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Robust Resilient Guaranteed Cost Control for Singular Impulsive Switched Systems with Time-varying Delay*

时变时滞奇异脉冲切换系统的鲁棒弹性保成本控制

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Abstract: This paper focuses on the problem of robust resilient guaranteed cost control for a class of singular impulsive switched systems with time-varying delay. Based on the multiple Lyapunov functional technique, some sufficient criteria, ensuring the regularity, causality, and asymptotic stability, are obtained initially for the nominal and unforced systems. Then the resilient controller is designed such that the corresponding closed-loop system, for all admissible uncertainties, is regular, causal and asymptotically stable, and the cost function does not exceed a cost upper bound. Further, a minimization approach of the largest singular value of matrices and a convex optimization method are introduced to seek the optimal robust resilient guaranteed cost controller. All the conditions are cast into the form of linear matrix inequalities (LMIs) through ingenious processing. Finally, two examples are presented to illustrate the less conservativeness and the effectiveness of the proposed results.

Key words: singular switched systems, impulsive switched systems, resilient guaranteed cost control, multiple Lyapunov technique, linear matrix inequalities (LMIs)

摘要:针对一类具有时变时滞的奇异脉冲切换系统,研究鲁棒弹性保成本控制问题.首先,基于多 Lyapunov 泛函技术,建立标称自由系统具有正则性、因果性及渐近稳定性的充分条件.然后,给出一个弹性保性能控制器的设计方案,该方案能保证对所有容许的不确定性,闭环系统是正则的、因果的和渐近稳定的,且成本函数不超过某个上界.并进一步运用矩阵最大奇异值的最小化方法和凸优化方法,求解最优鲁棒弹性保成本控制器.所有的充分条件均巧妙地表示为线性矩阵不等式形式.最后,运用两个仿真实例验证本研究方法较少的保守性和有效性.

关键词:奇异切换系统　脉冲切换系统　弹性保成本控制　多 Lyapunov 技术　线性矩阵不等式

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0 Introduction

Switched systems have attracted considerable attention in recent decades[1-7] which include a series of continuous-time or discrete-time subsystems and a switching rule that orchestrates the switching between subsystems. They can be found in various

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real-world systems such as transportation systems[8-9], electric power systems[10], communication networks[11-12], and chemical processes[13]. However, singular phenomena often exist in practical processes modeled by switching systems such as robotics, economics, chemistry and power systems. We call this kind of systems as singular switched systems. The past decades have witnessed considerable research on analysis and synthesis of singular switched systems[14-15]. In addition, impulses often take place in various applications modeled by switching systems, which makes it more intricate to analyze the property of impulsive switched systems. Recently, some theoretical results on impulsive switched systems are reported in literatures, respectively[16-21].

In actually physical processes, due to some physical restriction such as resistance errors, A/D or D/A conversion, finite word length in digital systems and rounding off errors in numerical computation, it is impossible to implement controller precisely, and it is important to take the controller gain perturbations into account during the designing process of the controller. On the other hand, the relatively small fluctuation of controller parameters may lead to the performance degradation or even instability. The two aspects above inspire us to design a controller that should be able to tolerate some levels of controller parameter perturbations. This kind of controllers are usually called as "resilient controllers". Therefore, it is extremely imperative to design a resilient controller, and at the same time, some techniques and approaches solving this problem have been proposed. In [22], the problem of non-fragile hybrid guaranteed cost control is addressed for a class of uncertain switched linear systems. An observer-based resilient controller is designed in [23] for singular time-delay systems. Up to now, just little attention has been paid to design a resilient guaranteed cost controller for singular impulsive switched systems with time-varying delay, which stimulates the authors' research interests.

Here, we mainly study the robust resilient guaranteed cost control problem for a class of singular impulsive switched systems with time-varying delay. The outstanding contributions lie in several aspects: Firstly, we consider the uncertainty, impulse, singularity and time delay in switched systems at the same time, which throw out the greater challenge for the authors; Secondly, for the singular impulsive switched systems with time-varying delay, the derived conclusions can apply to various systems such as singular switched systems, impulsive switched systems, and singular impulsive systems, which fully demonstrates the less conservativeness and the broader applicability; Thirdly, uncertainties exist not only in the system structure but also in the resilient controller, which make it more difficult to simplify and solve inequalities; Fourthly, we introduce a minimization approach of the largest singular value of matrices and a convex optimization method in this paper to seek the optimal robust resilient guaranteed cost controller; Finally, all the conditions are cast into linear matrix inequalities (LMIs), and two examples are provided to illustrate the effectiveness of the proposed results.

Notations Throughout this paper, T denotes the transpose. R^n represents the n-dimensional Euclidean space. Z^+ is a positive integer set, C stands for complex domain. Matrix $P>0 (P\geqslant 0)$ means that P is positive definite (positive semi-definite), and I is identity matrix with appropriate dimensions. $*$ stands for the symmetric part in a block symmetric matrix.

1 Problem formulation and preliminaries

Consider the following singular impulsive switched system with time-varying delay

$$\Sigma_{(1)}: E\dot{x}(t)=(A_{\sigma(t)}+\Delta A_{\sigma(t)})x(t)+(A_{\tau\sigma(t)}+\Delta A_{\tau\sigma(t)})x(t-\tau(t))+B_{\sigma(t)}u_{\sigma(t)}(t), t\neq t_k,$$
$$\Delta x(t)=(C_{\sigma(t)}+\Delta C_{\sigma(t)})x(t), t=t_k,$$
$$x(t)=\phi(t), t\in[-\tau_m, 0],$$

where $x(t)\in R^n$ is the state, $\sigma(t):[0,+\infty)\rightarrow M=\{1,2,\cdots,m\}, m\in Z^+$ is a piecewise constant switching signal to be designed which usually depends on time t or state $x(t)$, and $\sigma(t)=i$ implies that the i-th subsystem is activated. $u_{\sigma(t)}(t)\in R^q$ is the control input. $\tau(t)$ is the time-varying delay satisfying $0<\tau(t)\leqslant\tau_m$ and $0\leqslant\dot{\tau}(t)\leqslant\mu<1$. $\phi(t)$ is a differentiable initial function. $A_i, A_{\tau i}, B_i, C_i, i\in M$, are known real constant matrices of appropriate dimensions. $E \in R^{n\times n}$ is a singular matrix with $0 < \mathrm{rank}(E) = r < n$. $\Delta A_i, \Delta A_{\tau i}, \Delta C_i, i \in M$ are unknown real norm-bounded matrices representing time-varying parameter uncertainties and satisfying

$$\Delta A_i = N_{1i}F_{1i}(t)D_{1i}, F_{1i}^{\mathrm{T}}(t)F_{1i}(t) \leqslant I, \quad (1)$$

$$\Delta A_{\tau i} = N_{2i}F_{2i}(t)D_{2i}, F_{2i}^{\mathrm{T}}(t)F_{2i}(t) \leqslant I, \quad (2)$$

$$\Delta C_i = N_{5i}F_{5i}(t)D_{5i}, F_{5i}^{\mathrm{T}}(t)F_{5i}(t) \leqslant I, \quad (3)$$

where $N_{1i}, N_{2i}, N_{5i}, D_{1i}, D_{2i}, D_{5i}$ are known constant matrices of appropriate dimensions. $F_{1i}(t), F_{2i}(t), F_{5i}(t)$ are unknown matrix functions, and t_k is an impulsive switching point satisfying $0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots, k \in \{0,1,2,\cdots\}$. $x(t_k) = x(t_k^-) = \lim_{h\to 0^+} x(t_k - h), x(t_k^+) = \lim_{h\to 0^+} x(t_k + h), \Delta x(t_k) = x(t_k^+) - x(t_k^-) = x(t_k^+) - x(t_k)$.

