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Oscillation of Impulsive Partial Difference Equation*
脉冲偏差分方程的振动性

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Abstract By employing arithemetic mean-geometric mean inequality and partial difference inequality, we obtain sufficient conditions for oscillation of all solution of the impulsive partial difference equation

$$\begin{cases} A_{m+1,n} + A_{m,n+1} - A_{mn} + p_{mn} A_{m-r,n-l} = 0, m \geqslant m_0, n \geqslant n_0 - 1, m \neq m_k, \\ A_{m_k+1,n} + A_{m_k,n+1} - A_{m_k,n} = b_k A_{m_k,n}, \forall n \geqslant n_0 - 1, k \in N(1), \end{cases}$$

where $\{p_{mn}\}$ is a double sequence and $p_{mn} \geqslant 0$ and not identically zero, for $m \geqslant m_0, n \geqslant n_0 - 1$, $\{b_k\}$ is a real sequence, r, l are positive integers, $0 \leqslant m_0 \leqslant m_1 < \cdots < m_k < \cdots$ with $\lim_{k\to\infty} m_k = \infty$.

Key words partial difference equation, oscillatory solution, impulsive

摘要: 获得脉冲偏差分方程

$$\begin{cases} A_{m+1,n} + A_{m,n+1} - A_{mn} + p_{mn} A_{m-r,n-l} = 0, m \geqslant m_0, n \geqslant n_0 - 1, m \neq m_k, \\ A_{m_k+1,n} + A_{m_k,n+1} - A_{m_k,n} = b_k A_{m_k,n}, \forall n \geqslant n_0 - 1, k \in N(1), \end{cases}$$

所有解振动的充分条件,其中 $\{p_{mn}\}$ 是一个双指标序列,对 $m \geqslant m_0, n \geqslant n_0 - 1$,有 $p_{mn} \geqslant 0$ 且不恒为零, $\{b_k\}$ 是实数序列, r, l 是正整数, $0 \leqslant m_0 \leqslant m_1 < \cdots < m_k < \cdots$ 满足 $\lim_{k\to\infty} m_k = \infty$.

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The oscillatory behavior on partial difference equations without impulses has been investigated by some authors[1~4]. However, to the present time, there is no literature investigate the oscillation of impulsive partial difference equations.

Let N denote the set of all integers. For any $a \in$ N, define $N(a) = \{a, a+1, \cdots, \}$. For any $m, r \in N(1)$, define $N(m-r, m) = \{m-r, m-r+1, \cdots, m\}$. In this paper, we consider the sufficient conditions for oscillation of all solutions of the impulsive partial difference equation

$$\begin{cases} A_{m+1,n} + A_{m,n+1} - A_{mn} + p_{mn} A_{m-r,n-l} = 0, \\ \quad m \geqslant m_0, n \geqslant n_0 - 1, m \neq m_k, \\ A_{m_k+1,n} + A_{m_k,n+1} - A_{m_k,n} = b_k A_{m_k,n}, \\ \quad \forall n \geqslant n_0 - 1, k \in N(1), \end{cases} \tag{1}$$

where $\{p_{mn}\}$ is a double sequence and $p_{mn} \geqslant 0$ and not identically zero, for $m \geqslant m_0, n \geqslant n_0 - 1$, $\{b_k\}$ is a real sequence, r, l are positive integers, $0 \leqslant m_0 \leqslant m_1 < \cdots < m_k < \cdots$ with $\lim_{k\to\infty} m_k = \infty$. For any $m_0, n_0 \geqslant 0$, let $\Omega_{m_0,n_0} = \{h: K_0 \to R\}$, where

$$K_0 = \{(m,n) \mid m \geqslant m_0 - r, n \geqslant n_0 - (l+1)\} \setminus \{(m,n) \mid m \geqslant m_0, n \geqslant n_0 - 1\}.$$

1 Preliminaries

Definition 1 For given $m_0 \geqslant 0, n_0 \geqslant 0$ and $h \in \Omega_{m_0,n_0}$, a double sequence $\{A_{mn}\}$ is said to be a solution of equation (1) satisfying the initial condition

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$$A_{mn} = h_{mn}, (m,n) \in K_0, \quad (2)$$

if $\{A_{mn}\}$ is defined on $N(m_0 - r) \times N(n_0 - (l+1))$ and satisfies equation(1) and formulae(2).

