42} & 0 & 0 \end{bmatrix},$$

其中，$a_{11} = A^TP + PA + G + \tau R + 2\tau PCR^{-1}C^TP$，$a_{12} = PB - A^TPM$，$a_{13} = Pb + \frac{1}{2}\alpha d$，$a_{14} = Pe$，$a_{21} = B^TP - M^TPA$，$a_{22} = - B^TPM - M^TPB - G + 2\tau M^TPCR^{-1}C^TPM$，$a_{23} = - M^TPb + \frac{1}{2}\alpha f$，$a_{24} = - M^TPe$，$a_{31} = b^TP + \frac{1}{2}\alpha d^T$，$a_{32} = - b^TPM + \frac{1}{2}\alpha f^T$，$a_{33} = - \frac{\alpha}{K}$，$a_{41} = e^TP$，$a_{42} = - e^TPM$.

要想使系统渐近稳定，只需 $H < 0$. 但 $H < 0$ 不是关于 P,G,R 的非线性不等式，需要把它转化为一个等价的线性不等式. 令 $S_{21} = (\sqrt{2\tau}C^TP \quad 0 \quad \sqrt{2\tau}C^TPM \quad 0)$，$H_0 =$

$$\begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & 0 \\ b_{41} & b_{42} & 0 & 0 \end{bmatrix},$$

其中，$b_{11} = A^TP + PA + G + \tau R$，$b_{12} = PB - A^TPM$，$b_{13} = Pb + \frac{1}{2}\alpha d$，$b_{14} = Pe$，$b_{21} = B^TP - M^TPA$，$b_{22} = - B^TPM - M^TPB - G$，$b_{23} = - M^TPb + \frac{1}{2}\alpha f$，$b_{24} = - M^TPe$，$b_{31} = b^TP + \frac{1}{2}\alpha d^T$，$b_{32} = - b^TPM + \frac{1}{2}\alpha f^T$，$b_{33} = - \frac{\alpha}{K}$，$b_{41} = e^TP$，$b_{42} = - e^TPM$.

则由引理知 $H < 0$ 等价于 $Q < 0$. $Q < 0$ 是一个关于 P,G,R 的线性不等式，我们可以用 LMI 求解.

定理 1 若存在正定的矩阵 P,G,R 及正常数 α，使得矩阵 $Q = \begin{bmatrix} H_0 & S_{21}^T \\ S_{21} & -R \end{bmatrix} < 0$，则系统(2)是绝对稳定的.

定理 2 若存在正定的矩阵 P,G,R 及正常数 α，使得矩阵 $U\begin{bmatrix} K_0 & S_{21}^T \\ S_{21} & -R \end{bmatrix} < 0$，则系统(1)是绝对稳定的.

证明 由上述中的 $\dot{V}$ 得

$$\dot{V} \leqslant x(t)(A^TP + PA + G + \tau R)x(t) + x(t)(PB - A^TPM)x(t-h) + x^T(t-h)(B^TP - M^TPA)x(t) + x^T(t-h)(-B^TPM - M^TPB - G)x(t-h) + \phi^T(y)b^TPx(t) + x^T(t)Pb\phi(y) - \phi^T(y)b^TPMx(t-h) - x^T(t-h)M^TPb\phi(y) + 2\tau x^T(t)PCR^{-1}C^TPx(t) + x^T(t-h)2\tau M^TPCR^{-1}C^TPMx(t-h) + \phi(y)\alpha d^Tx(t) + \phi(y)\alpha fx(t-h) - \frac{\alpha}{k}\phi^2(y) + \omega^Te^TPx(t) - \omega^Te^TPMx(t-h) + x^T(t)Pe\omega - x^T(t-h)M^TPe\omega +$$

$$\phi(y)\alpha g\omega = \begin{bmatrix} x(t) \\ x(t-h) \\ \phi(y) \\ \omega \end{bmatrix}^T K \begin{bmatrix} x(t) \\ x(t-h) \\ \phi(y) \\ \omega \end{bmatrix},$$

$$K = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & 0 \end{bmatrix},$$

其中,$c_{11} = A^TP + PA + G + \tau R + 2\tau PCR^{-1}C^TP$, $c_{12} = PB - A^TPM$, $c_{13} = Pb + \frac{1}{2}\alpha d$, $c_{14} = Pe$, $c_{21} = B^TP - M^TPA$, $c_{22} = -B^TPM - M^TPB - G + 2\tau M^TPCR^{-1}C^TPM$, $c_{23} = -M^TPb + \frac{1}{2}\alpha f$, $c_{24} = -M^TPe$, $c_{31} = b^TP + \frac{1}{2}\alpha d^T$, $c_{32} = -b^TPM + \frac{1}{2}\alpha f^T$, $c_{33} = -\frac{\alpha}{K}$, $c_{34} = \frac{1}{2}\alpha g^T$, $c_{41} = e^TP$, $c_{42} = -e^TPM$, $c_{43} = \frac{1}{2}\alpha g$.

