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Ruin Probabilities in the Extended Compound Markov Binomial Model*

广义复合马尔可夫二项模型的破产概率

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Abstract The compound Markov binomial model which was first proposed by Cossette et al. (2003) is extended to the case where the premium income process, based on a binomial process, is no longer a linear function and its ruin probability is investigated. Recursive formulas are provided for the computation of the ruin probabilities. The Lundberg exponential bound is derived for the ruin probability.

Key words compound binomial model, Markov binomial, ruin probability, Lundberg exponential

摘要:将复合马尔可夫二项模型推广为保费收取过程而得到广义复合马尔可夫二项模型并研究其破产概率.给出该模型破产概率的递推公式及其 Lundberg型指数的上界.

关键词:复合二项模型　Markov二项　破产概率　Lundberg型指数
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The compound binomial risk model has been studied by various authors, and some extensions have been made on this model recently[1～6]. Reference [7] and reference [8] study a compound binomial risk model with time-correlated claims, that is each claim causes a by-claim which may be delayed to the next time period. The compound Markov binomial model, an extension to the compound binomial model, was first proposed by cossette et al.[9] as a discrete-time model which introduces time-dependence in the claim occurrence process[10].

In this paper, we extend the compound Markov binomial model to case where the premium income process, based on a binomial process, is no longer a linear function. We define the surplus process of an insurance company by

$$U_k = u + M_k - S_k, u \in N, k \in N^+, \tag{0.1}$$

where $u = u_0$ corresponds to the initial surplus, premiums are payable at a rate of 1 per time unit, $S_k = Y_1 + Y_2 + \cdots + Y_k$ and Y_k is the eventual claim amount in period k ($k \in N^+$). We suppose that one claim can occur per period at most. The r.v. Y_k is then defined as

$$Y_k = \begin{cases} X_k, I_k = 1, \\ 0, I_k = 0, \end{cases} \tag{0.2}$$

where the occurrence r.v. I_k and the individual claim amount r.v. X_k are independent in each time period. The r.v. I_k submits to Bernoulli distribution with mean $q \in (0,1)$. $\{I_k, k \in N\}$ is a Markov chain with a two-state transition probability matrix:

$$P = \begin{pmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{pmatrix} = \begin{pmatrix} 1-(1-c)q & (1-c)q \\ (1-c)(1-q) & c+(1-c)q \end{pmatrix}, \tag{0.3}$$

where $P(I_{k+1} = j | I_k = i) = p_{ij}$ for $i, j \in \{0,1\}$ and $k \in N^+$, initial probabilities $P(I_0 = 1) = q = 1 - P(I_0 = 0)$,

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$q \in (0, 1)$ and c is the dependence parameter ($0 \leqslant c < 1$). We assume X_k is a strictly positive discrete r. v. $\{X_k, k \in N^+\}$ is a sequence of i. i. d. r. v.'s with probability mass function (p. m. f.) f, cumulative distribution function (c. d. f.) F, probability generating function (p. g. f.) $\hat{f}$ and mean μ.

$\{M_k, k \in N^+\}$ is a binomial process with parameter d, independent of $\{I_k, k \in N\}$ and $\{X_k, k \in N^+\}$. $M_k = Z_1 + Z_2 + \cdots + Z_k, k \in N^+$, where M_k is corresponding to the number of the customers up to time k. We denote by $Z_k = 1$ the event where a payment occurs in $(k-1, k]$ and $Z_k = 0$ the event where no payment occurs in period $(k-1, k]$. Let $P(Z_k = 1) = d$, $P(Z_k = 0) = 1 - d$ $(0 < d \leqslant 1)$.

Moreover, we assume that the r. v.'s I_k and X_k are defined such that $(1+\theta) q\mu = d$, where θ is the strictly positive relative risk margin.

In this paper, we derive recursive formulas of the infinite ruin probabilities and a Lundberg exponential bound for the ruin probability in the extended compound Markov binomial model (0. 1).

