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两种群时滞脉冲竞争型系统的概周期解*

Existence of Almost Periodic Solutions of a Two Dimensional Delay Impulsive Systems

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摘要:应用不动点定理,给出两种群时滞脉冲 Lotka-Volterra竞争型系统存在唯一概周期解的 1组充分条件.

关键词:时滞系统 概周期解 脉冲 不动点定理

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Abstract By using the fixed point theorem, a set of sufficient conditions to the existence of almost periodic solutions of a two dimensional delay Lotka-Volterra competition system with impulsive is given.

Key words delay system, almost periodic solution, impulsive, fixed point theorem

1993年,文献[1]提出一个公开问题,即考虑时滞系统

$$\begin{cases} x_1'(t) = x_1(t)[r_1 - a_1x_1(t-\tau) - b_1x_2(t)], \\ x_2'(t) = x_2(t)[r_2 - a_2x_1(t) - b_2x_2(t)]. \end{cases} \tag{1}$$

的周期解,其中 $r_1, r_2, a_1, a_2, b_1, b_2$ 都是正常数.这类模型描述生态学中两个种群之间的动力学行为.从那时起,许多文献开始研究形如(1)式的解的各种性态,诸如稳定性,振动性,渐近性,周期性和概周期性等[2~8].如文献[9]所言,在实际的生态系统中考虑时滞的生态系统应更符合实际.同样,在生态系统中引入脉冲效应也具有实际意义[10].本文研究时滞且含脉冲效应的两种群 Lotka-Volterra竞争型系统:

$$\begin{cases} x_1'(t) = x_1(t)[r_1 - a_1x_1(t) - b_1x_2(t-\tau)], \\ x_2'(t) = x_2(t)[r_2 - a_2x_1(t-\tau) - b_2x_2(t)], \\ \qquad t \neq t_k; \\ x_1(t_k^+) = (1+\lambda_k)x_1(t_k), \\ x_2(t_k^+) = (1+\mu_k)x_2(t_k), \end{cases} \tag{2}$$

其中 $r_1, r_2, a_1, a_2, b_1, b_2$ 都是正常数,时滞 $\tau > 0$ 为常数; t_k 是 x_1, x_2 的第一类间断点,即 $x_i(t_k) = x_i(t_k^-)$ ($i = 1, 2$);脉冲效应时刻 $t_k < t_{k+1}$ ($k \in Z$), t_k 是 T-周期的,即 $t_{k+q} = t_k + T$, 且有 $\lim_{k\to\pm\infty} t_k = \pm\infty$.当设 $\lambda_k \neq -1, \mu_k \neq -1$,并且 λ_k, μ_k,都是概周期的,即对任意给定的 $\varepsilon > 0$,存在正整数 q 使得 $|\lambda_{k+q} - \lambda_k| < \varepsilon, |\mu_{k+q} - \mu_k| < \varepsilon$ 时,运用不动点定理,在适当的假设条件下,得到系统(2)存在概周期解的 1组充分条件.

1 预备知识

对系统(2)作变换 $x_i = \frac{1}{y_i}$ ($i = 1, 2$),则系统(2)等价于系统:

$$\begin{cases} y_1'(t) = -r_1y_1(t) + a_1 + b_1\dfrac{y_1(t)}{y_2(t-\tau)}, \\ y_2'(t) = -r_2y_2(t) + a_2\dfrac{y_2(t)}{y_1(t-\tau)} + b_2, t \neq t_k; \\ y_1(t_k^+) = \dfrac{1}{1+\lambda_k}y_1(t_k), \\ y_2(t_k^+) = \dfrac{1}{1+\mu_k}y_2(t_k). \end{cases} \tag{3}$$

显然,如果系统(3)具有概周期解,则系统(2)具有定正概周期解,所以只需要考虑系统(3)概周期解的存在性.

