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The Moment Stability and Stabilization of Hybrid Stochastic Retarded Systems under Asynchronous Markovian Switching*

异步马尔科夫切换混杂随机时滞系统的矩稳定与镇定

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Abstract: This paper investigates the pth moment stability and stabilization problems of a class of hybrid stochastic retarded systems under asynchronous Markovian switching. By exploring the relationship between the sizes of Markov switching signal detection time delay and the generator of Markov chain, a novel integral-inequality-estimation technique is developed to deal with time-varying delay. The Lyapunov stability criterion of asynchronous Markov switching time delay system is established. Then the criterion is applied to a class of Markovian jump time-delay systems, and the delay independent stability criterion and the design of stabilization controller are given. Finally, two numerical examples are provided to demonstrate the validity of the developed results.

Key words: hybrid stochastic systems, asynchronous switching control, Markovian switching, delay-independent stability

摘要:针对一类具有异步马尔科夫切换的混杂时滞随机系统,研究 p-阶矩稳定性与镇定问题。探究马尔科夫切换信号检测时延的大小与马尔科夫链生成元之间的关系,并发展一种不等式处理时变时滞的新型技术,建立了异步马尔科夫切换时滞系统的 Lyapunov 稳定性判据。然后将该稳定性判据应用于一类马尔科夫跳变时滞系统,给出了时滞无关的稳定性判据和镇定性控制器的设计方案。最后,运用两个数值实例验证本研究方法的有效性。

关键词:混杂随机系统　异步切换控制　马尔科夫切换　时滞无关稳定性

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0 Introduction

Switched systems are a class of hybrid systems

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which are widely applied in many fields such as failure-prone manufacturing, traffic management, power systems, and networked control systems. Stability properties and control synthesis are the crucial and fundamental problems of these successful applications, which have been extensively studied (see, for instance and the references therein[1-5]). It has been shown that mode-dependent controller is more general, flexible and less conservative in solving control synthesis problems. Unfortunately, most of the work done on the mode-dependent controller is built upon the assumption that the switching signal of controller and the system modes are strictly synchronized. The so-called "asynchronous switching" which takes into account the detected delay of switching signal when implementing the controller may be reasonable in reality[6-7]. However, asynchronous switching may lead to instability or performance degradation of the switched system due to the mismatched controller act on each subsystem. Therefore, asynchronous switching control is more realistic and challenging. Some salient results have been done on switched systems under asynchronous switching control, such as [6-12]. The stabilization problem of switched linear systems with detected delay of switching signal was studied in [8]. By using a novel Lyapunov-like function approach, the authors of [6] solved the stability and l_2-gain problems for discrete-time switched systems with average dwell time and asynchronous switching. Some further result on asynchronous switching control for continuous-time and discrete-time switched systems were obtained in [10]. Based on a new integral inequality and the piecewise Lyapunov-Krasovskii functional technique, the stabilization problem for a class of switched linear neutral systems under asynchronous switching was addressed in [12].

It should be pointed out that the above-mentioned results are only suitable for deterministic systems. As we all know, in real-world evolutionary processes, noise is unavoidable. The presence of noise can degrade the performance of the corresponding deterministic dynamics and even may drastically alter the system dynamics behaviors[13]. Consequently, stochastic modeling has played an important role in areas of automatic control. Recent years, hybrid stochastic systems which driven by continuous-time Markov chains and Wiener process have been studied by many works, see [13-22]. However, there is few work which has been done on hybrid stochastic systems with asynchronous switching control. For dwell-time-based asynchronous switching control, there is a general requirement (**R**): The detected delay of switching signal is less than the corresponding switching interval, see [23-25]. However, this requirement seems to be difficult to meet for randomly switched systems due to the switching signal is a stochastic process. For example, in Markovian switching control, it is well known that almost every sample path of Markov chain is a right-continuous step function, i. e., for every sample path, the switching intervals is almost sure greater than zeros, but it is not easily to obtain the lower bound of the switching interval for all sample paths. Although the asynchronous Markovian switching issues have been studied[26-28], there is further room for investigation. For example, in [26-27], the authors only investigated the stability and stabilization of discrete-time Markovian jump systems via a time-delayed controller. Under the assumption **R**, Razumikhin-type stability criteria for hybrid stochastic retarded systems were established in [28], but the stability conditions depend on a sequence of stopping time caused by Markov chain, and thus resulting in inconvenient application and verification.