1 Definitions and notations

Let $T = \{k \in N^+; U_k < 0\}$ be the time of ruin. The conditional and the unconditional infinite-time ruin probabilities are denoted by $\psi(u|i)$ and $\psi(u)$, where

$$\psi(u|i) = P(T < \infty | I_0 = i), i = 0, 1, \qquad (1.1)$$

$$\psi(u) = P(T < \infty). \qquad (1.2)$$

Denoted $\varphi(u|i)$ and $\varphi(u)$ the conditional infinite-time non-ruin probabilities and infinite-time non-ruin probabilities respectively, where

$$\varphi(u|i) = 1 - \psi(u|i) = P(U_k \geqslant 0, \forall k \in N^+ | I_0 = i), \; i = 0, 1, \qquad (1.3)$$

$$\varphi(u) = 1 - \psi(u) = P(U_k \geqslant 0, \forall k \in N^+). \qquad (1.4)$$

Clearly, we have

$$\varphi(u) = (1-q)\varphi(u|0) + q\varphi(u|1). \qquad (1.5)$$

Define

$$E(e^{-r(M_1 - Y_1)} | I_0 = i) = (1-d) p_{i0} + d p_{i0} e^{-r} + (1-d) p_{i1} E e^{rX_1} + d p_{i1} E e^{-r(1-X_1)} = d E(e^{-r(1-Y_1)} | I_0 = i) + (1-d) E(e^{rY_1} | I_0 = i). \qquad (1.6)$$

Also, let R_0 and R_1 be strictly positive real numbers such that

$$E(e^{-r(M_1 - Y_1)} | I_0 = 0) = 1 \qquad (1.7)$$

and

$$E(e^{-r(M_1 - Y_1)} | I_0 = 1) = 1. \qquad (1.8)$$

The solutions R_0 and R_1 of formulae (1. 6) and formulae (1. 7) exist if $E(Y_1 - M_1 | I_0 = i) < 1, i = 0, 1$.

2 Main results

Theorem 2. 1 The conditional infinite-time non-ruin probabilities are given by

$$\varphi(u|0) = ((p_{01} + d p_{00})\varphi(u-1|0) - (1-d) p_{01} \cdot \sum_{k=1}^{u-1} \varphi(u-1-k|1) f(k) - d p_{01} \sum_{k=1}^{u} \varphi(u-k|1) f(k)) / d p_{00}, \text{ for } u = 1, 2, 3, \cdots; \qquad (2.1)$$

$$\varphi(u|1) = (p_{10}\varphi(u|0) + (1-d) c \sum_{k=1}^{u} \varphi(u-k|1) \cdot f(k) + dc \sum_{k=2}^{u+1} \varphi(u+1-k|1) f(k)) / (p_{00} - c d f(1)), \text{ for } u = 1, 2, 3, \cdots; \qquad (2.2)$$

$$\varphi(0|0) = \frac{d - q\mu}{d(1-q)}; \qquad (2.3)$$

$$\varphi(0|1) = \frac{p_{10}}{p_{00} - c d f(1)} \varphi(0|0). \qquad (2.4)$$

Proof We consider $\varphi(u|0)$ and $\varphi(u|1)$, in the first period $(0, 1]$ and separate the four possible cases as follows

(1) $A_1 = \{$no premium arrives in $(0, 1]$ and no claim occurs either$\}$;

(2) $A_2 = \{$a premium arrives in $(0, 1]$ but no claim occurs$\}$;

(3) $A_3 = \{$no premium arrives in $(0, 1]$ but a claim occurs$\}$;

(4) $A_4 = \{$a premium arrives in $(0, 1]$ and a claim occurs too$\}$.

According to the laws of conditional probability, the conditional infinite-time non-ruin probabilities is equal to

$$\varphi(u|i) = \sum_{k=1}^{4} \varphi(u|i, A_i) P(A_i|i), i = 0, 1. \qquad (2.5)$$

So we have

$$\varphi(j|0) = (1-d) p_{00}\varphi(j|0) + d p_{00}\varphi(j+1|0) + (1-d) p_{01} \sum_{k=1}^{j} \varphi(j-k|1) f(k) + d p_{01} \sum_{k=1}^{j+1} \varphi(j+1-k|1) f(k), \qquad (2.6)$$

$$\varphi(j|1) = (1-d) p_{10}\varphi(j|0) + d p_{10}\varphi(j+1|0) + (1-d) p_{11} \sum_{k=1}^{j} \varphi(j-k|1) f(k) + d p_{11} \sum_{k=1}^{j+1} \varphi(j+1-k|1) f(k), j = 0, 1, 2, \cdots \qquad (2.7)$$

From which follows formulae (2. 1).

