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Complete Convergence and Strong Convergence for Weighted Sums of ρ^--Mixing Random Sequences*

ρ^--混合序列加权和的完全收敛性和强收敛性

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Abstract Some sufficient conditions of the complete convergence and strong convergence for weighted sums of ρ^--mixing random sequences are established. The results obtained extend the theorems of Stout and Thrum.

Key words ρ^--mixing random sequences, weighted sums, complete convergence, strong convergence

摘要: 给出 ρ^--混合序列加权和完全收敛和强收敛的充分条件, 所得结果推广了 Stout 和 Thrum 定理.

关键词: ρ^- 混合序列 加权和 完全收敛 强收敛
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Following the introductory concept of ρ^--mixing random variables in 1999[1], Zhang got moment inequalities of partial sums, central limit theorems, complete convergence and the strong law of large numbers[1~3]. Since ρ^--mixing random variables include NA and ρ^*-mixing random variables, which have a lot of applications, their limit properties have aroused wide interest recently. In this paper, we obtain some sufficient conditions of the complete convergence and strong convergence for weighted sums of ρ^--mixing random sequences. The results obtained extend the theorem of Stout[4] and Thrum[5].

1 Definitions and lemma

Definition 1.1[6] A sequence $\{X_k; k\in N\}$ is called negatively associated (NA) if for every pair of disjoint subsets S, T of N, $\mathrm{Cov}\{f(X_i; i\in S), g(X_j; j\in T)\}\leqslant 0$, where $f, g\in \mathcal{C}$, $\mathcal{C}$ is a class of functions which are coordinatewise increasing.

Definition 1.2[1] A sequence $\{X_k; k\in N\}$ is called ρ^*-mixing if $\rho^*(s) = \sup\{\rho(S,T); S,T\subset N, \mathrm{dist}(S,T)\geqslant s\}\to 0\ (s\to\infty)$, where $\rho(S,T) = \sup\{|E(f-Ef)(g-Eg)/(\|f-Ef\|_2\|g-Eg\|_2)|; f\in L_2(\mathcal{F}(S)), g\in L_2(\mathcal{F}(T))\}$.

Definition 1.3[1] A sequence $\{X_k; k\in N\}$ is called ρ^--mixing if $\rho^-(s) = \sup\{\rho^-(S,T); S,T\subset N, \mathrm{dist}(S,T)\geqslant s\}\to 0\ (s\to\infty)$, where $\rho^-(S,T) = 0\vee \sup$

$$\left\{\frac{\mathrm{Cov}\{f(X_i; i\in S), g(X_j; j\in T)\}}{\sqrt{\mathrm{Var}\{f(X_i; i\in S)\}\mathrm{Var}\{g(X_j; j\in T)\}}}; f,g\in\mathcal{C}\right\}.$$

It is easy to see that $\{X_k; k\in N\}$ is negatively associated if and only if $\rho^-(s)=0$, for $s\geqslant 1$. It is obvious that $\rho^-(s)\leqslant \rho^*(s)$, so ρ^--mixing is weaker than ρ^*-mixing.

Property 1.1[1] A subset of a ρ^--mixing sequences $\{X_i\}_{i\geqslant 1}$ with mixing coefficients $\rho^-(s)$ is also ρ^--mixing with coefficients not greater than $\rho^-(s)$.

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Property 1.2[1] Increasing functions defined on disjoint subsets of a $\tilde{\rho}$-mixing sequences $\{X_i\}_{i\geq 1}$ with mixing coefficients $\tilde{\rho}(s)$ are also $\tilde{\rho}$-mixing with coefficients not greater than $\tilde{\rho}(s)$.

Throughout this paper, C will represent a positive constant though its value may change from one appearance to another, and $a_n << b_n$ will mean $a_n \leq Cb_n$.

Lemma 1.1[7] For a positive integer $N\geq 1$, positive real numbers $p\geq 2$ and $0\leq r<(\frac{1}{6p})^{\frac{p}{2}}$, if $\{X_i; i\geq 1\}$ is a sequence of random variables with $\tilde{\rho}(N)\leq r$, $EX_i=0$ and $E|X_i|^p<\infty$ for every $i\geq 1$, then for all $n\geq 1$, there is a positive constant $C=C(p,N,r)$ such that

$$E(\max_{1\leq j\leq n}|S_j|^p)\leq C\{\sum_{i=1}^{n}E|X_i|^p+(\sum_{i=1}^{n}EX_i^2)^{p/2}\},$$

where $S_j=\sum_{i=1}^{j}X_i$.