This paper revisits asynchronous Markovian switching control problem of hybrid stochastic retarded systems through considering the relationship among the sizes of detected delay of switching signal and the transition probabilities of Markov chain. In contrast to Halanay's inequality or Razumikhin-type analysis technique, an integral-inequality-estimation technique is proposed to establish pth moment exponential stability criteria irrespective of the sizes of the state delay for hybrid stochastic retarded systems with an asynchronous Markovian switching controller. For its application, the obtained results are applied to a class of stochastic delayed systems with asynchronous Markovian switching that include linear stochastic systems, recurrent neural networks. Moreover, the stabilizing mode-dependent controller is designed by solving a set of linear matrix inequalities (LMIs). Compared with the case of synchronous switching results[16-18], our results do not impose any restriction on the derivative of state delay.

1 System description and preliminaries

Notation For a real symmetric matrix A, the notation $A>(<,\leqslant,\geqslant)0$ means that the matrix A is positive (negative, semi-negative, semi-positive) definite, and $\lambda_{\max}(A),\lambda_{\min}(A)$, respectively, denote the largest and least eigenvalue of A. The symmetric elements of a symmetric matrix are denoted by $*$. $\|\cdot\|$ denotes the Euclidean vector norm. For $\tau>0$, let $C([-\tau,0];\mathbf{R}^n)$ denote the space of continuous functions ϕ from $[-\tau,0]$ to $\mathbf{R}^n$ with norm $\|\phi\|_\tau=\sup_{-\tau\leqslant s\leqslant 0}\|\phi(t+s)\|$. For a given function $\phi(t)\in C([-\tau,b];\mathbf{R}^n)$ with $b>0$ and $t\in[0,b]$, the associated function $\phi_t\in C([-\tau,0];\mathbf{R}^n)$ is defined as $\phi_t(s)=\phi(t+s),s\in[-\tau,0]$. $C^{2,1}(\mathbf{R}^n\times\mathbf{R}_+;\mathbf{R}_+)$ denotes the family of all nonnegative functions $V(x,t)$ on $\mathbf{R}^n\times\mathbf{R}_+$ that are twice continuously differentiable in x and once in t. Given a complete probability space $(\Omega,\mathscr{F},\mathrm{P})$ with a natural filtration $\{\mathscr{F}_t\}_{t\geqslant 0}$ satisfying the usual conditions, let $w(t)=(w_1(t),\cdots,w_m(t))^{\mathrm{T}}\in\mathbf{R}^m$ be an m-dimensional Wiener process defined on the probability space.

Consider the following time-delay hybrid stochastic system

$$\begin{cases}\mathrm{d}x(t)=[f_{\sigma(t)}(x(t),x(t-\tau(t)),t)+\\ \quad h_{\sigma(t)}(x(t),t)u_{\sigma(t)}(x(t))]\mathrm{d}t+\\ \quad g_{\sigma(t)}(x(t),x(t-\tau(t)),t)\mathrm{d}w(t),t>t_0,\\ x_{t_0}(s)=\varphi(s),-\bar{\tau}\leqslant s\leqslant 0,\sigma(t_0)=\sigma_0,\end{cases}\tag{1}$$

where $x(t)\in\mathbf{R}^n$ is the state variable. $\{\sigma(t),t\geqslant t_0\}$ is a right-continuous Markov process defined on the probability space which takes values in the finite set $\mathscr{M}=\{1,2,\cdots,N\}$ with generator $\Pi=(\pi_{ij}),i,j\in\mathscr{M}$, given by

$$\mathrm{P}\{\sigma(t+\Delta t)=j\mid\sigma(t)=i\}=$$
$$\begin{cases}\pi_{ij}\Delta t+o(\Delta t),i\neq j,\\ 1+\pi_{ii}\Delta t+o(\Delta t),i=j,\end{cases}$$

where $\Delta t>0,\lim_{\Delta t\to 0}\dfrac{o(\Delta t)}{\Delta t}=0$, and $\pi_{ij}\geqslant 0$ for $i\neq j$, $\pi_{ii}\leqslant 0$ with $\sum_{j=1,j\neq i}^{N}\pi_{ij}=-\pi_{ii}$. $\tau(t)$ are state delay satisfies $0\leqslant\tau(t)\leqslant\bar{\tau}$. For each $i\in\mathscr{M}$, $f_i:\mathbf{R}^n\times\mathbf{R}^n\times\mathbf{R}_+\to\mathbf{R}^n$, $g_i:\mathbf{R}^n\times\mathbf{R}^n\times\mathbf{R}_+\to\mathbf{R}^{n\times m}$, $h_i:\mathbf{R}^n\times\mathbf{R}_+\to\mathbf{R}^{n\times n_0}$, and $u_i:\mathbf{R}^n\to\mathbf{R}^{n_0\times n}$ is mode-dependent control input which is used to achieve system stability or certain performances.