By substituting formulae (2. 6) in formulae (2. 7), we have

$$\varphi(j|1) = \frac{p_{10}}{p_{00}} [\varphi(j|0) - (1-d) p_{01} \sum_{k=1}^{j} \varphi(j-k|1) \cdot f(k) - d p_{01} \sum_{k=1}^{j+1} \varphi(j+1-k|1) f(k)] + (1-d) p_{11} \cdot$$

$$\sum_{k=1}^{j}\Phi(j-k|1)f(k)+dp_{11}\sum_{k=1}^{j+1}\Phi(j+1-k|1)f(k). \qquad (2.8)$$

Which leads to the recursive equation (2.2).

To prove formulae (2.3), we sum for each equality (2.6) and (2.7) which result in

$$\sum_{j=0}^{u-1}\Phi(j|0)=(1-d)p_{00}\sum_{j=0}^{u-1}\Phi(j|0)+dp_{00}\sum_{j=0}^{u-1}\Phi(j+1|0)+(1-d)p_{01}\sum_{j=0}^{u-1}\sum_{k=1}^{j}\Phi(j-k|1)f(k)+dp_{01}\sum_{j=0}^{u-1}\sum_{k=1}^{j+1}\Phi(j+1-k|1)f(k) \qquad (2.9)$$

and

$$\sum_{j=0}^{u-1}\Phi(j|1)=(1-d)p_{10}\sum_{j=0}^{u-1}\Phi(j|0)+dp_{10}\sum_{j=0}^{u-1}\Phi(j+1|0)+(1-d)p_{11}\sum_{j=0}^{u-1}\sum_{k=1}^{j}\Phi(j-k|1)f(k)+dp_{11}\sum_{j=0}^{u-1}\sum_{k=1}^{j+1}\Phi(j+1-k|1)f(k). \qquad (2.10)$$

Since $p_{00}=1-p_{01}$, $p_{11}=1-p_{10}$, we rewrite formulae(2.9) and formulae(2.10) as follows:

$$(1-d)\sum_{j=0}^{u-1}\Phi(j|0)+d\sum_{j=0}^{u-1}\Phi(j+1|0)-\sum_{j=0}^{u-1}\Phi(j|0)=p_{01}\Big[(1-d)\sum_{j=0}^{u-1}\Phi(j|0)+d\sum_{j=0}^{u-1}\Phi(j+1|0)-(1-d)\sum_{j=0}^{u-1}\sum_{k=1}^{j}\Phi(j-k|1)f(k)-d\sum_{j=0}^{u-1}\sum_{k=1}^{j+1}\Phi(j+1-k|1)f(k)\Big] \qquad (2.11)$$

and

$$\sum_{j=0}^{u-1}\Phi(j|1)-(1-d)\sum_{j=0}^{u-1}\sum_{k=1}^{j}\Phi(j-k|1)\cdot f(k)-d\sum_{j=0}^{u-1}\sum_{k=1}^{j+1}\Phi(j+1-k|1)f(k)=p_{10}\Big[(1-d)\sum_{j=0}^{u-1}\Phi(j|0)+d\sum_{j=0}^{u-1}\Phi(j+1|0)-(1-d)\sum_{j=0}^{u-1}\sum_{k=1}^{j}\Phi(j-k|1)f(k)-d\sum_{j=0}^{u-1}\sum_{k=1}^{j+1}\Phi(j+1-k|1)f(k)\Big]. \qquad (2.12)$$