2 Main results

Theorem 2.1 Let $0<\alpha\leq 1$, and $\{X_n; n\geq 1\}$ be a $\tilde{\rho}$-mixing sequence of identically distributed random variables with

$$EX_1=0, E|X_1|^{2/\alpha}<\infty. \quad (1)$$

And

$$|a_{ni}|\leq Cn^{-2/\alpha-\beta},\ n\geq 1, i\leq n \text{ and } 0<\beta<\alpha/2,$$
$$|a_{ni}|=0,\ i>n. \quad (2)$$

And there exists a constant $\theta>0$ such that

$$\sum_{i=1}^{n}a_{ni}^2\leq Cn^{-\theta}, \text{ for all } n. \quad (3)$$

Then

$$T_n \triangleq \sum_{i=1}^{n}a_{ni}X_i \xrightarrow{a.s.} 0, n\to\infty. \quad (4)$$

Proof Let

$$\hat{X}_{ni}=-n^{-\beta/2}I(a_{ni}X_i<-n^{-\beta/2})+X_iI(|a_{ni}X_i|\leq n^{-\beta/2})+n^{-\beta/2}I(a_{ni}X_i>n^{-\beta/2}),$$

then

$$T_n=\sum_{i=1}^{n}a_{ni}(X_i-\hat{X}_{ni})+\sum_{i=1}^{n}a_{ni}(\hat{X}_{ni}-E\hat{X}_{ni})+E(\sum_{i=1}^{n}a_{ni}\hat{X}_{ni})\triangleq T_{n1}+T_{n2}+T_{n3}.$$

Therefore, in order to prove the Theorem 2.1, we have to prove only that $T_{ni}\xrightarrow{a.s.}0, n\to\infty$, $i=1,2,3$. By formula (1) and formula (2), we have

$$\sum_{n=1}^{\infty}P(X_i\neq \hat{X}_{ni})=\sum_{n=1}^{\infty}P(|a_{ni}X_i|>n^{-\beta/2})\leq \sum_{n=1}^{\infty}n^{\beta/\alpha}E|a_{ni}X_i|^{2/\alpha}<<\sum_{n=1}^{\infty}n^{-1-\beta/\alpha}<\infty.$$

Thus, according to the Borel-Cantelli lemma, $T_{n1}\xrightarrow{a.s.}0, n\to\infty$.

By Property 1.1 and Property 1.2, $\{\hat{X}_i-E\hat{X}_{ni}, i\geq 1\}$ is also a $\tilde{\rho}$-mixing random sequence and satisfies the conditions of Lemma 1.1.

Let $q>\max\{2/\alpha, 2/\theta, (2\alpha-2\beta)/(\alpha\beta)\}$, by Lemma 1.1, Markov inequality, C_r inequality and formula (1)~(3),

$$P(\left|\sum_{i=1}^{n}a_{ni}(\hat{X}_{ni}-E\hat{X}_{ni})\right|>\varepsilon)<<E\left|\sum_{i=1}^{n}a_{ni}(\hat{X}_{ni}-E\hat{X}_{ni})\right|^q<<\sum_{i=1}^{n}E|a_{ni}(\hat{X}_{ni}-E\hat{X}_{ni})|^q+\left(\sum_{i=1}^{n}E(a_{ni}\hat{X}_{ni}-a_{ni}E\hat{X}_{ni})^2\right)^{q/2}<<$$
$$\sum_{i=1}^{n}E|a_{ni}X_i|^qI(|a_{ni}X_i|\leq n^{-\beta/2})+\sum_{i=1}^{n}E|a_{ni}n^{-\beta/2}|^q\cdot P(|a_{ni}X_i|>n^{-\beta/2})+(\sum_{i=1}^{n}a_{ni}^2)^{q/2}\leq$$
$$\sum_{i=1}^{n}E|a_{ni}X_i|^{2/\alpha}n^{-\beta(q-2/\alpha)/2}+\sum_{i=1}^{n}|a_{ni}|^{q+2/\alpha}n^{\beta/\alpha-\beta q/2}E|X_i|^{2/\alpha}+(\sum_{i=1}^{n}a_{ni}^2)^{q/2}<<$$
$$n^{-\beta q/2-\beta/\alpha}+n^{-\beta/\alpha-3\beta q/2-\alpha q/2}+n^{-\theta q/2}.$$