Associated with system $\Sigma_{(1)}$, the cost function is given by

$$J = \int_0^{+\infty} x^{\mathrm{T}}(t)Sx(t) + u_{\sigma(t)}^{\mathrm{T}}(t)Ru_{\sigma(t)}(t)\mathrm{d}t, \quad (4)$$

where S and R are positive definite weighted matrices.

For system $\Sigma_{(1)}$, a resilient controller

$$u_{\sigma(t)}(t) = (K_{\sigma(t)} + \Delta K_{\sigma(t)})x(t), \quad (5)$$

is considered, where $K_i, i \in M$ is a controller gain to be designed, and $\Delta K_i, i \in M$ represents a additive controller gain variation which has the following form

$$\Delta K_i = N_{3i}F_{3i}(t)D_{3i}, F_{3i}^{\mathrm{T}}(t)F_{3i}(t) \leqslant I, i \in M, \quad (6)$$

where N_{3i} and D_{3i} are known real constant matrices, and $F_{3i}(t)$ describes the uncertainty of the controller gain.

Definition 1[15] Consider the pair $(E, A_{\sigma(t)})$.

1. For a given $i \in M$, the pair (E, A_i) is said to be regular if $\det(sE - A_i) \neq 0, s \in C$.

2. For a given $i \in M$, the pair (E, A_i) is said to be causal if it is regular and $\deg(\det(sE - A_i)) = rank(E)$.

3. The pair $(E, A_{\sigma(t)})$ is said to be regular and causal if every pair (E, A_i) is regular and causal, $i \in M$.

Definition 2[15] The system $\Sigma_{(1)}$ with $\Delta A_i = 0, \Delta A_{\tau i} = 0, u_i(t) = 0, i \in M$ is said to be regular and causal if the pair $(E, A_{\sigma(t)})$ is regular and causal.

Remark 1 The existence and uniqueness of the solutions of systems $\Sigma_{(1)}$ with $\Delta A_i = 0, \Delta A_{\tau i} = 0, u_i(t) = 0$ for each $i \in M$, can be ensured by regularity and causality.

Definition 3 For system $\Sigma_{(1)}$, if there exist a switching signal $\sigma(t)$, a state feedback controller $u_{\sigma(t)}(t)$ in the form of (5), and a positive scalar J^* such that for all admissible uncertainties, the corresponding closed-loop system is regular, causal, asymptotically stable, and the value of the cost function (4) satisfies $J \leqslant J^*$, then J^* is said to be a guaranteed cost and the controller (5) is said to be a robust resilient guaranteed cost controller. If $J_{\min}^*$ is the minimal upper bound of the guaranteed cost, then $J_{\min}^*$ is known as an optimal guaranteed cost and the corresponding controller $u_{\sigma(t)}^*(t)$ is called an optimal robust resilient guaranteed cost controller.

The main object of this paper is to construct a switching signal, design a robust resilient guaranteed cost controller and give an upper bound of the cost function for systems $\Sigma_{(1)}$.

Lemma 1[24] Let $Y = Y^{\mathrm{T}}, H, E$ and F be real matrices of appropriate dimensions with $F^{\mathrm{T}}F \leqslant I$. The following statements are equivalent

a. $Y + HFE + E^{\mathrm{T}}F^{\mathrm{T}}H^{\mathrm{T}} < 0$,

b. there exists a scalar $\epsilon > 0$, satisfying $Y + \epsilon HH^{\mathrm{T}} + \epsilon^{-1}E^{\mathrm{T}}E < 0$.

Lemma 2[25] For matrix $Q \geqslant 0$, if there is a zero element q_i on the main diagonal line of Q, then the column and row which q_i lies on are both zero.

2 Main results

2.1 Stability analysis

In this section, we initially establish stability conditions for the following system $\Sigma_{(2)}$.

$$\Sigma_{(2)}: E\dot{x}(t) = A_{\sigma(t)}x(t) + A_{\tau\sigma(t)}x(t - \tau(t)), \; t \neq t_k,$$

$$\Delta x(t) = C_{\sigma(t)}x(t), t = t_k,$$

$$x(t) = \phi(t), t \in [-\tau_m, 0].$$

Theorem 1 Consider system $\Sigma_{(2)}$. If, for any $i \in M$, there exist constants $\beta_{ij} \geqslant 0 (j \in M)$, matrices $Q_i > 0, X_i \geqslant 0, Z_i > 0, P_i, Y_i$ such that

$$P_iE = E^{\mathrm{T}}P_i^{\mathrm{T}} \geqslant 0, \quad (7)$$

$$\Gamma_1 = \begin{bmatrix} \Gamma_{11} & \tau_m A_i^{\mathrm{T}}Z_iA_{\tau i} - Y_i + P_iA_{\tau i} \\ * & \tau_m A_{\tau i}^{\mathrm{T}}Z_iA_{\tau i} - (1-\mu)Q_i \end{bmatrix} < 0, \quad (8)$$

$$(I+C_j)^{\mathrm{T}}P_iE(I+C_j)-P_jE\leqslant 0,i\neq j,j\in M, \tag{9}$$

$$\begin{bmatrix} X_i & Y_i \\ * & (1-\mu)E^{\mathrm{T}}Z_iE \end{bmatrix}\geqslant 0, \tag{10}$$

where $\Gamma_{11}=P_iA_i+A_i^{\mathrm{T}}P_i^{\mathrm{T}}+Q_i+\tau_mX_i+Y_i+Y_i^{\mathrm{T}}+\tau_mA_i^{\mathrm{T}}Z_iA_i+\sum_{j=1}^{m}\beta_{ij}(P_j-P_i)E$,

then the system $\Sigma_{(2)}$ is regular, causal and asymptotically stable under a state-dependent switching signal

$$\sigma(t)=\arg\min\{x^{\mathrm{T}}(t)P_iEx(t),i\in M\}. \tag{11}$$

Proof Without loss of generality, let $E=\begin{bmatrix} I_r & 0 \\ * & 0 \end{bmatrix}$. Define the following multiple Lyapunov functional

$$V_{\sigma(t)}(t)=x^{\mathrm{T}}(t)P_{\sigma(t)}Ex(t)+\int_{t-\tau(t)}^{t}x^{\mathrm{T}}(s)Q_{\sigma(s)}x(s)\mathrm{d}s+\int_{-\tau(t)}^{0}\int_{t+\beta}^{t}\dot{x}^{\mathrm{T}}(\alpha)E^{\mathrm{T}}Z_{\sigma(\alpha)}E\dot{x}(\alpha)\mathrm{d}\alpha\mathrm{d}\beta,$$

and design the switching signal (11).

When $t\in(t_k,t_{k+1}]$, suppose that the i-th subsystem is activated. Then one obtains

$$V_i(t)=x^{\mathrm{T}}(t)P_iEx(t)+\int_{t-\tau(t)}^{t}x^{\mathrm{T}}(s)Q_{\sigma(s)}x(s)\cdot\mathrm{d}s+\int_{-\tau(t)}^{0}\int_{t+\beta}^{t}\dot{x}^{\mathrm{T}}(\alpha)E^{\mathrm{T}}Z_{\sigma(\alpha)}E\dot{x}(\alpha)\mathrm{d}\alpha\mathrm{d}\beta. \tag{12}$$

From (11) and the condition $\beta_{ij}\geqslant 0$, we get

$$\sum_{j=1}^{m}\beta_{ij}(P_j-P_i)E\geqslant 0. \tag{13}$$

In the following, we firstly prove that system $\Sigma_{(2)}$ is regular and causal.