Combining formulae(2.11) and formulae(2.12), we have

$$d\sum_{j=0}^{u-1}\Phi(j+1|0)-\sum_{j=0}^{u-1}\Phi(j|0)=\frac{p_{01}}{p_{10}}\Big[\sum_{j=0}^{u-1}\Phi(j|1)-(1-d)\sum_{j=0}^{u-2}\Phi(j|1)F(u-1-j)-d\sum_{j=0}^{u-1}\Phi(j|1)F(u-j)\Big], \qquad (2.13)$$

which becomes

$$\Phi(u|0)=\Phi(0|0)+\frac{p_{01}}{dp_{10}}\Big[(1-d)\sum_{j=0}^{u-1}\Phi(j|1)(1-F(u-1-j))+d\sum_{j=0}^{u-1}\Phi(j|1)F(u-j)\Big]. \qquad (2.14)$$

Since $\Phi(\infty|i)=\lim_{u\to\infty}\Phi(u|i)$, $i=0,1$ and $\Phi(\infty|0)=\Phi(\infty|1)=1$, then we take $u\to\infty$ in formulae(2.14) and find

$$\Phi(0|0)=1-\frac{p_{01}}{p_{10}}\frac{EX-d}{d}=\frac{d-qEX}{d(1-q)}, \qquad (2.15)$$

which corresponds to the formulae (2.3).

We derive formulae (2.4) by combining both formulae(2.5) at $j=0$ and formulae(2.6) at $j=0$.

$$\Phi(0|0)=(1-d)p_{00}\Phi(0|0)+dp_{00}\Phi(1|0)+dp_{01}\Phi(1|1)f(1), \qquad (2.16)$$

and

$$\Phi(0|1)=(1-d)p_{10}\Phi(0|0)+dp_{10}\Phi(1|0)+dp_{11}\Phi(1|1)f(1), \qquad (2.17)$$

that is formulae(2.4).

Remark:

$$\Phi(0)=(1-q)\Phi(0|0)+q\Phi(0|1)=\frac{(1-cdf(1))(d-qEX)}{d(1-cdf(1)-(1-c)q)}. \qquad (2.18)$$

From transition probability matrix, one can see that the c parameter introduces a positive correlation among the claim occurrence r. v. $\{I_k, k\in N^+\}$. We show in the following theorem that $\Phi(u)$ $(u\in N)$ decrease as the dependence parameter c increases resulting in the increase of $\Phi(u)$ $(u\in N)$ with c.

Theorem 2.2 The nonconditional infinite-time non-ruin probabilities $\Phi(u)$ $(u\in N)$ decrease when c increases.

Proof we prove $(d/dc)\Phi(u)\leqslant 0$ for $u\in N$ by induction.

Since the infinite-time non-ruin probability can be written as

$$\Phi(u)=(1-q)\Phi(u|0)+q\Phi(u|1). \qquad (2.19)$$

We prove that the derivative of both $\Phi(u|0)$ and $\Phi(u|1)$ $(u\in N)$ with respect to c are smaller than or equal to 0.

First, we demonstrate

$$(d/dc)\Phi(u|i)\leqslant 0, \text{ for } i=0,1. \qquad (2.20)$$

By formulae(2.3) and formulae(2.4), we have

$$(d/dc)\Phi(0|0)\leqslant 0, \qquad (2.21)$$

and

$$(d/dc)\Phi(0|1)=(d/dc)\Big(\frac{p_{10}}{p_{00}-cdf(1)}\Phi(0|0)\Big)=\frac{-(1-q)(1-df(1))}{(p_{00}-cdf(1))^2}\Phi(0|0)\leqslant 0. \qquad (2.22)$$

which imply $(d/dc)\Phi(0)\leqslant 0$. Now, we assume that

$$(d/dc)\Phi(j|0)\leqslant 0, \qquad (2.23)$$

and

$$(d/dc)\Phi(j|1)\leqslant 0 \qquad (2.24)$$

are verified for $j=1,\cdots,u-1$ and we prove that these inequalities also hold for $j = u$.