By $q>\max\{2/\alpha, 2/\theta, (2\alpha-2\beta)/(\alpha\beta)\}$, we have $-\beta q/2-\beta/\alpha<-1$, $-\beta/\alpha-3\beta q/2-\alpha q/2<-1$, $-\theta q/2<-1$. Therefore, we conclude that $\sum_{n=1}^{\infty}P(\left|\sum_{i=1}^{n}a_{ni}(\hat{X}_{ni}-E\hat{X}_{ni})\right|>\varepsilon)<\infty$. Thus, $T_{n2}\xrightarrow{a.s.}0, n\to\infty$. Finally, we prove that $T_{n3}\xrightarrow{a.s.}0, n\to\infty$. By formula (1)~(3) and Markov inequality,

$$|T_{n3}|=\left|\sum_{i=1}^{n}E(a_{ni}X_i-a_{ni}\hat{X}_{ni})\right|\leq \sum_{i=1}^{n}E|a_{ni}X_i|I(|a_{ni}X_i|>n^{-\beta/2})+\sum_{i=1}^{n}|a_{ni}n^{-\beta/2}|\cdot P(|a_{ni}X_i|>n^{-\beta/2})\leq \sum_{i=1}^{n}E|a_{ni}X_i|\frac{|a_{ni}X_i|^{2/\alpha-1}}{n^{-\beta(2/\alpha-1)/2}}+$$
$$\sum_{i=1}^{n}|a_{ni}|n^{-\beta/2}\frac{E|a_{ni}X_i|}{n^{-\beta/2}}\leq \sum_{i=1}^{n}E|a_{ni}X_i|^{2/\alpha}\cdot n^{\beta(2/\alpha-1)/2}\sum_{i=1}^{n}|a_{ni}|^2E(X_1)<<n^{-\beta/\alpha-\beta/2}+n^{-\theta}\to 0, n\to\infty.$$

Now we complete the proof of Theorem 2.1.

Theorem 2.2 Let $0<\alpha\le 1$, and $\{X_n; n\ge 1\}$ be a φ-mixing sequence of identically distributed random variables with

$$EX_1=0, E|X_1|^{2/\alpha}<\infty. \qquad (1)$$

And

$$|a_{ni}|\le Cn^{-\alpha-\delta}, n\ge 1, i\le n \text{ and } 0<\delta<\alpha/2, \quad |a_{ni}|=0,\ i>n. \qquad (5)$$

And there exists a constant $\theta>0$ such that

$$\sum_{i=1}^{n} a_{ni}^2\le Cn^{-\theta}, \text{ for all } n, \text{ as } 0<\alpha\le 1/2-\delta. \qquad (6)$$

Then

$$T_n \hat{=} \sum_{i=1}^{n} a_{ni}X_i \xrightarrow{c} 0, n\to\infty.$$

Proof Let

$$X_{ni}\hat{=} -n^{\delta}I(a_{ni}X_i<-n^{\delta})+X_iI(|a_{ni}X_i|\le n^{\delta})+n^{\delta}I(a_{ni}X_i>n^{\delta}),$$

then

$$\left\{\left|\sum_{i=1}^{n} a_{ni}X_i\right|>\varepsilon\right\}=\left\{\left|\sum_{i=1}^{n} a_{ni}X_i\right|>\varepsilon, \exists i: 1\le i\le n, |a_{ni}X_i|>n^{-\delta}\right\}\cup\left\{\left|\sum_{i=1}^{n} a_{ni}X_i\right|>\varepsilon, \exists\forall i: 1\le i\le n, |a_{ni}X_i|\le n^{-\delta}\right\}\subset\bigcup_{i=1}^{n}\{|a_{ni}X_i|>n^{-\delta}\}\cup\left\{\left|\sum_{i=1}^{n} a_{ni}X_{ni}\right|>\varepsilon\right\}\hat{=} A_n+B_n.$$