引理 1[11] 对线性系统

$$\begin{cases} x'(t) = A(t)x(t), t \neq t_k, \\ x(t_k^+) - x(t_k^-) = B_kx(t_k^-). \end{cases} \tag{4}$$

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以及

$$\begin{cases} x'(t) = A(t)x(t) + g(t), t \neq \tau_k, \\ x(\tau_k^+) - x(\tau_k^-) = B_k x(\tau_k) + h_k. \end{cases} \quad (5)$$

如果 τ_k, h_k 是概周期序列, $A(t)$ 是连续的概周期函数矩阵, B_k 是概周期序列矩阵, 并且 $\det(E + B_k) \neq 0$, $|W(t,s)| \leqslant Ke^{-\alpha(t-s)}$, $t \geqslant s$, $t, s \in R$, 其中 $W(t,s)$ 是系统(4)的柯西矩阵, 则系统(5)存在唯一概周期解

$$\varphi(t) = \int_{-\infty}^{t} W(t,s)g(s)\mathrm{d}s + \sum_{\tau_k < t} W(t,\tau_k^+)h_k.$$

2 主要结果

定理 1 假设条件:

(Ⅰ) λ_k, μ_k 都是概周期序列, 对任意 k 有 $1 + \lambda_k > 0$, $1 + \mu_k > 0$, 并且对任意 $t \in R$, 存在正常数 M 使得对任意 $\tau_k < t$ 有 $\frac{15M}{17} \leqslant \prod_{\tau_k < t} \frac{1}{1 + \lambda_k} \leqslant M$, $\frac{15M}{17} \leqslant \prod_{\tau_k < t} \frac{1}{1 + \mu_k} \leqslant M$.

(Ⅱ) $r_1, r_2, a_1, a_2, b_1, b_2$ 为正常数, 满足条件 $\frac{a_1}{5} \leqslant b_1 \leqslant \frac{a_1}{4}$, $\frac{b_2}{5} \leqslant a_2 \leqslant \frac{b_2}{4}$, $b_2 r_1 = a_1 r_2$

则系统(3)存在唯一概周期解.

证明 如果记 $y = (y_1, y_2)^{\mathrm{T}}$, $A = \begin{pmatrix} -r_1 & 0 \\ 0 & -r_2 \end{pmatrix}$, $B_k = \begin{pmatrix} \frac{-\lambda_k}{1+\lambda_k} & 0 \\ 0 & \frac{-\mu_k}{1+\mu_k} \end{pmatrix}$, 则(3)式对应的齐线性系统可以简记为

$$\begin{cases} y'(t) = Ay(t), t \neq \tau_k, \\ y(\tau_k^+) - y(\tau_k) = B_k y(\tau_k). \end{cases} \quad (6)$$

由刘维尔公式知(6)式的柯西矩阵 $W(t,s)$ 满足

$$\det W(t,s) = \prod_{s < \tau_k < t} \det(E + B_k)\exp\left(\int_s^t \mathrm{Tr}A(r)\mathrm{d}r\right),$$

其中 $\det W$ 表示 W 的行列式, $\mathrm{Tr}A$ 表示矩阵 A 的迹. 既然 $1 + \lambda_k > 0$, $1 + \mu_k > 0$, 如果记 $\alpha = \min\{r_1, r_2\}$, $\beta = \min\{\inf_{k \in Z}\{1 + \lambda_k\}, \inf_{k \in Z}\{1 + \mu_k\}\}$, 再令 $K = \frac{1}{\beta^2}$, 则

$$|W(t,s)| \leqslant Ke^{-\alpha(t-s)}. \quad (7)$$

假设 $X = \{\varphi; \varphi = (\varphi_1, \varphi_2)^{\mathrm{T}}$, φ_1, φ_2 是 R 到 R, 除了脉冲点以外连续的概周期函数, 满足 $\frac{Ma_1}{r_1} \leqslant \varphi_1 \leqslant \frac{3Ma_1}{2r_1}$, $\frac{Mb_2}{r_2} \leqslant \varphi_2 \leqslant \frac{3Mb_2}{2r_2}\}$. φ_i 的范数定义为 $\|\varphi_i\| = \sup_{t \in R}|\varphi_i(t)|$ $(i = 1, 2)$. 于是 $\|(\varphi_1, \varphi_2)\| = \max\{\|\varphi_1\|, \|\varphi_2\|\}$, 在此范数下 X 构成一个 Banach 空间. 任取 $\psi = (\psi_1, \psi_2)^{\mathrm{T}} \in X$, 并记 $G(\psi(t)) = \begin{pmatrix} a_1 + b_1 \frac{\psi_1(t)}{\psi_2(t - \tau)} \\ a_2 \frac{\psi_2(t)}{\psi_1(t - \tau)} + b_2 \end{pmatrix}$.