From formulae(2.14), one can find

$$\frac{d}{dc}\psi(u|0) = \frac{d}{dc}\{\psi(0|0) + \frac{p_{01}}{dp_{10}}[(1-d)\sum_{j=0}^{u-1}\psi(j|1)(1-F(u-1-j)) + d\sum_{j=0}^{u-1}\psi(j|1)F(u-j)]\},$$

and given formulae(2.24), concludes that

$$(d/dc)\psi(u|0) \leqslant 0. \quad (2.25)$$

From equation(2.2), one can find

$$\frac{d}{dc}\psi(u|1) = \frac{d}{dc}[(p_{10}\psi(u|0) + (1-d)c\sum_{k=1}^{u}\psi(u-k|1)f(k) + dc\sum_{k=2}^{u+1}\psi(u+1-k|1)f(k))/(p_{00} - cdf(1))] = \frac{d}{dc}[\frac{p_{10}}{p_{00}-cdf(1)}\psi(u|0) + \sum_{j=0}^{u-1}\frac{(1-d)cf(u-j) + dcf(u+1-j)}{p_{00}-cdf(1)}\psi(j|1)] \leqslant$$

$$[\psi(u|0)\frac{d}{dc}\frac{p_{10}}{p_{00}-cdf(1)} + \sum_{j=0}^{u-1}[(1-d)f(u-j) + df(u+1-j)]\psi(j|1)\frac{d}{dc}\frac{c}{p_{00}-cdf(1)}]. \quad (2.26)$$

Since $p_{00} - cdf(1) = (1-c)(1-q) + c(1-df(1)) \geqslant 0$.

The right-hand side of formulae(2.26) can also be written as

$$-\frac{(1-q)(1-df(1))}{(p_{00}-cdf(1))^2}\psi(u|0) + \sum_{j=0}^{u-1}[(1-d)\cdot f(u-j) + df(u+1-j)]\psi(j|1)\frac{1-q}{(p_{00}-cdf(1))^2},$$

which is equivalent to

$$\frac{1-q}{(p_{00}-cdf(1))^2}[-\psi(u|0)(1-df(1)) + \sum_{j=0}^{u-1}[(1-d)f(u-j) + df(u+1-j)]\psi(j|1)] = \frac{1-q}{c(p_{00}-cdf(1))^2}[-c\psi(u|0)(1-df(1)) + (p_{00} - cdf(1))\psi(u|1) - p_{10}\psi(u|0)]. \quad (2.27)$$

Replacing formulae(2.27) in formulae(2.26) and after further simplifications, we obtain

$$(d/dc)\psi(u|1) \leqslant \frac{1-q}{c(p_{00}-cdf(1))}\cdot[\psi(u|1) - \psi(u|0)]. \quad (2.28)$$

In order to compare $\psi(u|1)$ and $\psi(u|0)$, we rearrange formulae(2.6) and formulae(2.7) as follows:

$$p_{01}[(1-d)\sum_{k=1}^{u}\psi(u-k|1)f(k) + d\sum_{k=1}^{u+1}\psi(u+1-k|1)f(k) - (1-d)\psi(u|0) - d\psi(u+1|0)] = \psi(u|0) - (1-d)\psi(u|0) - d\psi(u+1|0) \quad (2.29)$$

and

$$p_{11}[(1-d)\sum_{k=1}^{u}\psi(u-k|1)f(k) + d\sum_{k=1}^{u+1}\psi(u+1-k|1)f(k) - (1-d)\psi(u|0) - d\psi(u+1|0)] = \psi(u|1) - (1-d)\psi(u|0) - d\psi(u+1|0). \quad (2.30)$$

The combination of equations formulae(2.29) and formulae(2.30) leads to

$$\frac{\psi(u|1) - (1-d)\psi(u|0) - d\psi(u+1|0)}{\psi(u|0) - (1-d)\psi(u|0) - d\psi(u+1|0)} = \frac{p_{11}}{p_{01}} = 1 + \frac{c}{(1-c)q},$$

from which we deduct

$$\psi(u|1) - \psi(u|0) = \frac{cd}{(1-c)q}(\psi(u|0) - \psi(u+1|0)) \geqslant 0.$$

So we obtain

$$(d/dc)\psi(u|1) \leqslant 0. \quad (2.31)$$

Given formulae(2.25) and formulae(2.31), we conclude that $\psi(u)$ $(u \in N^+)$ decrease as the dependence parameter c increases since

$$(d/dc)\psi(u) = (d/dc)[(1-q)\psi(u|0) + q\psi(u|1)] = (1-q)(d/dc)\psi(u|0) + q(d/dc)\psi(u|1) \leqslant 0. \quad (2.32)$$

We derive an exponential bound on the ruin probability within the extended compound Markov binomial model which is similar to the Lundberg exponential bound within the classical risk model.