Therefore, in order to prove the Theorem 2.2, we have to prove only that

$$\sum_{n=1}^{\infty} P(A_n)<\infty, \qquad (7)$$

$$\sum_{n=1}^{\infty} P(B_n)<\infty. \qquad (8)$$

By formula (1) and formula (5),

$$\left|E\left(\sum_{i=1}^{n} a_{ni}X_{ni}\right)\right| = \left|\sum_{i=1}^{n} E(a_{ni}X_i - a_{ni}X_{ni})\right|\le \sum_{i=1}^{n} E|a_{ni}X_i|\, I(|a_{ni}X_i|>n^{-\delta})+\sum_{i=1}^{n}|a_{ni}n^{-\delta}|\cdot P(|a_{ni}X_i|>n^{-\delta})\le\sum_{i=1}^{n}E|a_{ni}X_i|^{2/\alpha}n^{(2/\alpha-1)\delta}+\sum_{i=1}^{n}|a_{ni}n^{-\delta}|\frac{E|a_{ni}X_i|^{2/\alpha}}{n^{-2\delta/\alpha}} << n^{-1-\delta}+n^{-1-\alpha-\delta}\to 0, n\to\infty.$$

$$\sum_{n=1}^{\infty}P(A_n)\le\sum_{n=1}^{\infty}\sum_{i=1}^{n}P(|a_{ni}X_i|>n^{-\delta})\le\sum_{n=1}^{\infty}nP(|X_1|>cn^{\alpha})=\sum_{n=1}^{\infty}n\sum_{j=n}^{\infty}P(cj^{\alpha}<|X_1|\le c(j+1)^{\alpha})=\sum_{j=1}^{\infty}\sum_{n=1}^{j}nEI(cj^{\alpha}<|X_1|\le c(j+1)^{\alpha}) << \sum_{j=1}^{\infty}j^2E(|X_1|/j^{\alpha})^{2/\alpha}I(cj^{\alpha}<|X_1|\le c(j+1)^{\alpha})=\sum_{j=1}^{\infty}E|X_1|^{2/\alpha}I(cj^{\alpha}<|X_1|\le c(j+1)^{\alpha}) << E|X_1|^{2/\alpha}<\infty.$$

In order to prove formula (8), first we show that

$$E\left(\sum_{i=1}^{n} a_{ni}X_{ni}\right)\to 0, n\to\infty. \qquad (9)$$

By formula (1), formula (5) and Markov inequality,

$$\left|E\left(\sum_{i=1}^{n} a_{ni}X_{ni}\right)\right|=\left|\sum_{i=1}^{n}E(a_{ni}X_i-a_{ni}X_{ni})\right|\le\sum_{i=1}^{n}E(a_{ni}X_i)I(|a_{ni}X_i|>n^{-\delta})+\sum_{i=1}^{n}|a_{ni}n^{-\delta}|\cdot P(|a_{ni}X_i|>n^{-\delta})\le\sum_{i=1}^{n}E|a_{ni}X_i|^{2/\alpha}n^{2/\alpha-\delta}+\sum_{i=1}^{n}|a_{ni}n^{-\delta}|\frac{E|a_{ni}X_i|^{2/\alpha}}{n^{-2\delta/\alpha}} << n^{-1-\delta}+n^{-1-\alpha-\delta}\to 0, n\to\infty.$$

Hence, we need only to prove that

$$\sum_{n=1}^{\infty}P\left(\left|\sum_{i=1}^{n}(a_{ni}X_i-a_{ni}X_{ni})\right|>\varepsilon/2\right)<\infty, \forall\varepsilon<0. \qquad (10)$$

Let $q>\max\{2/\alpha, 2/\theta, 1/(\alpha+\delta-1/2)\}$, by Lemma 1.1, Markov inequality, C_r inequality, formula (1) and foumula (3),