要想使系统绝对稳定,只需$K<0$.但$K<0$不是关于P,G,R的非线性不等式,下面我们把它转化为一个等价的线性不等式.令

$$S_{21} = (\sqrt{2\tau}C^TP \quad 0 \quad \sqrt{2\tau}C^TPM \quad 0),$$

$$K_0 = \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \\ d_{31} & d_{32} & d_{33} & d_{34} \\ d_{41} & d_{42} & d_{43} & 0 \end{bmatrix},$$

其中,$d_{11} = A^TP + PA + G + \tau R$, $d_{12} = PB - A^TPM$, $d_{13} = Pb + \frac{1}{2}\alpha d$, $d_{14} = Pe$, $d_{21} = B^TP - M^TPA$, $d_{22} = -B^TPM - M^TPB - G$, $d_{23} = -M^TPb + \frac{1}{2}\alpha f$, $d_{24} = -M^TPe$, $d_{31} = b^TP + \frac{1}{2}\alpha d^T$, $d_{32} = -b^TPM + \frac{1}{2}\alpha f^T$, $d_{33} = -\frac{\alpha}{K}$, $d_{34} = \frac{1}{2}\alpha g^T$, $d_{41} = e^TP$, $d_{42} = -e^TPM$, $d_{43} = \frac{1}{2}\alpha g$.

则由引理知$K<0$等价于$U = \begin{bmatrix} K_0 & S_{21}^T \\ S_{21} & -R \end{bmatrix} < 0$.$U<0$是一个关于$P,G,R$的线性不等式,我们可以用LMI求解.定理2得证.

考虑具有时滞反馈控制系统

$$\dot{x}(t) - M\dot{x}(t-h) = Ax(t) + Bx(t-h) + C\int_{t-\tau}^{t} x(s)\mathrm{d}s + b\phi(y(t-\mu)) + e\omega, y = d^Tx(t). \quad (3)$$

首先定义

$$V(x(t),x(t-h)) = (x(t) - Mx(t-h))^TP(x(t) - Mx(t-h)) + \alpha\int_0^t (y\phi(y) - \frac{\phi^2(y)}{k})\mathrm{d}\theta + \beta\int_{t-\mu}^{t} \phi^2(d^Tx(\theta))\mathrm{d}\theta + \int_{t-h}^{t} x^T(\theta)Gx(\theta)\mathrm{d}\theta + \int_0^{\tau}\int_{t-s}^{t} x^T(\theta)Rx(\theta)\mathrm{d}\theta\mathrm{d}s,$$

于是

$$\dot{V} \leqslant x(t)(A^TP + PA + G + \tau R)x(t) + x(t)(PB - A^TPM)x(t-h) + x^T(t-h)(B^TP - M^TPA)x(t) + x^T(t-h)(-B^TPM - M^TPB - G)x(t-h) + \phi^T(y(t-\mu))b^TPx(t) + x^T(t)Pb\phi(y(t-\mu)) - \phi^T(y(t-\mu))b^TPMx(t-h) + \beta\phi^2(y) - \beta\phi^2(y(t-\mu)) - x^T(t-h)M^TPb\phi(y(t-\mu)) + 2\tau x^T(t)PCR^{-1}C^TPx(t) + x^T(t-h)2\tau M^TPCR^{-1}C^TPMx(t-h) + \phi(y)\alpha d^Tx(t) - \frac{\alpha}{k}\phi^2(y) + \omega^Te^TPx(t) - \omega^Te^TPMx(t-h) + x^T(t)Pe\omega - x^T(t-h)M^TPe\omega = \zeta L\zeta^T,$$

其中 $\zeta = (x^T(t) \quad x^T(t-h) \quad \phi^T(y) \quad \phi^T(y(t-\mu)) \quad \omega^T)$, $L =$

$$\begin{bmatrix} e_{11} & e_{12} & e_{13} & Pb & Pe \\ e_{21} & e_{22} & 0 & -M^TPb & -M^TPe \\ a_{31} & 0 & -\frac{\alpha}{K}+\beta & 0 & 0 \\ b^TP & -b^TPM & 0 & -\beta & 0 \\ e^TP & -e^TPM & 0 & 0 & 0 \end{bmatrix},$$

其中,$e_{11} = A^TP + PA + G + \tau R + 2\tau PCR^{-1}C^TP$, $e_{12} = PB - A^TPM$, $e_{13} = \frac{1}{2}\alpha d$, $e_{21} = B^TP - M^TPA$, $e_{22} = -B^TPM^T - M^TPB - G + 2\tau M^TPCR^{-1}C^TPM$, $a_{31} = \frac{1}{2}\alpha d^T$.