Lemma 2.1 In the extended compound Markov binomial model(0.1), we have

$$R^* = \min(R_0, R_1) = R_1. \quad (2.33)$$

Proof Define

$$g_i(r) = E(e^{-r(M_1 - Y_1)}|I_0 = i), \text{ for } i = 0, 1 \quad (2.34)$$

The functions $g_0(r)$ and $g_1(r)$ are equal when $r = 0$.

$$g_0(0) = g_1(0) = 1. \quad (2.35)$$

Since $p_{01} \leqslant p_{11}$,

we have

$$g_0(r) = (1-r) + re^{-r} + p_{01}[(1-r)Ee^{rX_1} + rEe^{-r(1-X_1)} - (1-r) - re^{-r}] \leqslant (1-r) + re^{-r} + p_{11}[(1-r)Ee^{rX_1} + rEe^{-r(1-X_1)} - (1-r) - re^{-r}] = g_1(r), r \geqslant 0. \quad (2.36)$$

Furthermore, we have

$$g_0'(0) = E((Y_1 - M_1)|I_0 = 0) < 1 \quad (2.37)$$

and

$$g_1'(0) = E((Y_1 - M_1)|I_0 = 0) < 1. \quad (2.38)$$

Which indicates that the function $g_0(r)$ and $g_1(r)$ cross the straight line with slope 1 at another point r greater than 1 and by formulae(2.36), this crossing point is first obtained by $g_0(r)$. Consequently, we get $R_1 \leqslant R_0$.

Theorem 2.3 If there exist $R_0, R_1 > 0$ satisfying formulae(1.7) and formulae(1.8) respectively, then

$$\psi(u) \leqslant e^{-R^* u}, \qquad (2.39)$$

where $R^* = R_1$ and $u \in \{0, 1, 2, \cdots\}$.

Proof We first prove that $\{e^{-R^* U_k}, k \in N^+\}$ corresponds to a supermartingale.

Letting $\bar{Y}_{k-1} = (Y_1 = y_1, \cdots, Y_{k-1} = y_{k-1})$, it follows that $\bar{Y}_{k-1}$ summarize all relevant information about the surplus process during the $k-1$ first periods. We have

$E(e^{-R^* U_k} | I_0 = i, \bar{Y}_{k-1}) = E(e^{-R^* (U_{k-1} + Z_k - Y_k)} | I_0 = i, \bar{Y}_{k-1}) = e^{-R^* U_{k-1}} E(e^{-R^* (Z_k - Y_k)} | I_0 = i, \bar{Y}_{k-1}) = e^{-R^* U_{k-1}} [\delta E(e^{-r(1-Y_k)} | I_0 = i, \bar{Y}_{k-1}) + (1-\delta) E(e^{rY_k} | I_0 = i, \bar{Y}_{k-1})] = e^{-R^* U_{k-1}} [\delta E(e^{-r(1-Y_k)} | I_{k-1} = i_{k-1}) + (1-\delta) E(e^{rY_k} | I_{k-1} = i_{k-1})]$.

Since $R^* = R_1$, we have

$[\delta E(e^{-R^* (1-Y_k)} | I_{k-1} = 1) + (1-\delta) E(e^{R^* Y_k} | I_{k-1} = 1)] = 1$,

and from formulae(2.36), we obtain that

$\delta E(e^{-R^* (1-Y_k)} | I_{k-1} = 0) + (1-\delta) E(e^{R^* Y_k} | I_{k-1} = 0) \leqslant \delta E(e^{-R^* (1-Y_k)} | I_{k-1} = 1) + (1-\delta) E(e^{R^* Y_k} | I_{k-1} = 1) = 1$.