It is noted that in the controlled system (1), the control input is coincident with the switching rule. However, just as [6-7] point out that this requirement is hard to be satisfied in the physical systems, the control input may exist a time delay which is induced by the identification of the system modes or the implementation of the matched controller. That is, the control input $u_{\sigma(t)}(x(t))$ should be modified by $u_{\sigma(t-\delta)}(x(t))$, where $\delta>0$ and $\sigma(t-\delta)=\sigma_0$ if $t\leqslant\delta$. Hence, the resulting closed-loop system is given by

$$\begin{cases}\mathrm{d}x(t)=[f_{\sigma(t)}(x(t),x(t-\tau(t)),t)+\\ \quad h_{\sigma(t)}(x(t),t)u_{\sigma(t-\delta)}(x(t))]\mathrm{d}t+\\ \quad g_{\sigma(t)}(x(t),x(t-\tau(t)),t)\mathrm{d}w(t),t>t_0,\\ x_{t_0}(s)=\varphi(s),-\bar{\tau}\leqslant s\leqslant 0,\sigma(t_0)=\sigma_0,\end{cases}\tag{2}$$

Obviously, because of the existence of the mismatched control input, it may degrade the performances and even cause instability if applying the matched control input and the switching signal designed for system (1) to system (2). Therefore, this paper attempts to establish general stability criteria for the hybrid stochastic system under asynchronous Markovian switching, and quantitatively analyze the effect of the detected delay of switching signal on the stability performance. For this purpose, we always assume that for each $i,j\in\mathscr{M}$, $\tilde{f}_{ij}(0,0,t)\equiv 0$, $g_i(0,0,t)\equiv 0$, for all $t\geqslant t_0$, and both $\tilde{f}_{ij}(x,y,t)$ and $g_i(x,y,t)$ satisfy the local Lipschitz condition and linear growth condition, where $\tilde{f}_{ij}(x,y,t)=f_i(x,y,t)+h_i(x,t)u_j(x)$. Hence, it follows from Theorem 8.3 of [13] that system (2) has a unique global solution, and $x(t;t_0,0)=0$ is the trivial solution.

For each $i\in\mathscr{M}$, if $V_i\in C^{2,1}(\mathbf{R}^n\times\mathbf{R}_+;\mathbf{R}_+)$, define an operator associated with (2) by

$$\mathscr{L}V(x_t,t,i,j)=G_i(x(t),x(t-\tau(t)),t)+\Delta G_{i,j}(x(t),t),$$

where

$$G_i(x,y,t)=\frac{\partial V_i(x,t)}{\partial t}+\frac{\partial V_i(x,t)}{\partial x}\cdot$$
$$[f_i(x,y,t)+h_i(x,t)u_i(x)]+$$
$$\frac{1}{2}\mathrm{trace}[g_i^{\mathrm{T}}(x,y,t)\frac{\partial^2 V_i(x,t)}{\partial x^2}g_i(x,y,t)]+$$
$$\sum_{j=1}^{N}\pi_{ij}V_j(x,t),$$
$$\Delta G_{i,j}(x,t)=\frac{\partial V_i(x,t)}{\partial x}h_i(x,t)(u_j(x)-$$
$$u_i(x)).$$

Then the generalized Itô formula[13] can be given as follows

$$\mathrm{E}V_{\sigma(t_2)}(x(t_2),t_2)=\mathrm{E}V_{\sigma(t_1)}(x(t_1),t_1)+$$
$$\mathrm{E}\int_{t_1}^{t_2}\mathscr{L}V(x_s,s,\sigma(s),\sigma(s-\delta))\mathrm{d}s,t_2\geqslant t_1\geqslant t_0. \qquad (3)$$

To make this paper more readable, we give the definition of pth moment exponential stability.

Definition 1 The zero solution of system (2) is pth-moment exponentially stable, if there exist two positive scalars M and γ such that

$$\mathrm{E}\|x(t)\|^p\leqslant M\mathrm{E}\|\varphi\|_\tau^p\mathrm{e}^{-\gamma(t-t_0)},t\geqslant t_0.$$

2 Stability analysis

Theorem 1 Consider hybrid stochastic system (2), if there exist nonnegative functions $V_i\in C^{2,1}(\mathrm{R}^n\times\mathrm{R}_+;\mathrm{R}_+)$, positive constants $\underline{c},\bar{c},\xi_1,\xi_2,c_{i,j},j\neq i$, such that for any $x,y\in\mathrm{R}^n,i,j\in\mathscr{M}$, the following conditions hold:

$$\underline{c}\|x\|^p\leqslant V_i(x,t)\leqslant\bar{c}\|x\|^p, \qquad (4)$$
$$G_i(x,y,t)\leqslant-\xi_1\|x\|^p+\xi_2\|y\|^p, \qquad (5)$$
$$\Delta G_{i,j}(x,t)\leqslant c_{i,j}\|x\|^p,j\neq i, \qquad (6)$$
$$\bar{c}\xi_2+\underline{c}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}c_{i,j}\mathrm{P}_{ji}(\delta)<\underline{c}\xi_1, \qquad (7)$$

where $\mathrm{e}^{\Pi\delta}=(\mathrm{P}_{ij}(\delta))_{N\times N}$, then system (2) is pth-moment exponentially stable.