For given $m_0 \geqslant 0, n_0 \geqslant 0$ and $h \in \Omega_{m_0,n_0}$, by means of method of steps, the solution of equation(1) exists and unique.

Definition 2 A solution of equation(1) is said to be oscillatory if it is neither eventually positive nor eventually negative.

When $\{m_k\} = \emptyset$, *i.e.* $\{m_k\}$ is an empty set, equation (1) reduces to the partial difference equation

$$A_{m+1,n} + A_{m,n+1} - A_{mn} + p_{mn} A_{m-r,n-l} = 0, m \geqslant m_0, n \geqslant n_0 - 1. \quad (3)$$

If there is a sequence $\{m_{k_i}\}$ of positive integers such that $m_{k_i} \to \infty$ when $i \to \infty$, and $b_{m_{k_i}} \leqslant -1$, then, it is easy to see that every solution of equation(1) is oscillatory. Therefore, we always assume that $b_k > -1$ for any $k \in N(1)$.

2 Main results

The following theorem 1 provides a sufficient condition for oscillation of all solutions of equation (1).

Theorem 1 Assume that

(i) $$\lim_{m,n\to\infty} \sup [(1+b_s)^{-l}_{m=m_s \in \{m_k\}} \prod_{m_k \in N(m-r,m-1)} (1+b_k)^{-1}] < \infty, \quad (4)$$

(ii) $$\lim_{m,n\to\infty} \inf [\sum_{\substack{i \in N(m-r,m-1) \\ i \notin \{m_k\}}} p_{i,n-l} + \sum_{\substack{j \in N(n-l,n-1) \\ i \notin \{m_k\}}} p_{m,j}] \times [(1+b_s)^{-l}_{m=m_s \in \{m_k\}} \prod_{m_k \in N(m-r,m-1)} (1+b_k)^{-1}] > \frac{(r+l)^{r+l+1}}{(r+l+1)^{r+l+1}}. \quad (5)$$

Then every solution of equation(1) is oscillatory.

Proof Suppose on the contrary, there is a non-oscillatory solution $\{A_{mn}\}$ of equation (1) which is eventually positive. Without loss of generality, we assume that $A_{mn} > 0$ for $m \geqslant m_0^1 - r, n \geqslant n_0^1 - (l+1)$. Let

$$w_{mn} = \frac{A_{m-r,n-l}}{A_{mn}}, m \geqslant m_0^1, n \geqslant n_0^1 - 1. \quad (6)$$

By equation(1), we have

$$\frac{A_{m+1,n} + A_{m,n+1}}{A_{mn}} = 1 - p_{mn} w_{mn}, m \neq m_k,$$

and $$\frac{A_{m_k+1,n} + A_{m_k,n+1}}{A_{m_k n}} = 1 + b_k,$$ so

$$\frac{A_{mn}}{A_{m+1,n}} \geqslant [1 - p_{mn} w_{mn}]^{-1}, m \neq m_k, \quad (7)$$

$$\frac{A_{mn}}{A_{m,n+1}} \geqslant [1 - p_{mn} w_{mn}]^{-1}, m \neq m_k, \quad (8)$$

and

$$\frac{A_{m_k,n}}{A_{m_k+1,n}} \geqslant (1+b_k)^{-1}, \frac{A_{m_k,n}}{A_{m_k,n+1}} \geqslant (1+b_k)^{-1}. \quad (9)$$

By formulae(6)~(9), we have

$$w_{mn} = \frac{A_{m-r,n-l}}{A_{m-r+1,n-l}} \frac{A_{m-r+1,n-l}}{A_{m-r+2,n-l}} \cdots\cdots \frac{A_{m,n-l}}{A_{m,n-l+1}} \cdot \frac{A_{m,n-l+1}}{A_{m,n-l+2}} \cdots \frac{A_{m,n-1}}{A_{mn}} \geqslant \prod_{\substack{i \in N(m-r,m-1) \\ i \notin \{m_k\}}} (1 - p_{i,n-l} w_{i,n-l})^{-1} \times \prod_{\substack{j \in N(n-l,n-1) \\ m \notin \{m_k\}}} (1 - p_{mj} w_{mj})^{-1} (1+b_s)^{-l}_{m=m_s \in \{m_k\}} \prod_{m_k \in N(m-r,m-1)} (1+b_k)^{-1}.$$