Then, it follows that

$E(e^{-R^* U_k} | I_0 = i, \bar{Y}_{k-1}) = e^{-R^* U_{k-1}} | [\delta E(e^{-r(1-Y_k)} | I_{k-1} = i_{k-1}) + (1-\delta) E(e^{rY_k} | I_{k-1} = i_{k-1})] \leqslant e^{-R^* U_{k-1}}$.

From the Kolmogorov's inequality for positive supermartingales, one can find that

$P(\max_{k \in \{0,1,\cdots\}} \{e^{-R^* U_k} \geqslant 1 | I_0 = i_0) \leqslant e^{-R^* u}, u \in \{0, 1, \cdots\}$.

Since

$\psi(u | i_0) = P(\min_{u \in \{0,1,\cdots\}} \{U_k\} < 0 | I_0 = i_0) \leqslant P(\max_{k \in \{0,1,\cdots\}} \{e^{-R^* U_k} \geqslant 1\} | I_0 = i_0) \leqslant e^{-R^* u}$,

we obtain

$$\psi(u | i_0) \leqslant e^{-R^* u}, u \in N, \qquad (2.40)$$

From formulae(2.40), the nonconditional ruin probability must satisfy the following inequality

$$\psi(u) = (1-q)\psi(u | 0) + q\psi(u | 1) \leqslant e^{-R^* u}.$$

References

[1] Gerber H U. Mathematical fun with the compound binomial process[J]. ASTIN Bulletin, 1998, 18: 161–168.

[2] Willmot G E. Ruin probabitilies in the compound binomial model [J]. Insurance: Mathematics and Economics12, 1993, 12: 133–142.

[3] Dickson D C M. Some comments on the compound binomial model[J]. Astin Bulletin, 1994, 24: 33–45.

[4] De Vylder F, Marceau E. Classical numerical ruin probabilities[J]. Scandination Actuarial Journal, 1996, 2: 109–123.

[5] Cheng S, Zhu R. The asymptotic formulas and Lundberg upper bound in fully discrete risk model[J]. Applied Mathematics: A Journal of Chinese Universities Series A, 2001, 16(3): 348–358.

[6] Pavlova K P, Willmot G E. The discrete stationary renewal risk model and the Gerber-Shiu discounted penalty function [J]. Insurance: Mathematics and Economics, 2004, 35: 267–277.

[7] Yuen K C, Guo J Y. Ruin probabilities for time-correlated claims in the compound binomial model[J]. Insurance: Mathematics and Economics, 2001, 35: 47–57.

[8] Xiao Y T, Guo J Y. The compound binomial risk model with time-correlated claims[J]. Insurance: Mathematics and Economics, 2007, 41: 124–133.

[9] Cossette H, Landriault D, Marceau E. Ruin probabilities in the compound Markov biomial model[J]. Scandination Actuarial Journal, 2003, 4: 301–323.

[10] Cossette H, Landriault D, Marceau E. Exact expressions and upper bound for ruin probabilities in the compound Markov biomial model[J]. Insurance: Mathematics and Economics, 2004, 34: 449–466.

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参考文献:

[1] 张昌应.网上评卷误差控制的方法与实施[J].高教探索, 2003(3): 77–79.

[2] 丁文,杨卫东,刘继来.基于神经网络技术的评卷误差控制模型及其应用[J].浙江工业大学学报, 2003, 31(4): 419–423.

[3] 高爱国.利用数学建模评阅竞赛试卷[J].高师理科学刊, 2004, 24(1): 8–11.

[4] 李志学.建立公平绩效评价的分值转换模型研究[J].中国管理科学, 2005, 13(10): 126–130.

[5] 张厚粲,刘昕.考试改革与标准参照测验[M].辽宁:辽宁教育出版社, 1992.

[6] 刘应成.考试系统中成绩正态分布检验的设计与实现[J].重庆工学院学报, 2004, 18(6): 188–191.

[7] 袁卫,庞皓,曾五一,等.统计学[M].北京:高等教育出版社, 2004.

[8] 魏宗舒.概率论与数理统计教程[M].北京:高等教育出版社, 2004.

[9] 卢纹岱. SPSS for Windows统计分析[M].北京:电子工业出版社, 2005.

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