Proof Set $W(t)=\mathrm{e}^{\gamma(t-t_0)}V_{\sigma(t)}(x(t),t)$, where $\gamma=\dfrac{\xi_1-\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}c_{i,j}\mathrm{P}_{ji}(\delta)}{\bar{c}}$. Applying the generalized Itô formula to $W(t)$ and utilizing conditions (5) and (6), we have

$$\mathrm{E}W(t)=\mathrm{E}W(t_0)+\mathrm{E}\int_{t_0}^{t}\mathrm{e}^{\gamma(s-t_0)}[\gamma V_{\sigma(s)}(x(s),s)+$$
$$\mathscr{L}V(x_s,s,\sigma(s),\sigma(s-\delta))]\mathrm{d}s\leqslant\mathrm{E}W(t_0)+$$
$$\mathrm{E}\int_{t_0}^{t}e^{\gamma(s-t_0)}[(\gamma\bar{c}-\xi_1+c_{\sigma(s),\sigma(s-\delta)})\|x(s)\|^p+$$
$$\xi_2\|x(s-\tau(s))\|^p]\mathrm{d}s.$$

Let $c_{i,i}=0,i\in\mathscr{M}$, then for any $t\geqslant t_0$,

$$c_{\sigma(t),\sigma(t-\delta)}=\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}c_{i,j}I_{\{\sigma(t)=i\}}I_{\{\sigma(t-\delta)=j\}}=$$
$$\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}c_{i,j}\mathrm{E}\{I_{\{\sigma(t)=i\}}I_{\{\sigma(t-\delta)=j\}}\mid\mathscr{F}_{t-\delta}\}=$$
$$\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}c_{i,j}I_{\{\sigma(t-\delta)=j\}}\mathrm{E}\{I_{\{\sigma(t)=i\}}\mid\mathscr{F}_{t-\delta}\}=$$
$$\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}c_{i,j}I_{\{\sigma(t-\delta)=j\}}\mathrm{P}\{\sigma(t)=i\mid\sigma(t-\delta)=j\}=$$
$$\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}c_{i,j}I_{\{\sigma(t-\delta)=j\}}\mathrm{P}_{ji}(\delta)\leqslant\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}c_{i,j}\mathrm{P}_{ji}(\delta).$$

Based on this fact, we have

$$\mathrm{E}W(t)\leqslant\mathrm{E}W(t_0)+\xi_2\int_{t_0}^{t}\mathrm{e}^{\gamma(s-t_0)}\mathrm{E}\|x(s-$$
$$\tau(s))\|^p\mathrm{d}s.$$

Applying (4) again, the above inequality can be transferred to

$$\mathrm{e}^{\gamma(t-t_0)}\mathrm{E}\|x(t)\|^p\leqslant\frac{\bar{c}}{\underline{c}}\mathrm{E}\|\varphi\|_\tau^p+$$
$$\frac{\xi_2}{\underline{c}}\int_{t_0}^{t}e^{\gamma(s-t_0)}\mathrm{E}\|x(s-\tau(s))\|^p\mathrm{d}s. \qquad (8)$$

Define a strictly monotonically increasing function $\underline{\rho}(t)=\underline{c}\bar{c}t+\bar{c}\xi_2\mathrm{e}^{t\bar{\tau}}+\underline{c}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}c_{i,j}\mathrm{P}_{ji}(\delta)-\underline{c}\xi_1$, then it follows from (7) that $\underline{\rho}(0)<0$ and $\underline{\rho}(\gamma)=\bar{c}\xi_2\mathrm{e}^{\gamma\bar{\tau}}>0$. According to the intermediate value theorem, there exist a constant $\eta\in(0,\gamma)$ such that $\underline{\rho}(\eta)=0$, whence $\dfrac{\xi_2\mathrm{e}^{\eta\bar{\tau}}}{\underline{c}(\gamma-\eta)}=1$. In the sequel, we will show that for any $\epsilon>1$,