要想使系统(3)绝对稳定,只需$L<0$.但$L<0$不是关于P,G,R的非线性不等式.我们把它转化为一个等价的线性不等式.令

$$S_{21} = (\sqrt{2\tau}C^TP \quad 0 \quad \sqrt{2\tau}C^TPM \quad 0 \quad 0),$$

$$L_0 = \begin{bmatrix} f_{11} & f_{12} & \frac{1}{2}\alpha d & Pb & Pe \\ f_{21} & f_{22} & 0 & f_{23} & f_{24} \\ \frac{1}{2}\alpha d^T & 0 & -\frac{\alpha}{K}+\beta & 0 & 0 \\ b^TP & -b^TPM & 0 & -\beta & 0 \\ e^TP & -e^TPM & 0 & 0 & 0 \end{bmatrix},$$

其中,$f_{11} = A^TP + PA + G + \tau R$, $f_{12} = PB - A^TPM$, $f_{21} = B^TP - M^TPA$, $f_{22} = -B^TPM^T - M^TPB - G$, $f_{23} = -M^TPb$, $f_{24} = -M^TPe$.

则由引理知$L<0$等价于$J = \begin{bmatrix} L_0 & S_{21}^T \\ S_{21} & -R \end{bmatrix} < 0$.$J<$

0是一个关于P,G,R的线性不等式. 我们可以用LMI求解.

定理3 若存在正定的矩阵P,G,R及正常数α,β使得矩阵$J<0$,则系统(3)是绝对稳定的.

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$$|x|_\infty \leqslant D + \frac{T}{2}|x'|_\infty \leqslant D + \frac{T}{2}R_1 \triangleq R_2, \quad (18)$$

取$M > \max\{R_1, R_2\}$,令$\Omega = \{x \in X: \|x\| < M\}$,由(5)、(6)式易知$N$在$\overline{\Omega}$上是$L$-紧的. 从上面的证明知引理1的条件(a)成立. 当$x \in \partial\Omega \cap \mathrm{Ker}L$时,$x$为常数且$|x| = M$,注意到$M > D$,则$QNx = -\frac{1}{T}\int_0^T [g(t,x) - p(t)]\mathrm{d}t \neq 0$,即引理1的条件(b)满足.

定义连续映射$H(x,\mu)$为

$$H(x,\mu) = -\mu x - \frac{1-\mu}{T}\int_0^T [g(t,x) - p(t)]\mathrm{d}t,$$

$\mu \in [0,1]$,则当$x \in \mathrm{Ker}L \cap \partial\Omega$及$\mu \in [0,1]$时,有

$$xH(x,\mu) = -\mu x^2 - \frac{1-\mu}{T}\int_0^T x[g(t,x) - p(t)]\mathrm{d}t < 0,$$

故$H(x,\mu)$为同伦映射,由于$\mathrm{Im}Q = \mathrm{Ker}L = R$,可取$J$为自然同构,因而有

$$\deg\{JQN, \Omega \cap \mathrm{Ker}L, 0\} = \deg\{-\frac{1}{T}\int_0^T [g(t,x) - p(t)]\mathrm{d}t, \Omega \cap \mathrm{Ker}L, 0\} = \deg\{-x, \Omega \cap \mathrm{Ker}L, 0\} \neq 0.$$

由引理1知,方程(3)至少存在一个T-周期解.

类似定理1的证明,可得下面结论成立:

定理2 若引理2的条件(A_2)成立,并且满足下列条件:

(B_2)存在常数$a \geqslant 0, b > 0$,使得下列条件之一成立:

(i) 当$t \in R, x < -D$时,$g(t,x) - p(t) \leqslant -ax + b$,且$\forall x \in R, h_i(x) \leqslant 0, i = 1,2,\cdots,n$;

(ii) 当$t \in R, x > D$时,$g(t,x) - p(t) \geqslant -ax - b$,且$\forall x \in R, h_i(x) \geqslant 0, i = 1,2,\cdots,n$.

则当$aT^2 + 2FT < 1$时,方程(3)至少一个T-周期解,其中$F = \sup\limits_{(t,x,y)\in R^3} |f(t,x,y)|$.

注 本文的结论不要求条件(H_0)成立,且当$a = 0$时,有$F = \sup\limits_{(t,x,y)\in R^3} |f(t,x,y)| < \frac{1}{2T}$. 把$F = \sup\limits_{(t,x,y)\in R^3} |f(t,x,y)| < \frac{1}{4T}$放大了两倍,因此本文的结果推广和改进了文献[1,2]的相关结果.

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