由引理 1 的结论, 系统

$$\begin{cases} y'(t) = Ay(t) + G(\psi(t)), t \neq \tau_k, \\ y(\tau_k^+) - y(\tau_k) = B_k y(\tau_k) \end{cases} \quad (8)$$

存在唯一概周期解

$$y_\psi(t) = \int_{-\infty}^{t} W(t,s)G(\psi(s))\mathrm{d}s, \quad (9)$$

其中 $W_{11}(t,s) = \prod_{s < \tau_k < t} \frac{1}{1 + \lambda_k} e^{-r_1(t-s)}$, $W_{22}(t,s) = \prod_{s < \tau_k < t} \frac{1}{1 + \mu_k} e^{-r_2(t-s)}$, $W_{12}(t,s) = W_{21}(t,s) = 0$. 定义映射

$$Ty_\psi(t) = \begin{pmatrix} \int_{-\infty}^{t} \prod_{s < \tau_k < t} \frac{1}{1 + \lambda_k} e^{-r_1(t-s)} \left[a_1 + b_1 \frac{\psi_1(s)}{\psi_2(s - \tau)}\right]\mathrm{d}s \\ \int_{-\infty}^{t} \prod_{s < \tau_k < t} \frac{1}{1 + \mu_k} e^{-r_2(t-s)} \left[a_2 \frac{\psi_2(s)}{\psi_1(s - \tau)} + b_2\right]\mathrm{d}s \end{pmatrix}. \quad (10)$$

然后证明 T 是 X 到自身的压缩映射.

(ⅰ) 证明 $TX \subset X$. 对任意的 $\psi = (\psi_1, \psi_2)^{\mathrm{T}} \in X$, 由范数的定义并注意到 $\sup_{t \in R}|\psi_i(t)| = \sup_{t \in R}|\psi_i(t - \tau)|$ $(i = 1, 2)$, 又由于 $\psi \in X$ 定正, 于是有

$$Ty_{\psi_1}(t) = \int_{-\infty}^{t} \prod_{s < \tau_k < t} \frac{1}{1 + \lambda_k} e^{-r_1(t-s)} \left[a_1 + b_1 \frac{\psi_1(s)}{\psi_2(s - \tau)}\right]\mathrm{d}s \leqslant M\int_{-\infty}^{t} e^{-r_1(t-s)} \left[a_1 + \frac{a_1}{4} \cdot \frac{\psi_1(s)}{\psi_2(s - \tau)}\right]\mathrm{d}s \leqslant M\int_{-\infty}^{t} e^{-r_1(t-s)} \left[a_1 + \frac{a_1}{4} \cdot \frac{3a_1 r_2}{2b_2 r_1}\right]\mathrm{d}s = \frac{11Ma_1}{8r_1} < \frac{3Ma_1}{2r_1}. \quad (11)$$

同理有

$$Ty_{\psi_2}(t) \leqslant \frac{11Mb_2}{8r_2} < \frac{3Mb_2}{2r_2}. \quad (12)$$

由于 $\psi \in X$ 定正, 于是又有

$$Ty_{\psi_1}(t) \geqslant \int_{-\infty}^{t} \prod_{s < \tau_k < t} \frac{1}{1 + \lambda_k} e^{-r_1(t-s)} \left[a_1 + \frac{a_1}{5} \cdot \frac{\psi_1(s)}{\psi_2(s - \tau)}\right]\mathrm{d}s \geqslant \frac{15M}{17}\int_{-\infty}^{t} e^{-r_1(t-s)} \left[a_1 + \frac{a_1}{5} \cdot \frac{2a_1 r_2}{3b_2 r_1}\right]\mathrm{d}s = \frac{Ma_1}{r_1}. \quad (13)$$

同理有

$$Ty_{\psi_2}(t) \geqslant \frac{Mb_2}{r_2}. \quad (14)$$

结合(11)~(14)式可知 $TX \subset X$.

(ⅱ) 证明 T 是 X 上的压缩映射.