$$\mathrm{E}\|x(t)\|^p\leqslant\frac{\bar{c}}{\underline{c}}\mathrm{E}\|\varphi\|_\tau^p\mathrm{e}^{-\eta(t-t_0)}<$$
$$\epsilon M\mathrm{e}^{-\eta(t-t_0)},\forall t\geqslant t_0-\bar{\tau}, \qquad (9)$$

where $M=\dfrac{\bar{c}}{\underline{c}}\mathrm{E}\|\varphi\|_\tau^p$. If (9) is not true, then by the continuity of $\mathrm{E}\|x(t)\|^p$ on $(t_0,+\infty)$, there exists $t_1>t_0$ such that $\mathrm{E}\|x(t_1)\|^p\geqslant\epsilon M\mathrm{e}^{-\eta(t_1-t_0)}$. Let $t^*=\inf\{t\in(t_0,t_1],\mathrm{E}\|x(t)\|^p\geqslant\epsilon M\mathrm{e}^{-\eta(t-t_0)}\}$, we have

$$\mathrm{E}\|x(t^*)\|^p=\epsilon M\mathrm{e}^{-\eta(t^*-t_0)},\mathrm{E}\|x(s)\|^p<$$
$$\epsilon M\mathrm{e}^{-\eta(s-t_0)},\text{ for all }s\in[t_0-\bar{\tau},t^*).$$

Then inequality (8) has the estimation at t^*

$$\mathrm{e}^{\gamma(t^*-t_0)}\mathrm{E}\|x(t^*)\|^p\leqslant M+$$
$$\frac{\xi_2}{\underline{c}}\int_{t_0}^{t^*}\mathrm{e}^{\gamma(s-t_0)}\epsilon M\mathrm{e}^{-\eta(s-\tau(s)-t_0)}\mathrm{d}s\leqslant M+$$
$$\frac{\xi_2}{\underline{c}}\mathrm{e}^{\eta\bar{\tau}}\epsilon M\int_{t_0}^{t^*}\mathrm{e}^{(\gamma-\eta)(s-t_0)}\mathrm{d}s<\epsilon M(1-\frac{\xi_2\mathrm{e}^{\eta\bar{\tau}}}{\underline{c}(\gamma-\eta)})+$$
$$\frac{\xi_2\mathrm{e}^{\eta\bar{\tau}}}{\underline{c}(\gamma-\eta)}\epsilon M\mathrm{e}^{(\gamma-\eta)(t^*-t_0)}=\mathrm{E}\|x(t^*)\|^p\mathrm{e}^{\gamma(t^*-t_0)},$$

which leads to a contradiction. Therefore, (9) holds. Letting $\epsilon \to 1^+$ in (9), it follows that

$$\mathrm{E}\|x(t)\|^p \leqslant \frac{\bar{c}}{\underline{c}}\mathrm{E}\|\varphi\|_\tau^p \mathrm{e}^{-\eta(t-t_0)}.$$

Therefore, system (2) is pth-moment exponentially stable.

Remark 1 Theorem 1 reveals that if the synchronous switched controlled system (1) is pth-moment exponentially stable (i. e. the conditions (4), (5) and (7) with $\delta=0$ hold), then there exists a small enough δ such that the asynchronous switched controlled system (2) is also pth-moment exponentially stable. Actually, it follows from (7) that the admissible time delay of mismatched switching can be easily calculated for the synchronous switched controlled systems with the pre-designed controller and the generator of Markov chain. On the other hand, it is interesting to compare our result with existing results for the case of synchronous switched systems. Compared with results [16-18], the conditions of Theorem 1 is not only irrespective of the sizes of the state delay but also does not impose any restriction on the derivative of the delay. Different from the Razumikhin-type results[13,19,22], Theorem 1 is established by utilizing proofs by contradiction to estimate an integral inequality, and thus leading to simpler stability conditions.

For the constant delay case, condition (7) of Theorem 1 can be relaxed. That is, if we set $\tau(t) \equiv \tau$, then we have the following result.

Corollary 1 Consider hybrid stochastic system (2), if there exist nonnegative functions $V_i \in C^{2,1}(\mathbf{R}^n \times \mathbf{R}_+; \mathbf{R}_+)$, positive constants $\underline{c}, \bar{c}, \xi_1, \xi_2, c_{i,j}, j \neq i$, such that for any $x, y \in \mathbf{R}^n, i, j \in \mathscr{M}$, the following conditions hold:

$$\underline{c}\|x\|^p \leqslant V_i(x,t) \leqslant \bar{c}\|x\|^p, \tag{10}$$

$$G_i(x,y,t) \leqslant -\xi_1\|x\|^p + \xi_2\|y\|^p, \tag{11}$$

$$\Delta G_{i,j}(x,t) \leqslant c_{i,j}\|x\|^p, j \neq i, \tag{12}$$

$$\xi_2 + \sum_{i=1}^N \sum_{j=1,j\neq i}^N c_{i,j}\mathrm{P}_{ji}(\delta) < \xi_1, \tag{13}$$

where $\mathrm{e}^{\Pi\delta} = (\mathrm{P}_{ij}(\delta))_{N\times N}$, then system (2) is pth-moment exponentially stable.