Corresponding to the blocks of matrix E, one denotes

$$X_i=\begin{bmatrix} X_{i1} & X_{i2} \\ * & X_{i3} \end{bmatrix},Y_i=\begin{bmatrix} Y_{i1} & Y_{i2} \\ Y_{i3} & Y_{i4} \end{bmatrix},$$

$$Z_i=\begin{bmatrix} Z_{i11} & Z_{i12} \\ * & Z_{i13} \end{bmatrix}, \tag{14}$$

$$P_i=\begin{bmatrix} P_{i1} & P_{i2} \\ P_{i3} & P_{i4} \end{bmatrix},A_i=\begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix}.$$

Substituting (14) into (10), we obtain

$$\begin{bmatrix} X_{i1} & X_{i2} & Y_{i1} & Y_{i2} \\ * & X_{i3} & Y_{i3} & Y_{i4} \\ * & * & (1-\mu)Z_{i11} & 0 \\ * & * & * & 0 \end{bmatrix}\geqslant 0,$$

which implies from Lemma 2 that

$$Y_i=\begin{bmatrix} Y_{i1} & 0 \\ Y_{i3} & 0 \end{bmatrix}. \tag{15}$$

From (7), there holds

$$P_i=\begin{bmatrix} P_{i1} & P_{i2} \\ 0 & P_{i4} \end{bmatrix}. \tag{16}$$

From (8), we have

$$P_iA_i+A_i^{\mathrm{T}}P_i^{\mathrm{T}}+Q_i+\tau_mX_i+Y_i+Y_i^{\mathrm{T}}+\tau_mA_i^{\mathrm{T}}Z_iA_i+\sum_{j=1}^{m}\beta_{ij}(P_j-P_i)E<0. \tag{17}$$

Bearing (13) and $Q_i>0,X_i\leqslant 0,Z_i>0,\tau_m>0$ in mind, one gets from (17)

$$P_iA_i+A_i^{\mathrm{T}}P_i^{\mathrm{T}}+Y_i+Y_i^{\mathrm{T}}<0. \tag{18}$$

Substituting (15), (16), $A_i=\begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix}$ into (18) implies $P_{i4}A_{i4}+A_{i4}^{\mathrm{T}}P_{i4}^{\mathrm{T}}<0$. Hence, A_{i4} is invertible. This shows that system $\Sigma_{(2)}$ is regular and causal[4].

Subsequently, we will focus on our attention to asymptotical stability analysis.

Due to the fact $x(t)-x(t-\tau(t))=\int_{t-\tau(t)}^{t}\dot{x}(\alpha)\mathrm{d}\alpha$, one can rewrite $\Sigma_{(2)}$ as

$$\Sigma'_{(2)}:E\dot{x}(t)=(A_i+A_{\tau i})x(t)-A_{\tau i}\int_{t-\tau(t)}^{t}\dot{x}(\alpha)\mathrm{d}\alpha,t\neq t_k,$$

$$\Delta x(t)=C_{\sigma(t)}x(t),t=t_k,$$

$$x(t)=\phi(t),t\in[-\tau_m,0].$$

Calculating the derivative of $V_i(t)$ in form of (12) along with the solution of system $\Sigma'_{(2)}$, one has

$$\dot{V}_i(t)=2x^{\mathrm{T}}(t)P_i[(A_i+A_{\tau i})x(t)-A_{\tau i}\int_{t-\tau(t)}^{t}\dot{x}(\alpha)\mathrm{d}\alpha]+x^{\mathrm{T}}(t)Q_ix(t)-(1-\dot{\tau}(t))x^{\mathrm{T}}(t-\tau(t))Q_ix(t-\tau(t))+\tau(t)\dot{x}^{\mathrm{T}}(t)E^{\mathrm{T}}Z_iE\dot{x}(t)-\int_{t-\tau(t)}^{t}\dot{x}^{\mathrm{T}}(\alpha)E^{\mathrm{T}}Z_iE\dot{x}(\alpha)\mathrm{d}\alpha-(-\dot{\tau}(t))\int_{t-\tau(t)}^{t}\dot{x}^{\mathrm{T}}(\alpha)E^{\mathrm{T}}Z_iE\dot{x}(\alpha)\mathrm{d}\alpha. \tag{19}$$

From (10), it is easy to derive

$$\begin{bmatrix} X_i & Y_i \\ * & (1-\dot{\tau}(t))E^{\mathrm{T}}Z_iE \end{bmatrix}\geqslant 0,$$

which implies

$$-2x^{\mathrm{T}}(t)P_iA_{\tau i}\int_{t-\tau(t)}^{t}\dot{x}(\alpha)\mathrm{d}\alpha\leqslant\int_{t-\tau(t)}^{t}\begin{pmatrix} x(t) \\ \dot{x}(\alpha) \end{pmatrix}^{\mathrm{T}}\begin{pmatrix} X_i & Y_i-P_iA_{\tau i} \\ * & (1-\dot{\tau}(t))E^{\mathrm{T}}Z_iE \end{pmatrix}\begin{pmatrix} x(t) \\ \dot{x}(\alpha) \end{pmatrix}\mathrm{d}\alpha=\tau(t)x^{\mathrm{T}}(t)X_ix(t)+2x^{\mathrm{T}}(t)(Y_i-P_iA_{\tau i})\cdot\int_{t-\tau(t)}^{t}\dot{x}(\alpha)\mathrm{d}\alpha+\int_{t-\tau(t)}^{t}\dot{x}^{\mathrm{T}}(\alpha)E^{\mathrm{T}}Z_iE\dot{x}^{\mathrm{T}}(\alpha)\mathrm{d}\alpha-$$

$$\dot{\tau}(t)\int_{t-\tau(t)}^{t}\dot{x}^{\mathrm{T}}(\alpha)E^{\mathrm{T}}Z_iE\dot{x}(\alpha)\mathrm{d}\alpha. \quad (20)$$

Substituting (20) into (19) gives

$$\dot{V}_i(t)\leqslant\begin{pmatrix}x(t)\\x(t-\tau(t))\end{pmatrix}^{\mathrm{T}}\begin{pmatrix}\Gamma_{12} & \tau_mA_i^{\mathrm{T}}Z_iA_{\tau i}-Y_i+P_iA_{\tau i}\\ * & \tau_mA_{\tau i}^{\mathrm{T}}Z_iA_{\tau i}-(1-\mu)Q_i\end{pmatrix}\begin{pmatrix}x(t)\\x(t-\tau(t))\end{pmatrix}, \quad (21)$$

where $\Gamma_{12}=P_iA_i+A_i^{\mathrm{T}}P_i^{\mathrm{T}}+Q_i+\tau_mX_i+Y_i+Y_i^{\mathrm{T}}+\tau_mA_i^{\mathrm{T}}Z_iA_i$. From (8) and (13), we conclude that $\dot{V}_i(t)<0$.

In order to ensure the asymptotic stability of system $\Sigma_{(2)}$, we shall verify that $V_{\sigma(t)}(t)$ do not increase along with the switching instants. It is supposed that system $\Sigma_{(2)}$ switches to i-th subsystem from j-th subsystem at $t=t_k$. That is to say, $\sigma(t)=j,t\in(t_{k-1},t_k]$, and $\sigma(t)=i,t\in(t_k,t_{k+1}]$. Substituting $t=t_k$ into $\Delta x(t)=C_{\sigma(t)}x(t)$, we have $x(t_k^+)-x(t_k)=C_jx(t_k)$, that is, $x(t_k^+)=(I+C_j)x(t_k)$. This equation, together with (12), yields

$$V_i(t_k^+)=x^{\mathrm{T}}(t_k^+)P_iEx(t_k^+)+\int_{t_k^+-\tau(t_k^+)}^{t_k^+}x^{\mathrm{T}}(s)Q_{\sigma(s)}x(s)\mathrm{d}s+\int_{-\tau(t_k^+)}^{0}\int_{t_k^++\beta}^{t_k^+}\dot{x}^{\mathrm{T}}(\alpha)\cdot E^{\mathrm{T}}Z_{\sigma(\alpha)}E\dot{x}(\alpha)\mathrm{d}\alpha\mathrm{d}\beta=x^{\mathrm{T}}(t_k)(I+C_j)^{\mathrm{T}}P_iE(I+C_j)x(t_k)+\int_{t_k-\tau(t_k)}^{t_k}x^{\mathrm{T}}(s)Q_{\sigma(s)}x(s)\mathrm{d}s+\int_{-\tau(t_k)}^{0}\int_{t_k+\beta}^{t_k}\dot{x}^{\mathrm{T}}(\alpha)E^{\mathrm{T}}Z_{\sigma(\alpha)}E\dot{x}(\alpha)\mathrm{d}\alpha\mathrm{d}\beta,$$