By employing arithemetic mean-geometric mean inequality, we get

$$w_{mn} \geqslant \{1 - \frac{1}{r+l} [\sum_{\substack{i \in N(m-r,m-1) \\ i \notin \{m_k\}}} p_{i,n-l} w_{i,n-l} + \sum_{\substack{j \in N(n-l,n-1) \\ i \notin \{m_k\}}} p_{m,j} w_{m,j}]^{-r-l}\} (1+b_s)^{-l}_{m=m_s \in \{m_k\}} \cdot \prod_{m_k \in N(m-r,m-1)} (1+b_k)^{-1}.$$

By equation (1), we get

$$0 \leqslant \sum_{\substack{i \in N(m-r,m-1) \\ i \notin \{m_k\}}} p_{i,n-l} w_{i,n-l} + \sum_{\substack{j \in N(n-l,n-1) \\ i \notin \{m_k\}}} p_{m,j} w_{m,j} < r + l.$$

Using the inequality[5] $(1 - \frac{c}{r+l})^{-r-l} \geqslant \frac{(r+l+1)^{r+l+1}}{(r+l)^{r+l+1}} c \quad (0 \leqslant c < r+l)$, we have

$$w_{mn} \geqslant \frac{(r+l+1)^{r+l+1}}{(r+l)^{r+l+1}} [\sum_{\substack{i \in N(m-r,m-1) \\ i \notin \{m_k\}}} p_{i,n-l} \cdot w_{i,n-l} + \sum_{\substack{j \in N(n-l,n-1) \\ i \notin \{m_k\}}} p_{m,j} w_{m,j}] (1+b_s)^{-l}_{m=m_s \in \{m_k\}} \cdot \prod_{m_k \in N(m-r,m-1)} (1+b_k)^{-1} \geqslant \frac{(r+l+1)^{r+l+1}}{(r+l)^{r+l+1}} \cdot [\sum_{\substack{i \in N(m-r,m-1) \\ i \notin \{m_k\}}} p_{i,n-l} + \sum_{\substack{j \in N(n-l,n-1) \\ i \notin \{m_k\}}} p_{m,j}] \cdot (1+b_s)^{-l}_{m=m_s \in \{m_k\}} \prod_{m_k \in N(m-r,m-1)} (1+b_k)^{-1} \times$$

$\{\min\{w_{i,j} | (i,j) \in N(m-r,m) \times N(n-l,n-$

$1)\}\}$.

By formulae(5), we can choose constants $\theta, M_0, N_0 > 0$ such that

$$\frac{(r+l+1)^{r+l+1}}{(r+l)^{r+l+1}}\Big[\sum_{\substack{i\in N(m-r,m-1)\\ i\notin\{m_k\}}} p_{i,n-l} + \sum_{\substack{j\in N(n-l,n-1)\\ i\notin\{m_k\}}} p_{m,j}\Big]\times(1+b_s)^{-l}_{m=m_s\in\{m_k\}}\cdot$$

$$\prod_{m_k\in N(m-r,m-1)}(1+b_k)^{-1} > \theta > 1, m > M_0, n > N_0.$$

Thus

$$w_{mn}\geqslant\theta\min\{w_{i,j}|(i,j)\in N(m-r,m)\times N(n-l,n-1)\}, m > M_0, n > N_0. \quad (10)$$

Let $\lim\limits_{m,n\to\infty}\inf w_{mn}=\lambda_0$. By formulae(4) and formulae (5), we obtain

$$\lim_{m,n\to\infty}\inf\Big[\sum_{\substack{i\in N(m-r,m-1)\\ i\notin\{m_k\}}} p_{i,n-l} + \sum_{\substack{j\in N(n-l,n-1)\\ m\notin\{m_k\}}} p_{m,j}\Big] > 0.$$

Then we can choose constants $a, M_1, N_1 > 0$ such that

$$\sum_{\substack{i\in N(m-r,m-1)\\ i\notin\{m_k\}}} p_{i,n-l} + \sum_{\substack{j\in N(n-l,n-1)\\ m\notin\{m_k\}}} p_{m,j}\geqslant a > 0$$

for $m > M_1, n > N_1$.