$$P\left(\left|\sum_{i=1}^{n}a_{ni}(X_{ni}-EX_{ni})\right|>\varepsilon/2\right) << E\left|\sum_{i=1}^{n}a_{ni}(X_{ni}-EX_{ni})\right|^{q} << \sum_{i=1}^{n}E|a_{ni}(X_{ni}-EX_{ni})|^{q}+\left(\sum_{i=1}^{n}E[(a_{ni}X_{ni}-a_{ni}EX_{ni})^2]\right)^{q/2} << \sum_{i=1}^{n}E|a_{ni}X_i|^{q}I(|a_{ni}X_i|\le n^{-\delta})+\sum_{i=1}^{n}|a_{ni}\cdot n^{-\delta}|^{q}P(|a_{ni}X_i|>n^{-\delta})+\left(\sum_{i=1}^{n}a_{ni}^2\right)^{q/2}\le\sum_{i=1}^{n}E|a_{ni}X_i|^{2/\alpha}n^{-\delta(q-2/\alpha)}+\sum_{i=1}^{n}|a_{ni}|^{q+2/\alpha}\cdot n^{2\delta/\alpha-\delta q}E|X_1|^{2/\alpha}+\left(\sum_{i=1}^{n}a_{ni}^2\right)^{q/2} << n^{-1-\delta q}+n^{-1-\alpha q}+\left(\sum_{i=1}^{n}a_{ni}^2\right)^{q/2}. \qquad (11)$$

By formula (6), when $0<\alpha\le 1/2-\delta$, then

$$\left(\sum_{i=1}^{n}a_{ni}^2\right)^{q/2}\le n^{-\theta q/2}. \qquad (12)$$

By formula (5), when $\alpha>1/2-\delta$, then

$$\left(\sum_{i=1}^{n}a_{ni}^2\right)^{q/2}\le n^{-q(1-2\alpha-2\delta)/2}. \qquad (13)$$

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这就说明不存在优于 $AY+A_0$ 的非齐次线性预测,故 $AY+A_0\overset{tH}{\sim} SY_0(T)$.

推论 1　考虑多元线性模型 $(Y, XB, U|(B,U)\in T)$.设 SY_0 是条件可预测,则 $AY+A_0\overset{tI}{\sim} SY_0(T)$ 的充要条件是 (i) $\_(A_0)\subseteq\_(AX-SX_0)$, (ii) 对所有 $\Gamma\in C^*$, $\_(\Gamma)\subseteq\_((AX-SX_0)')$,有 $tr(\Gamma'(AX-SX_0)^+A_0)\geqslant 0$, (iii) $(A-S\Sigma'V^+)X[(X'D^+X)^- - I](SX_0-S\Sigma'V^+X)'\geqslant(A-S\Sigma'V^+)X[(X'D^+X)^- - I]X'(A-S\Sigma'X^+)$, (iv) $rk(AX-SX_0)(X'D^+X-I)X'=rk(AX-SX_0)$,其中 $D=V+XX'$.

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Combining $q>\max\{2/\tau, 2\theta, 1/|\tau+W-1/2|\}$ and formula(11)~(13), we know the formula(10) is proved. So we complete the proof of Theorem 2.2.

Because $\tilde{\rho}$-mixing sequences are more general than NA sequences or ρ^*-mixing sequences. So we have the following two corollaries.

Corollary 2.1　Let $0<\tau\leqslant 1$, and $\{X_n; n\geqslant 1\}$ be a $\tilde{\rho}$-mixing or NA sequence of identically distributed random variables with $EX_1=0, E|X_1|^{2/\tau}<\infty$. And $|a_{ni}|\leqslant Cn^{-2/\tau-W}$, $n\geqslant 1$, $i\leqslant n$ and $0<W<\tau/2$, $|a_{ni}|=0$, $i>n$. And there exists a constant $\theta>0$ such that $\sum_{i=1}^{n}a_{ni}^2\leqslant Cn^{-\theta}$, for all n. Then $\hat{T}_n=\sum_{i=1}^{n}a_{ni}X_i\xrightarrow{a.s.}0, n\to\infty$.

Corollary 2.2　Let $0<\tau\leqslant 1$, and $\{X_n; n\geqslant 1\}$ be a $\tilde{\rho}$-mixing or NA sequence of identically distributed random variables with $EX_1=0, E|X_1|^{2/\tau}<\infty$. And $|a_{ni}|\leqslant Cn^{-2/\tau-W}$, $n\geqslant 1$, $i\leqslant n$ and $0<W<\tau/2$, $|a_{ni}|=0$, $i>n$. And there exists a constant $\theta>0$ such that $\sum_{i=1}^{n}a_{ni}^2\leqslant Cn^{-\theta}$, for all n, as $0<\tau\leqslant 1/2-W$. Then $\hat{T}_n=\sum_{i=1}^{n}a_{ni}X_i\xrightarrow{c}0, n\to\infty$.

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