Proof Choose a Lyapunov function for system (2): $\overline{W}(t) = \mathrm{e}^{\bar{\gamma}(t-t_0)}V_{\sigma(t)}(x(t),t)$, where $\bar{\gamma}$ is a unique positive solution of $\bar{c}\bar{\gamma} + \xi_2\mathrm{e}^{\bar{\gamma}\tau} + \sum_{i=1}^N \sum_{j=1,j\neq i}^N c_{i,j}\mathrm{P}_{ji}(\delta) - \xi_1 = 0$. Applying the same technique used above, we have

$$\mathrm{E}\,\overline{W}(t) \leqslant \mathrm{E}\,\overline{W}(t_0) + \mathrm{E}\int_{t_0}^t \mathrm{e}^{\bar{\gamma}(s-t_0)}[(\bar{c}\bar{\gamma} - \xi_1 + \sum_{i=1}^N \sum_{j=1,j\neq i}^N c_{i,j}\mathrm{P}_{ji}(\delta))\|x(s)\|^p + \xi_2\|x(t-\tau)\|^p]\mathrm{d}s \leqslant \mathrm{E}\,\overline{W}(t_0) + (\bar{c}\bar{\gamma} - \xi_1 + \sum_{i=1}^N \sum_{j=1,j\neq i}^N c_{i,j}\mathrm{P}_{ji}(\delta))\mathrm{E}\int_{t_0}^t \mathrm{e}^{\bar{\gamma}(s-t_0)}\|x(s)\|^p\mathrm{d}s + \xi_2\mathrm{e}^{\bar{\gamma}\tau}\mathrm{E}[\int_{t_0-\tau}^{t_0}\mathrm{e}^{\bar{\gamma}(s-t_0)}\|x(s)\|^p\mathrm{d}s + \int_{t_0}^{t-\tau}\mathrm{e}^{\bar{\gamma}(s-t_0)}\|x(s)\|^p\mathrm{d}s] \leqslant [\bar{c} + (1 - e^{-\bar{\gamma}\tau})\xi_2\mathrm{e}^{\bar{\gamma}\tau}/\bar{\gamma}]\mathrm{E}\|\varphi\|_\tau^p.$$

This implies that $\mathrm{E}\|x(t)\|^p \leqslant \frac{\bar{c} + (1-\mathrm{e}^{-\bar{\gamma}\tau})\xi_2\mathrm{e}^{\bar{\gamma}\tau}/\bar{\gamma}}{\underline{c}}\mathrm{E}\|\varphi\|_\tau^p\mathrm{e}^{-\bar{\gamma}(t-t_0)}$. The proof is complete.

3 Application

Some theoretical results of asynchronous Markovian switching control for a nonlinear hybrid stochastic system are established in the previous section. We now apply these results to establish an LMI-based stability condition and solve the controller design problem for a class of nonlinear stochastic systems under asynchronous Markovian switching. The considered nonlinear stochastic systems include linear systems, recurrent neural networks, some chaotic systems and so forth, which is given by the following stochastic differential equations

$$\begin{cases} \mathrm{d}x(t) = [C_{\sigma(t)}x(t) + A_{0,\sigma(t)}f_{\sigma(t)}(x(t),t) + \\ \quad A_{1,\sigma(t)}f_{\sigma(t)}(x(t-\tau),t) + \\ \quad B_{\sigma(t)}u(t)]\mathrm{d}t + g_{\sigma(t)}(x(t), \\ \quad x(t-\tau),t)\mathrm{d}w(t), t > t_0, \\ x_{t_0}(s) = \varphi(s), s \in [-\tau, 0], \sigma(t_0) = \sigma_0, \end{cases} \tag{14}$$

where $C_i, A_{0,i}, A_{1,i} \in \mathbf{R}^{n\times n}$, and $B_i \in \mathbf{R}^{n\times n_0}$ are known constant matrices. The control input is constructed by a mode-dependent state-feedback controller with delayed switching signal:

$$u(t) = K_{\sigma(t-\delta)}x(t), \tag{15}$$

where $K_i \in \mathbf{R}^{n_0\times n}$ and $\delta > 0$.