$$V_j(t_k)=x^{\mathrm{T}}(t_k)P_jEx(t_k)+\int_{t_k-\tau(t_k)}^{t_k}x^{\mathrm{T}}(s)Q_{\sigma(s)}x(s)\mathrm{d}s+\int_{-\tau(t_k)}^{0}\int_{t_k+\beta}^{t_k}\dot{x}^{\mathrm{T}}(\alpha)E^{\mathrm{T}}Z_{\sigma(\alpha)}E\dot{x}(\alpha)\mathrm{d}\alpha\mathrm{d}\beta,$$

$$V_i(t_k^+)-V_j(t_k)=x^{\mathrm{T}}(t_k)[(I+C_j)^{\mathrm{T}}P_iE(I+C_j)-P_jE]x(t_k).$$

By (9), there clearly holds $V_i(t_k^+)-V_j(t_k)\leqslant 0$. Hence, we conclude that system $\Sigma_{(2)}$ is asymptotically stable. This completes the proof.

Remark 2 For the nominal and unforced form of the singular impulsive switched system with time-varying delay, this theorem designs the state-dependent switching signal, under which the given system is regular, causal, and asymptotically stable based on the multiple Lyapunov functional technique. Further, it should be observed that this result can also apply to various systems such as singular switched systems, impulsive switched systems and singular impulsive systems. For impulsive switched systems and singular impulsive systems, the following corollaries state the related conclusions, which can fully demonstrate the universality and practicability of the theorem.

Corollary 1 Consider the following impulsive switched system with time delay

$$\Sigma''_{(2)}:\dot{x}(t)=A_{\sigma(t)}x(t)+A_{\tau\sigma(t)}x(t-h), t\neq t_k,$$
$$\Delta x(t)=C_{\sigma(t)}x(t), t=t_k,$$
$$x(t)=\phi(t), t\in[-h,0].$$

If, for any $i\in M$, there exist constants $\beta_{ij}\geqslant 0(j\in M)$, matrices $Q_i>0,P_i>0$ such that

$$\begin{bmatrix}P_iA_i+A_i^{\mathrm{T}}P_i+Q_i+\sum_{j=1}^{m}\beta_{ij}(P_j-P_i) & P_iA_{\tau i}\\ * & -Q_i\end{bmatrix}<0, \quad (22)$$

$$(I+C_j)^{\mathrm{T}}P_i(I+C_j)-P_j\leqslant 0, i\neq j, j\in M,$$

then the system $\Sigma''_{(2)}$ is regular, causal and asymptotically stable under a state-dependent switching signal $\sigma(t)=\arg\min\{x^{\mathrm{T}}(t)P_ix(t), i\in M\}$.

Remark 3 Theorem 2 in paper [18] requires that the energy function decreases on the whole space R^n, that is, every subsystem is stable on the whole space R^n, while this corollary just requires that the energy function decreases on the corresponding area Ω_i, which can stand out the merit of the result proposed in the paper.

Corollary 2 Consider the following singular time-varying delay system

$$\Sigma'''_{(2)}:E\dot{x}(t)=Ax(t)+A_\tau x(t-\tau(t)),$$
$$x(t)=\phi(t), t\in[-\tau_m,0].$$

If there exist matrices $Q>0, X\geqslant 0, Z>0, P$ and Y such that

$$PE=E^{\mathrm{T}}P^{\mathrm{T}}\geqslant 0,$$

$$\begin{bmatrix}\Gamma & \tau_mA^{\mathrm{T}}ZA_\tau-Y+PA_\tau\\ * & \tau_mA_\tau^{\mathrm{T}}ZA_\tau-(1-\mu)Q\end{bmatrix}<0,$$

$$\begin{bmatrix}X & Y\\ * & (1-\mu)E^{\mathrm{T}}ZE\end{bmatrix}\geqslant 0,$$

with $\Gamma=PA+A^{\mathrm{T}}P^{\mathrm{T}}+Q+\tau_mX+Y+Y^{\mathrm{T}}+\tau_mA^{\mathrm{T}}ZA$, then systems $\Sigma'''_{(2)}$ is regular, causal and asymptotically stable.

Remark 4 Lemma 2 in literature [23] studies the constant time delay while this corollary presents the corresponding results for the time-varying delay.

Corollary 3 Consider system $\Sigma_{(2)}$. If, for any $i \in M$, there exist constants $\beta_{ij} \leqslant 0 (j \in M)$, matrices $Q_i > 0, X_i \geqslant 0, Z_i > 0, P_i, Y_i$ satisfying (7), (8), (9), (10), then system $\Sigma_{(2)}$ is regular, causal and asymptotically stable under a switching signal

$$\sigma(t) = \arg\max\{x^{\mathrm{T}}(t)P_i Ex(t), i \in M\}. \quad (23)$$

Remark 5 When $\beta_{ij} \leqslant 0$, this corollary designs a new state-dependent switching signal (23), which differs from Theorem 1. In a word, this corollary, together with Theorem 1, shows two different cases.

2.2 Performance analysis

Based on Theorem 1, we are now in the position to provide the sufficient conditions on the existence of a robust resilient guaranteed cost controller for system $\Sigma_{(1)}$.

Theroem 2 Consider system $\Sigma_{(1)}$ with the cost function (4). If, for $i \in M$, there exist scalars $\beta_{ij} \geqslant 0 (j \in M)$, matrices $Q_i > 0, X_i \geqslant 0, Z_i > 0, P_i, Y_i$ satisfying (7), (10) and

$$\Gamma_2 = \begin{bmatrix} \Gamma_{21} & \tau_m(A_{ki}+\Delta A_{ki})^{\mathrm{T}} Z_i (A_{\tau i}+\Delta A_{\tau i}) - Y_i + P_i(A_{\tau i}+\Delta A_{\tau i}) \\ * & \tau_m(A_{\tau i}+\Delta A_{\tau i})^{\mathrm{T}} Z_i (A_{\tau i}+\Delta A_{\tau i}) - (1-\mu)Q_i \end{bmatrix} < 0, \quad (24)$$

$$(I + C_j + \Delta C_j)^{\mathrm{T}} P_i E (I + C_j + \Delta C_j) - P_j E \leqslant 0, i \neq j, j \in M, \quad (25)$$

where

$$\Gamma_{21} = P_i(A_{ki}+\Delta A_{ki}) + (A_{ki}+\Delta A_{ki})^{\mathrm{T}} P_i^{\mathrm{T}} + Q_i + \tau_m X_i + Y_i + Y_i^{\mathrm{T}} + \tau_m (A_{ki}+\Delta A_{ki})^{\mathrm{T}} Z_i (A_{ki}+\Delta A_{ki}) + S + (K_i+\Delta K_i)^{\mathrm{T}} R (K_i + \Delta K_i) + \sum_{j=1}^{m} \beta_{ij}(P_j - P_i)E,$$

$$A_{ki} = A_i + B_i K_i, \Delta A_{ki} = \Delta A_i + B_i \Delta K_i,$$

and a state-dependent switching signal (11), then controller (5) is a robust resilient guaranteed cost controller for system $\Sigma_{(1)}$ with the performance upper bound

$$J^* = \phi^{\mathrm{T}}(0) P_{\sigma(0)} E \phi(0) + \int_{-\tau(0)}^{0} \phi^{\mathrm{T}}(s) Q_{\sigma(s)} \phi(s) \cdot \mathrm{d}s + \int_{-\tau(0)}^{0} \int_{\beta}^{0} \dot{\phi}^{\mathrm{T}}(\alpha) E^{\mathrm{T}} Z_{\sigma(\alpha)} E \dot{\phi}(\alpha) \mathrm{d}\alpha \mathrm{d}\beta.$$