Thus, for any $m > M_1, n > N_1$, there are positive integers m^*, n^* such that

$$\frac{a}{r+l}\leqslant p_{m^*,n-l} = -\frac{A_{m^*+1,n-l}+A_{m^*+1,n-l+1}}{A_{m^*-r,n-2l}} + \frac{A_{m^*,n-l}}{A_{m^*-r,n-2l}}\leqslant\frac{A_{m^*,n-l}}{A_{m^*-r,n-2l}} = w^{-1}_{m^*,n-l}$$

or

$$\frac{a}{r+l}\leqslant p_{m,n^*} = -\frac{A_{m+1,n^*}+A_{m+1,n^*+1}}{A_{m-r,n^*-l}} + \frac{A_{m,n^*}}{A_{m-r,n^*-l}}\leqslant\frac{A_{m,n^*}}{A_{m-r,n^*-l}} = w^{-1}_{m,n^*}.$$

So we have $\lambda_0 < \infty$. We will show $\lambda_0 > 0$. If it is on the contrary, we set $\lim\limits_{m,n\to\infty}\inf w_{mn} = 0$, there are positive integers sequence $\{s_k\},\{t_k\}, s_k < s_{k+1}, t_k < t_{k+1}$, $s_k, t_k\to\infty$, $k\to\infty$,

$$w_{s_k,t_k} = \min\{w_{mn}|(m,n)\in N(m_0,s_k)\times N(n_0-1,t_k)\}.$$

By formulae(10), we get $w_{s_k,t_k}\geqslant\theta w_{s_k,t_k}$. This is a contradiction, so $0 < \lambda_0 < \infty$. By $\lim\limits_{m,n\to\infty}\inf w_{mn} = \lambda_0$, for every real number $Z(0 < Z < 1)$, there are $M, N > 0$ such that $w_{mn}\geqslant Z\lambda_0$, for $m > M, n > N$. By formulae(10), we have $w_{mn}\geqslant\theta Z\lambda_0$, $m > \max\{M_0, M+r\}$, $n > \max\{N_0, N+l\}$. Therefore, we have $\lim\limits_{m,n\to\infty}\inf w_{mn} = \theta Z\lambda_0$. Let $Z\to 1$, we obtain $\lambda_0\geqslant\theta\lambda_0$.

This is a contradiction, then we complete the proof.

Corollary 1 Assume that

(i) $m_{k+1}-m_k\geqslant T$, $rl < T$, $0\leqslant b_k\leqslant L$, $k=1,2,\cdots$,

(ii) $\lim\limits_{mn\to\infty}\inf\frac{(r+l+1)^{r+l+1}}{(r+l)^{r+l+1}}p_{mn} > 1+L$.

Then every solution of equation(1) is oscillatory.

Corollary 2 Assume that

(i) $m_{k+1}-m_k\geqslant T$, $rl < T$, $b_k\geqslant 0$, where $k=1,2,\cdots$, $\lim\limits_{k\to\infty}b_k = 0$, $p(x,y)\equiv p$,

(ii) $p\frac{(r+l+1)^{r+l+1}}{(r+l)^{r+l+1}} > 1$.

Then every solution of equation(1) is oscillatory.

Example Consider the oscillation of the equation

$$\begin{cases} A_{m+1,n}+A_{m,n+1}-A_{mn}+\frac{3^5}{4^4}A_{m-1,n-2}=0, \\ \quad m\geqslant m_0, n\geqslant n_0-1, m\neq 3k, \\ A_{3k+1,n}+A_{3k,n+1}-A_{3k,n}=\frac{1}{2}A_{3k,n}, \\ \quad\forall n\in[n_0-1,\infty), k\in N(1). \end{cases}$$

By corollary 1, we know that every solution of this equation is oscillatory.

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