In the following, we will assume that the nonlinear functions $f_i(x,t)$ and $g_i(x,y,t), i \in \mathscr{M}$, satisfy the following assumptions.

(**A1**) For each $i \in \mathscr{M}$, there exists a matrix L_i with appropriate dimension such that

$$\| f_i(x,t) \| \leqslant \| L_i x \|, \forall x \in \mathbf{R}^n, \forall t \in \mathbf{R}.$$

(**A2**) For each $i \in \mathscr{M}$, there exist real matrices $G_{1i} \geqslant 0$ and $G_{2i} \geqslant 0$ such that

$$\mathrm{trace}[g_i^{\mathrm{T}}(x,y,t) g_i(x,y,t)] \leqslant x^{\mathrm{T}} G_{1i} x + y^{\mathrm{T}} G_{2i} y, \forall x,y \in \mathbf{R}^n, \forall t \in \mathbf{R}.$$

3.1 Mean-square stability criterion

Theorem 2 Consider the Markovian jump system (14) with (15) satisfying (**A1**) and (**A2**). For given $K_i \in \mathbf{R}^{n_0 \times n}$ and detected delay of switching signal δ, if there exist positive matrices $P_i \in \mathbf{R}^{n \times n}$, and positive scalars $\beta_i, \xi_l, \alpha_{li}$, and $c_{i,j}$, $i,j \in \mathscr{M}, j \neq i, l = 1,2$, such that (13) and the following LMIs hold:

$$P_i \leqslant \beta_i I, \tag{16}$$

$$\Xi_i \triangleq \begin{bmatrix} \Phi_i & 0 & P_i A_{0,i} & P_i A_{1,i} \\ * & \Psi_i & 0 & 0 \\ * & * & -\alpha_{1i} I & 0 \\ * & * & * & -\alpha_{2i} I \end{bmatrix} < 0, i \in \mathscr{M}, \tag{17}$$

$$P_i B_i (K_j - K_i) + (K_j - K_i)^{\mathrm{T}} B_i^{\mathrm{T}} P_i - c_{i,j} I \leqslant 0, \quad i,j \in \mathscr{M}, j \neq i, \tag{18}$$

where

$$\Phi_i = P_i (C_i + B_i K_i) + (C_i + B_i K_i)^{\mathrm{T}} P_i + \beta_i G_{1i} + \alpha_{1i} L_i^{\mathrm{T}} L_i + \xi_1 I + \sum_{j=1}^{N} \pi_{ij} P_j,$$

$$\Psi_i = -\xi_2 I + \beta_i G_{2i} + \alpha_{2i} L_i^{\mathrm{T}} L_i,$$

then system (14) with (15) is mean-square exponentially stable.

Proof Define a stochastic Lyapunov function candidate $V_{\sigma(t)}(x(t)) = x^{\mathrm{T}}(t) P_{\sigma(t)} x(t)$ for system (14). Firstly, we compute $G_i(x,y,t)$ along the trajectories of system (15).

$$G_i(x,y,t) = x^{\mathrm{T}}(t)[P_i(C_i + B_i K_i) + (C_i + B_i K_i)^{\mathrm{T}} P_i + \sum_{j=1}^{N} \pi_{ij} P_j] x(t) + 2x^{\mathrm{T}}(t) P_i A_{0,i} f_i(x,t) + 2x^{\mathrm{T}}(t) P_i A_{1,i} f(x(t-\tau),t) + \mathrm{trace}[g_i^{\mathrm{T}}(x(t),x(t-\tau),t) P_i g_i(x(t),x(t-\tau),t)]. \tag{19}$$

Using condition (16) and (**A2**), we have

$$\mathrm{trace}[g_i^{\mathrm{T}}(x(t),x(t-\tau),t) P_i g_i(x(t),x(t-\tau),t)] \leqslant \beta_i x^{\mathrm{T}}(t) G_{1i} x(t) + \beta_i x^{\mathrm{T}}(t-\tau) G_{2i} x(t-\tau). \tag{20}$$

On the other hand, it can be deduced from (**A1**) that

$$0 \leqslant \sum_{l=1}^{2} \alpha_{li} [x^{\mathrm{T}}(t-\tau_l) L_i^{\mathrm{T}} L_i (t-\tau_l) - f_i^{\mathrm{T}}(x(t-\tau_l),t) f_i(x(t-\tau_i),t)]. \tag{21}$$

where $\tau_1 = 0$ and $\tau_2 = \tau$.