Proof When $t \in (t_k, t_{k+1}]$, assume that the i-th subsystem is activated. Applying the controller (5) to systems $\Sigma_{(1)}$ results in the following closed-loop system

$$\Sigma_{(3)}: E\dot{x}(t) = (A_{ki} + \Delta A_{ki})x(t) + (A_{\tau i} + \Delta A_{\tau i})x(t-\tau(t)), t \neq t_k,$$

$$\Delta x(t) = (C_j + \Delta C_j)x(t), t = t_k,$$

$$x(t) = \phi(t), t \in [-\tau_m, 0].$$

Based on Theorem 1 and $R > 0, S > 0$, it is easy to obtain that the closed-loop system $\Sigma_{(3)}$ is also regular, causal and asymptotically stable by replacing $A_i, A_{\tau i}, C_j$ with $A_{ki} + \Delta A_{ki}, A_{\tau i} + \Delta A_{\tau i}, C_j + \Delta C_j$. In the next, we shall prove that there exists a positive scalar J^* such that the value of the cost function (4) satisfies $J \leqslant J^*$. Similar to the proof of Theorem 1, when $t \in (t_k, t_{k+1}]$, one has

$$\dot{V}_i(t) \leqslant \begin{pmatrix} x(t) \\ x(t-\tau(t)) \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} \Gamma_{22} & \Gamma_{23} \\ * & \Gamma_{24} \end{pmatrix} \begin{pmatrix} x(t) \\ x(t-\tau(t)) \end{pmatrix},$$

where

$$\Gamma_{22} = P_i(A_{ki}+\Delta A_{ki}) + (A_{ki}+\Delta A_{ki})^{\mathrm{T}} P_i^{\mathrm{T}} + Q_i + \tau_m X_i + Y_i + Y_i^{\mathrm{T}} + \tau_m (A_{ki}+\Delta A_{ki})^{\mathrm{T}} Z_i (A_{ki}+\Delta A_{ki}) + \sum_{j=1}^{m} \beta_{ij}(P_j - P_i)E,$$

$$\Gamma_{23} = \tau_m (A_{ki}+\Delta A_{ki})^{\mathrm{T}} Z_i (A_{\tau i}+\Delta A_{\tau i}) - Y_i + P_i(A_{\tau i}+\Delta A_{\tau i}),$$

$$\Gamma_{24} = \tau_m (A_{\tau i}+\Delta A_{\tau i})^{\mathrm{T}} Z_i (A_{\tau i}+\Delta A_{\tau i}) - (1-\mu)Q_i.$$

From (24), we derive

$$\dot{V}_i(t) < -x^{\mathrm{T}}(t)[S + (K_i+\Delta K_i)^{\mathrm{T}} R (K_i + \Delta K_i)]x(t), \quad (26)$$

which gives rise to

$$J = \int_0^{+\infty} x^{\mathrm{T}}(t) S x(t) + u_{\sigma(t)}^{\mathrm{T}}(t) R u_{\sigma(t)}(t) \mathrm{d}t = \lim_{\rho \to \infty} \sum_{k=0}^{\rho} \int_{t_k^+}^{t_{k+1}} x^{\mathrm{T}}(t)[S + (K_{\sigma(t)} + \Delta K_{\sigma(t)})^{\mathrm{T}} R (K_{\sigma(t)} + \Delta K_{\sigma(t)})]x(t)\mathrm{d}t < -\lim_{\rho \to \infty} \sum_{k=0}^{\rho} \int_{t_k^+}^{t_{k+1}} \dot{V}_{\sigma(t)}(t)\mathrm{d}t = -\lim_{\rho \to \infty}[-V_{\sigma(0)}(0) + \sum_{k=0}^{\rho}(V_{\sigma(t_k)}(t_k) - V_{\sigma(t_k^+)}(t_k^+)) + V_{\sigma(t_{\rho+1})}(t_{\rho+1})] \leqslant V_{\sigma(0)}(0) = J^*.$$

Therefore, by Definition 3, controller (5) is a robust resilient guaranteed cost controller for system $\Sigma_{(1)}$ with the performance upper bound J^*. The proof is completed.

Remark 6 Based on the Theorem 1, this theorem further analyzes the performance of the singular impulsive switched system with time-varying delay. It is necessary to point out that the controller designed in Theorem 2 is not only a guaranteed cost controller but also a resilient controller. In addition, there exist uncertainties in the system structure, which, together with uncertainties in resilient controller, make it more difficult to simplify inequalities. The corresponding process will be stated in detail.

Remark 7 The paper [23] designs a robust resilient guaranteed cost controller for the uncertain singular time-delay system, but the main results in [23] fail to work when impulsive phenomena or switching behaviors occur. On the contrast, this theorem is feasible for the case that impulsive phenomena and switching behaviors take place at the same time. Obviously, Theorem 1 in literature [23] is the special case of this theorem, which shows that this conclusion has the broader application and less conservativeness.

Remark 8 It should be observed that the paper [18] ignores uncertainties of impulses. Here, it is more meaningful to add the uncertain term $\Delta C_{\sigma(t)}$ to the system matrix, which to some extent can reflect some uncertainties of impulsive phenomena. Besides, compared with the paper [18], the more complex systems are considered and the more information in Lyapunov functional are added in this theorem.

Corollary 4 Consider system $\Sigma_{(1)}$ with the cost function (4). If, for $i \in M$, there exist scalars $\beta_{ij} \leqslant 0 (j \in M)$, matrices $Q_i > 0, X_i \geqslant 0, Z_i > 0, P_i, Y_i$ satisfying (7), (10), (24), (25), and a state-dependent switching signal satisfying (23), then controller (5) is a robust resilient guaranteed cost controller for system $\Sigma_{(1)}$ with the performance upper bound J^* in the form of (26).

2.3 The robust resilient guaranteed cost controller design

In Theorem 2, uncertain terms $\Delta A_i, \Delta A_{\tau i}, \Delta K_i, \Delta C_j$ exist in conditions, which makes it impossible to solve inequalities. Therefore, how to remove uncertain terms is the key to overcome this problem. Here, by the LMIs technique, the feasible conditions solving a robust resilient guaranteed cost controller for systems $\Sigma_{(1)}$ are presented in Theorem 3.

Theorem 3 Consider system $\Sigma_{(1)}$ with the cost function (4). If, for any $i \in M$, the following conditions hold

a. there exist scalars $\beta_{ij} \geqslant 0 (j \in M), \lambda_i > 0, \epsilon_i > 0, \rho_j > 0$, matrices $Q_i > 0, X_i \geqslant 0, Z_i > 0, P_i, Y_i, G_i$ satisfying (7), (10) and

$$\Gamma_3 = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ * & \Lambda_{22} \end{bmatrix} < 0, \tag{27}$$

$$\bar{\Gamma}_3 = \begin{bmatrix} -I & G_i(I + C_j) & G_i N_{5j} \\ * & -P_j E + \rho_j D_{5j}^{\mathrm{T}} D_{5j} & 0 \\ * & * & -\rho_j I \end{bmatrix} \leqslant 0,$$

$$i \neq j, j \in M, \tag{28}$$

where

$$\Lambda_{11} = \begin{bmatrix} \Gamma_{31} & \Gamma_{32} & \Gamma_{33} & I & \lambda_i P_i B_i \\ * & \Gamma_{34} & \tau_m A_{\tau i}^{\mathrm{T}} & 0 & 0 \\ * & * & -\tau_m Z_i^{-1} & 0 & 0 \\ * & * & * & -S^{-1} & 0 \\ * & * & * & * & -R^{-1} \end{bmatrix},$$