任意取 $\varphi = (\varphi_1, \varphi_2)^{\mathrm{T}}$, $\phi = (\phi_1, \phi_2)^{\mathrm{T}} \in X$, 其中 $\frac{Ma_1}{r_1} \leqslant \varphi_1 \leqslant \frac{3Ma_1}{2r_1}$, $\frac{Mb_2}{r_2} \leqslant \varphi_2 \leqslant \frac{3Mb_2}{2r_2}$, $\frac{Ma_1}{r_1} \leqslant \phi_1 \leqslant$

$\frac{3Ma_1}{2r_1}$, $\frac{Mb_2}{r_2} \leqslant j_2 \leqslant \frac{3Mb_2}{2r_2}$, 注意到 $\sup_{\in R} | h_i(t) - j_i(t) | = \sup_{\in R} | h_i(t - f) - j_i(t - f) |$ $(i = 1, 2)$,于是有

$$| Th_1 - Tj_1 | \leqslant M\int_{-\infty}^{t} e^{-r_1(t-s)} (b_1 | \frac{h_1(s)}{h_2(s - f)} - \frac{j_1(s)}{j_2(s - f)} |) ds \leqslant \frac{Ma_1}{4}\int_{-\infty}^{t} e^{-r_1(t-s)} [j_2(s - f) | h_1(s) - j_1(s) | + j_1(s) | h_2(s - f) - j_2(s - f) |] / [h_2(s - f) j_2(s - f)] ds \leqslant \frac{Ma_1}{4r_1} (\frac{r_2}{Mb_2} \| h_1 - j_1 \| + \frac{3a_1 r_2^2}{2Mb_2^2 r_1} \cdot \| h_2 - j_2 \|) = \frac{1}{4} \| h_1 - j_1 \| + \frac{3}{8} \| h_2 - j_2 \|. \qquad (15)$$

同理有

$$| Th_2 - Tj_2 | \leqslant M\int_{-\infty}^{t} e^{-r_2(t-s)} (a_2 | \frac{h_2(s)}{h_1(s - f)} - \frac{j_2(s)}{j_1(s - f)} |) ds \leqslant \frac{Mb_2}{4r_2} (\frac{3b_2 r_1^2}{2Ma_1^2 r_2} \| h_1 - j_1 \| + \frac{r_1}{Ma_1} \| h_2 - j_2 \|) = \frac{3}{8} \| h_1 - j_1 \| + \frac{1}{4} \| h_2 - j_2 \|. \qquad (16)$$

故有 $\| Th - Tj \| < \frac{3}{4} \| h - j \|$.因此 T是 X上的压缩映射 ,从而 T在 X上有唯一不动点 ,该不动点为系统 (3) 亦即系统 (2)的概周期解 ,并且满足$\frac{2r_1}{3Ma_1} \leqslant x \leqslant \frac{r_1}{Ma_1}$, $\frac{2r_2}{3Mb_2} \leqslant x_2 \leqslant \frac{r_2}{Mb_2}$.证明完毕 .

3 算例

对 $T = 2\pi$ 和自然数 k,任意给定 $X > 0$,当 k为偶数时取 $\lambda_k = \sin\frac{1}{8(k + \theta)^2}$, $\_{k} = \sin\frac{\sqrt{2}}{16(k + \theta)^2}$,当 k为奇数时取 $\lambda_k = -\sin\frac{1}{8(k + \theta)^2}$, $\_{k} = -\sin\frac{\sqrt{2}}{16(k + \theta)^2}$,其中 $\theta = [\frac{1}{X}]$.易见 λ_k, $\_{k}$ 是概周期序列 ,则定理 1的条件都满足 ,从而系统

$$\begin{cases} x_1'(t) = x_1(t) [1.6 - 0.8x_1(t) - \\ \quad 0.18x_2(t - 0.1)], \\ x_2'(t) = x_2(t) [3 - 0.35x_1(t - 0.1) - \\ \quad 1.5x_2(t)], t \neq kT; \\ x_1(kT^+) = (1 + \lambda_k) x_1(kT), \\ x_2(kT^+) = (1 + \_{k}) x_2(kT). \end{cases}$$

存在唯一概周期解 .

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