$$\Gamma_{31} = P_i A_i + A_i^{\mathrm{T}} P_i^{\mathrm{T}} + Q_i + \tau_m X_i + Y_i + Y_i^{\mathrm{T}} + \sum_{j=1}^{m} \beta_{ij}(P_j - P_i)E + \epsilon_i(D_{1i}^{\mathrm{T}} D_{1i} + 2 D_{3i}^{\mathrm{T}} D_{3i}),$$

$$\Gamma_{32} = P_i A_{\tau i} - Y_i,$$

$$\Gamma_{33} = \tau_m (A_i^{\mathrm{T}} + \lambda_i P_i B_i B_i^{\mathrm{T}}),$$

$$\Gamma_{34} = \epsilon_i D_{2i}^{\mathrm{T}} D_{2i} - (1 - \mu) Q_i,$$

$$\Lambda_{12} = \begin{bmatrix} P_i N_{1i} & P_i B_i N_{3i} & P_i N_{2i} & 0 & \sqrt{2\lambda_i} P_i B_i \\ 0 & 0 & 0 & 0 & 0 \\ \tau_m N_{N_{1i}} & \tau_m B_i N_{3i} & \tau_m N_{2i} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & N_{3i} & 0 \end{bmatrix},$$

$$\Lambda_{22} = \begin{bmatrix} -\epsilon_i I & 0 & 0 & 0 & 0 \\ * & -\epsilon_i I & 0 & 0 & 0 \\ * & * & -\epsilon_i I & 0 & 0 \\ * & * & * & -\epsilon_i I & 0 \\ * & * & * & * & -I \end{bmatrix},$$

$$P_i E = G_i^{\mathrm{T}} G_i,$$

b. there exists a state-dependent switching signal satisfying (11), then controller (5) is a robust resilient guaranteed cost controller for system $\Sigma_{(1)}$. Here, the controller gain is

$$K_i = \lambda_i B_i^{\mathrm{T}} P_i^{\mathrm{T}}, \tag{29}$$

and the performance upper bound J^* can be given in the form of (26).

Proof Using (27), (29), and Schur complement lemma, we obtain that $\Gamma_3 < 0$ is equivalent to

$$\begin{bmatrix} \Delta_{11} & \Delta_{12} \\ * & \Delta_{22} \end{bmatrix} < 0, \tag{30}$$

where

$$\Delta_{11}=$$

$$\begin{bmatrix} \Gamma_{35} & P_iA_{\tau i}-Y_i & \tau_m A_{ki}^{\mathrm{T}} & I & K_i^{\mathrm{T}} \\ * & \Gamma_{34} & \tau_m A_{\tau i}^{\mathrm{T}} & 0 & 0 \\ * & * & -\tau_m Z_i^{-1} & 0 & 0 \\ * & * & * & -S^{-1} & 0 \\ * & * & * & * & -R^{-1} \end{bmatrix},$$

$$\Gamma_{35}=P_iA_{ki}+A_{ki}^{\mathrm{T}}P_i^{\mathrm{T}}+Q_i+\tau_m X_i+Y_i+Y_i^{\mathrm{T}}+\sum_{j=1}^{m}\beta_{ij}(P_j-P_i)E+\epsilon_i(D_{1i}^{\mathrm{T}}D_{1i}+2D_{3i}^{\mathrm{T}}D_{3i}),$$

$$A_{ki}=A_i+B_iK_i,$$

$$\Delta_{12}=$$

$$\begin{bmatrix} P_iN_{1i} & P_iB_iN_{3i} & P_iN_{2i} & 0 \\ 0 & 0 & 0 & 0 \\ \tau_m N_{1i} & \tau_m B_iN_{3i} & \tau_m N_{2i} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & N_{3i} \end{bmatrix},$$

$$\Delta_{22}=$$

$$\begin{bmatrix} -\epsilon_i I & 0 & 0 & 0 \\ * & -\epsilon_i I & 0 & 0 \\ * & * & -\epsilon_i I & 0 \\ * & * & * & -\epsilon_i I \end{bmatrix}.$$

From (30), we can derive

$$H_1+\varepsilon_i DD^{\mathrm{T}}+\epsilon_i^{-1}N^{\mathrm{T}}N<0, \tag{31}$$

where

$$H_1=$$

$$\begin{bmatrix} \Gamma_{36} & P_iA_{\tau i}-Y_i & \tau_m A_{ki}^{\mathrm{T}} & I & K_i^{\mathrm{T}} \\ * & -(1-\mu)Q_i & \tau_m A_{\tau i}^{\mathrm{T}} & 0 & 0 \\ * & * & -\tau_m Z_i^{-1} & 0 & 0 \\ * & * & * & -S^{-1} & 0 \\ * & * & * & * & -R^{-1} \end{bmatrix},$$

$$\Gamma_{36}=P_iA_{ki}+A_{ki}^{\mathrm{T}}P_i^{\mathrm{T}}+Q_i+\tau_m X_i+Y_i+Y_i^{\mathrm{T}}+\sum_{j=1}^{m}\beta_{ij}(P_j-P_i)E,$$

$$D^{\mathrm{T}}=\begin{bmatrix} D_{1i} & 0 & 0 & 0 & 0 \\ D_{3i} & 0 & 0 & 0 & 0 \\ 0 & D_{2i} & 0 & 0 & 0 \\ D_{3i} & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$N^{\mathrm{T}}=\begin{bmatrix} P_iN_{1i} & P_iB_iN_{3i} & P_iN_{2i} & 0 \\ 0 & 0 & 0 & 0 \\ \tau_m N_{1i} & \tau_m B_iN_{3i} & \tau_m N_{2i} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & N_{3i} \end{bmatrix}.$$

Define

$$F(t)=\begin{bmatrix} F_{1i}^{\mathrm{T}}(t) & 0 & 0 & 0 \\ * & F_{3i}^{\mathrm{T}}(t) & 0 & 0 \\ * & * & F_{2i}^{\mathrm{T}}(t) & 0 \\ * & * & * & F_{3i}^{\mathrm{T}}(t) \end{bmatrix}.$$

After some manipulations, by Lemma 1, we get from (1),(2),(6)

$$\begin{bmatrix} \Gamma_{37} & \Gamma_{38} & \tau_m(A_{ki}+\Delta A_{ki})^{\mathrm{T}} & I & (K_i+\Delta K_i)^{\mathrm{T}} \\ * & -(1-\mu)Q_i & \tau_m(A_{\tau i}+\Delta A_{\tau i})^{\mathrm{T}} & 0 & 0 \\ * & * & -\tau_m Z_i^{-1} & 0 & 0 \\ * & * & * & -S^{-1} & 0 \\ * & * & * & * & -R^{-1} \end{bmatrix}<0, \tag{32}$$

where $\Gamma_{37}=P_i(A_{ki}+\Delta A_{ki})+(A_{ki}+\Delta A_{ki})^{\mathrm{T}}P_i^{\mathrm{T}}+Q_i+\tau_m X_i+Y_i+Y_i^{\mathrm{T}}+\sum_{j=1}^{m}\beta_{ij}(P_j-P_i)E$, $\Gamma_{38}=P_i(A_{\tau i}+\Delta A_{\tau i})-Y_i$. Obviously, we can see that (32) is equivalent to (24). From (28), by Schur complement lemma, one has

$$\begin{bmatrix} -I & G_i(I+C_j) \\ * & -P_jE \end{bmatrix}+\rho_j\begin{bmatrix} 0 \\ D_{5j}^{\mathrm{T}} \end{bmatrix}[0 \quad D_{5j}]+\rho_j^{-1}\begin{bmatrix} G_iN_{5j} \\ 0 \end{bmatrix}[N_{5j}^{\mathrm{T}}G_i^{\mathrm{T}} \quad 0]\leqslant 0. \tag{33}$$

By Lemma 1, (3) and (33), we get

$$\begin{bmatrix} -I & G_i(I+C_j+\Delta C_j) \\ * & -P_jE \end{bmatrix}\leqslant 0. \tag{34}$$

Utilizing Schur complement lemma again and replacing $G_i^{\mathrm{T}}G_i$ with P_iE, we can see that inequality (34) is equivalent to (25). This completes the proof.

Remark 9 It is easy to see that various techniques are utilized to simplify inequalities of Theorem 2. Eventually, uncertain terms are successfully removed from conditions. Meanwhile, all the conditions are cast into LMIs for the given scalars β_{ij}, λ_i, which can be solved by the LMIs toolbox.

Remark 10 We state briefly the solving sequence of inequalities of Theorem 3.

Step 1 Calculate the P_i, Q_i, X_i, Z_i, Y_i by (7), (10) and (27).

Step 2 Decompose the positive semi-definite matrix P_iE into the product of G_i^{T} and G_i.

Step 3 Verify the condition (28), and solve the controller gain by (29).

Corollary 5 Consider system $\Sigma_{(1)}$ with the cost function (4). If, for any $i\in M$, the following conditions hold

a. there exist scalars $\beta_{ij}\leqslant 0(j\in M)$, $\lambda_i>0$, $\epsilon_i>$

$0, \rho_j > 0$, matrices $Q_i > 0, X_i \geqslant 0, Z_i > 0, P_i, Y_i, G_i$ satisfying (7), (10), (27), (28),

b. there exists a state-dependent switching signal satisfying (23), then controller (5) is a robust resilient guaranteed cost controller for system $\Sigma_{(1)}$. Here, the controller gain is (29), and a performance upper bound J^* can be given in the form of (26).

2.4 The optimal robust resilient guaranteed cost controller design

Theorem 3 factually presents a set of parameter representations of guaranteed cost controllers. From the expression of J^*, the upper bound of the performance not only depends on the selection of guaranteed cost controllers but also matrices Q_i, Z_i. Therefore, it is imperative to optimize the values of matrices in order to achieve the minimal guaranteed cost of the corresponding closed-loop system.

Theorem 4 For system $\Sigma_{(1)}$ with $\Delta C_{\sigma(t)} = 0$, and the cost function (4), if the following optimization problem Ω_{opt}

$$\min_{\beta_{ij}, \lambda_i, \epsilon_i, P_i, Q_i, Z_i, X_i, Y_i} C_1\alpha_i + C_2\beta_i + C_3\gamma_i \quad \text{s. t.}$$

(a) (7), (9), (10), (27),

(b) $\beta_{ij} \geqslant 0 (j \in M)$,

$$\text{(c) } \lambda_i > 0, \epsilon_i > 0, \tag{35}$$

$$\text{(d) } \begin{bmatrix} -\alpha_i I & P_i E \\ * & -\alpha_i I \end{bmatrix} < 0, \tag{36}$$

$$\text{(e) } \begin{bmatrix} -\beta_i I & Q_i \\ * & -\beta_i I \end{bmatrix} < 0, \tag{37}$$

$$\text{(f) } \begin{bmatrix} -\gamma_i I & E^{\mathrm{T}} Z_i E \\ * & -\gamma_i I \end{bmatrix} < 0, \tag{38}$$

has a solution $(\tilde{\beta}_{ij}, \tilde{\lambda}_i, \tilde{\epsilon}_i, \tilde{P}_i, \tilde{Q}_i, \tilde{Z}_i, \tilde{X}_i, \tilde{Y}_i)$, then under switching signal (11), there exists an optimal resilient guaranteed cost control controller $u^*_{\sigma(t)}(t) = (\tilde{K}_{\sigma(t)} + \Delta K_{\sigma(t)}) x(t)$ for system $\Sigma_{(1)}$. Here, the controller gain is $\tilde{K}_i = \tilde{\lambda}_i B_i^{\mathrm{T}} \tilde{P}_i^{\mathrm{T}}$, and the minimal cost upper bound is $J^*_{\min} = \min_{i\in M} C_1\alpha_i + C_2\beta_i + C_3\gamma_i$, where $\tilde{K}_i = \tilde{\lambda}_i B_i^{\mathrm{T}} \tilde{P}_i^{\mathrm{T}}$, $C_1 = \phi^{\mathrm{T}}(0)\phi(0)$, $C_2 = \int_{-\tau(0)}^{0} \phi^{\mathrm{T}}(s)\phi(s)\mathrm{d}s$, $C_3 = \int_{-\tau(0)}^{0}\int_{\beta}^{0} \dot{\phi}^{\mathrm{T}}(\alpha)\dot{\phi}(\alpha)\mathrm{d}\alpha\mathrm{d}\beta$.

Proof If $(\tilde{\beta}_{ij}, \tilde{\lambda}_i, \tilde{\epsilon}_i, \tilde{P}_i, \tilde{Q}_i, \tilde{Z}_i, \tilde{X}_i, \tilde{Y}_i)$ is a solution of the optimization problem Ω_{opt}, then it is also a feasible solution under the constraint conditions (a), (b), and (c). From Theorem 3, $u^*_{\sigma(t)}(t) = (\tilde{K}_{\sigma(t)} + \Delta K_{\sigma(t)}) x(t)$ is a robust resilient guaranteed cost controller. Observe that

(36) $\Leftrightarrow \sigma_{\max}(P_i E) < \alpha_i$, $\phi^{\mathrm{T}}(0) P_i E \phi(0) \leqslant \sigma_{\max}(P_i E) C_1$,

(37) $\Leftrightarrow \sigma_{\max}(Q_i) < \beta_i$, $\int_{-\tau(t)}^{0} \phi^{\mathrm{T}}(s) Q_{\sigma(s)} \phi(s) \mathrm{d}s \leqslant \sigma_{\max}(Q_i) C_2$,

(38) $\Leftrightarrow \sigma_{\max}(E^{\mathrm{T}} Z_i E) < \gamma_i$, $\int_{-\tau(0)}^{0}\int_{\beta}^{0} \dot{\phi}^{\mathrm{T}}(\alpha) E^{\mathrm{T}} Z_{\sigma(\alpha)} E \dot{\phi}(\alpha)\mathrm{d}\alpha\mathrm{d}\beta \leqslant \sigma_{\max}(E^{\mathrm{T}} Z_i E) C_3$.

Therefore, the minimization of $C_1\alpha_i + C_2\beta_i + C_3\gamma_i$ implies the minimization of the guaranteed cost J^*. The optimal solution of problem Ω_{opt} can be derived from the convexity of the objective function and constraint conditions. This completes the proof.

Remark 11 In order to obtain the optimal robust resilient guaranteed cost controller, a minimization approach of the largest singular value of matrices and a convex optimization method are introduced, which play an important role in the proof. In addition, Theorem 3 provides a feasible solution of solving a robust resilient guaranteed cost controller while this theorem further gives a optimal robust resilient guaranteed cost controller. To some extent, this theorem improves the conclusion of Theorem 3.

3 Numerical examples

Example 1 Consider the impulsive switched systems $\Sigma''_{(2)}$ with parameters given below

$$A_1 = \begin{bmatrix} -1 & 0 \\ 0 & -1.2 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 0 \\ 0 & -0.1 \end{bmatrix},$$

$$A_{\tau 1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} -0.1 & 0 \\ 0 & 1 \end{bmatrix}, A_{\tau 2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

For the given system, the linear matrix inequalities have not a feasible solution by Theorem 2 in [18]. Therefore, we are unable to judge the stability of the above system and Theorem 2 in [18] fails to work. However, Corollary 1 in this paper can be worked well to check the stability of the given system. Choosing $\beta_{12} = -0.2$, $\beta_{21} = -0.1$, we can see that the nonlinear matrix inequality (22) becomes the linear matrix inequality which can be solved by LMIs toolbox as following

$$P_1 = \begin{bmatrix} 192.1580 & -120.9833 \\ -120.9833 & 76.4228 \end{bmatrix},$$

$$P_2=\begin{bmatrix}4.6268 & -7.4789\\ -7.4789 & 12.0992\end{bmatrix}.$$

Under the switching signal $\sigma(t)=\arg\min\{x^{\mathrm T}(t)P_ix(t), i\in\{1,2\}\}$, the given system is asymptotically stable from Fig. 1, which can verify the feasibility of Corollary 1. In conclusion, both the theoretical analysis in Remark 3 and simulation result can show the fact that Corollary 1 has the wider application and the less conservativeness than the result in [18].

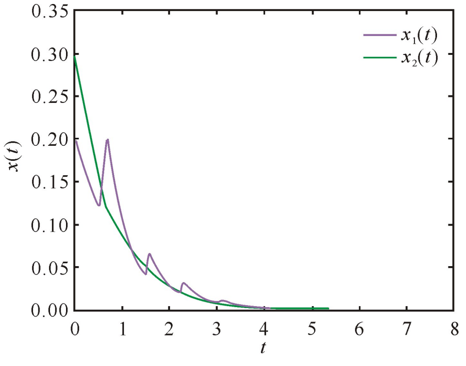

Fig. 1 The state trajectory $x(t)$ of the given system

Example 2 Consider the uncertain impulsive switched singular time-varying delay system $\Sigma_{(1)}$ with parameters given below

$$A_1=\begin{bmatrix}-0.7 & 0.1 & 0.1\\ -0.01 & -1 & 0.02\\ 0.1 & 0.1 & 1\end{bmatrix},$$

$$A_2=\begin{bmatrix}-1 & 0.1 & 2\\ 0.2 & -1.2 & -0.1\\ 0 & -0.1 & -1\end{bmatrix},$$

$$A_{\tau1}=\begin{bmatrix}-0.3 & 0 & 0\\ 0 & -0.1 & 0\\ 0 & 0 & -0.1\end{bmatrix},$$

$$A_{\tau2}=\begin{bmatrix}-0.1 & 0 & 0\\ 0 & -0.2 & 0\\ 0 & 0.1 & 0\end{bmatrix},$$

$$B_1=\begin{bmatrix}-1 & 1 & 0\\ 0 & 0.1 & 0\\ 0 & 0 & 0.2\end{bmatrix},$$

$$B_2=\begin{bmatrix}-2 & 0.1 & 0\\ 0 & 0.1 & 0\\ 0 & 0 & 0.1\end{bmatrix},$$

$$C_1=C_2=\begin{bmatrix}-0.9 & -0.8 & 0\\ -0.6 & -0.9 & 0\\ 0 & 0 & -1\end{bmatrix},$$

$$E=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{bmatrix},$$

$S=R=I$,

$N_{1i}=N_{2i}=N_{3i}=D_{1i}=D_{2i}=D_{3i}=0.1I, N_{5i}=D_{5i}=0, i=1,2$,

$F_{1i}=F_{2i}=F_{3i}=F_{5i}=0.1\sin(t)I, i=1,2$, $\tau(t)=0.1\sin t$.

Choose $\tau_m=1, \mu=0.1, \lambda_1=\lambda_2=0.01$, and give the initial function $\phi(t)=[1\quad t\quad 0]^{\mathrm T}$.

By Theorem 3, we can obtain

$$P_1=\begin{bmatrix}8.6770 & 0.1519 & -0.9043\\ 0.1519 & 8.9089 & 0.3313\\ 0 & 0 & -13.7769\end{bmatrix},$$

$$P_2=\begin{bmatrix}7.1749 & 0.0622 & 14.5406\\ 0.0622 & 7.9172 & -0.5976\\ 0 & 0 & 12.4873\end{bmatrix},$$

a robust resilient guaranteed cost controller $u_{\sigma(t)}(t)=(K_{\sigma(t)}+\Delta K_{\sigma(t)})x(t)$ with

$$K_1=\begin{bmatrix}-0.0868 & -0.0015 & 0\\ 0.0869 & 0.0104 & 0\\ -0.0018 & 0.0007 & -0.0276\end{bmatrix},$$

$$K_2=\begin{bmatrix}-0.1435 & -0.0012 & 0\\ 0.0072 & 0.0080 & 0\\ 0.0145 & -0.0006 & 0.0125\end{bmatrix},$$

and a performance upper bound $J^*=8.6770$. The above results derived from Theorem 3 just present a feasible solution. In the following, we aim at seeking the optimal controller and the optimal performance upper bound of systems $\Sigma_{(1)}$ by Theorem 4. By solving optimization problem Ω_{opt}, one gets

$$\widetilde{P}_{1opt}=\begin{bmatrix}0.9498 & 0.0129 & -0.3601\\ 0.0129 & 0.8243 & -0.3620\\ 0 & 0 & -3.7143\end{bmatrix},$$

$$\widetilde{P}_{2opt}=\begin{bmatrix}0.7835 & 0 & 0.1360\\ 0 & 0.7832 & 4.3182\\ 0 & 0 & 34.2693\end{bmatrix}.$$

The switching signal is designed by

$$\sigma(t)=\begin{cases}1 & x(t)\in\widetilde{\Omega}_1,\\ 2 & x(t)\in\widetilde{\Omega}_2\backslash\widetilde{\Omega}_1,\end{cases} \tag{39}$$

where $\widetilde{\Omega}_1=\{x(t)\in R^n \mid x^{\mathrm T}(t)(\widetilde{P}_{2opt}-\widetilde{P}_{1opt})x(t)\geqslant 0, x(t)\neq 0\}$, $\widetilde{\Omega}_2=\{x(t)\in R^n \mid x^{\mathrm T}(t)(\widetilde{P}_{1opt}-\widetilde{P}_{2opt})x(t)\geqslant 0, x(t)\neq 0\}$.

The optimal robust resilient guaranteed cost controller is designed as $u^*_{\sigma(t)}(t)=(\widetilde{K}_{\sigma(t)}+\Delta K_{\sigma(t)})x(t)$ with

$$\widetilde{K}_1=\begin{bmatrix}-0.0095 & -0.0001 & 0\\ 0.0095 & 0.0010 & 0\\ -0.0007 & -0.0007 & -0.0074\end{bmatrix},$$

$$\widetilde{K}_2=\begin{bmatrix}-0.0157 & 0 & 0\\ 0.0008 & 0.0008 & 0\\ 0.0001 & 0.0043 & 0.0343\end{bmatrix},$$

and the optimal performance upper bound $J^*_{\min}=0.7835$.

From Fig. 2, under the switching signal (39), the closed-loop system is asymptotically stable, which can illustrate the correctness of Theorem 4.

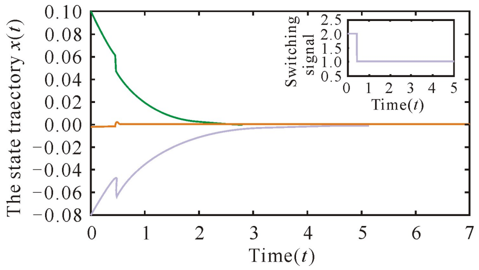

Fig. 2 The state trajectory $x(t)$ of the closed-loop system

4 Conclusions

In this paper, we have investigated the problem of the robust resilient guaranteed cost control for the uncertain impulsive switched singular system with time-varying delay. A robust resilient guaranteed cost controller and a state-dependent switching signal have been established, which guarantee that the closed-loop system is regular, causal, asymptotically stable, and satisfies a cost upper bound. Further, a minimization approach and a convex optimization method have been presented to seek the optimal robust resilient guaranteed cost controller. For the sake of the computation, all the conditions have been cast into LMIs, which can be easily solved by the LMIs toolbox. Finally, two examples have been provided to show the effectiveness of the main conclusions.

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