Applying the inequalities (20)-(21) to (19) yields

$$G_i(x,x(t-\tau),t) \leqslant \zeta_i^{\mathrm{T}}(t) \Xi_i \zeta_i(t) - \xi_1 \| x(t) \|^2 + \xi_2 \| x(t-\tau) \|^2.$$

where $\zeta_i(t) = \mathrm{col}(x(t), x(t-\tau), f_i(x(t),t), f_i(x(t-\tau),t))$. Then, by (17), we obtain

$$G_i(x,x(t-\tau),t) \leqslant -\xi_1 \| x(t) \|^2 + \xi_2 \| x(t-\tau) \|^2.$$

Next, we calculate $\Delta G_{ij}(x(t),t)$.

$$\Delta G_{ij}(x(t),t) = x^{\mathrm{T}}(t)[P_i B_i (K_j - K_i) + (K_j - K_i)^{\mathrm{T}} B_i^{\mathrm{T}} P_i - c_{ij} I] x(t) + c_{ij} \| x(t) \|^2.$$

Inequality (18) implies that $\Delta G_{ij}(x(t),t) \leqslant c_{ij} \| x(t) \|^2$. Therefore, by Theorem 1, we conclude that the zero solution of system (14) is mean-square exponentially stable.

3.2 Controller design

Based on the mean-square stability criterion, this subsection will solve controller synthesis problem. For this purpose, assume $G_{li} = \bar{G}_{li}^{\mathrm{T}} \bar{G}_{li}$, $i \in \mathscr{M}, l = 1,2$, and then we introduce two lemmas.

Lemma 1 [29] For matrices $\mathscr{A} \in \mathbf{R}^{n \times N}$, $\mathscr{B} \in \mathbf{R}^{N \times n}$, $\tilde{\Xi}_i \in \mathbf{R}^{N \times N}$, $\tilde{\Xi}_i = \tilde{\Xi}_i^{\mathrm{T}}$, $X_0, X_i, H_i \in \mathbf{R}^{n \times n}$, $i = 1,2,\cdots,p$, if they satisfy the following inequalities for all $i \in \{1,\cdots,p\}$:

$$\begin{bmatrix} \tilde{\Xi}_i + \mathscr{B} X_0 \mathscr{A} + (\mathscr{B} X_0 \mathscr{A})^{\mathrm{T}} & ((X_i - X_0)\mathscr{A})^{\mathrm{T}} + \mathscr{B} H_i \\ * & -\mathrm{H_i} - \mathrm{H_i}^T \end{bmatrix} < 0, \tag{22}$$

then it holds that

$$\tilde{\Xi}_i + \mathscr{B} X_i \mathscr{A} + (\mathscr{B} X_i \mathscr{A})^{\mathrm{T}} < 0, i = 1,\cdots,p. \tag{23}$$

Lemma 2 [30] For any $n \times n$ matrices $X > 0, U$, scalar $\varepsilon > 0$, the following matrix inequality holds:

$$U X^{-1} U^{\mathrm{T}} \geqslant \varepsilon (U + U^{\mathrm{T}}) - \varepsilon^2 X.$$

Theorem 3 Given the detected delay of switching signal δ, consider the Markovian jump system (14). If for some prescribed positive scalars κ_i and ε_{li}, there exist matrices $0 < X_i \in \mathbf{R}^{n \times n}$, $\bar{K}_i \in \mathbf{R}^{n \times n_0}$, $X_0 \in \mathbf{R}^{n \times n}$, and positive scalars $\bar{\beta}_i, \bar{\xi}_l, \bar{\alpha}_{li}$, and $\bar{c}_{i,j}$, $i,j \in \mathscr{M}$, $j \neq i, l = 1,2$ such that (13) and the following LMIs hold:

$$\bar{\beta}_i I \leqslant X_i, \tag{24}$$

$$\bar{\Xi}_i \triangleq \begin{bmatrix} \bar{\Phi}_i & 0 & \Gamma_i & A_{0,i}\bar{\alpha}_{1i} & A_{1,i}\bar{\alpha}_{2i} & X_i\mathscr{G}_i & X_i\mathscr{R}_i \\ * & \bar{\Psi}_i & 0 & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_{1i}(X_0+X_0^{\mathrm{T}}) & 0 & 0 & 0 & 0 \\ * & * & * & -\bar{\alpha}_{1i}I & 0 & 0 & 0 \\ * & * & * & * & -\bar{\alpha}_{2i}I & 0 & 0 \\ * & * & * & * & * & -\Lambda_i & 0 \\ * & * & * & * & * & * & -\chi \end{bmatrix} < 0, \quad (25)$$

$$\begin{bmatrix} \Omega_{ij} & X_i - X_0^{\mathrm{T}} + \varepsilon_{2i}B_i(\bar{K}_j - \bar{K}_i) \\ * & -\varepsilon_{2i}(X_0+X_0^{\mathrm{T}}) \end{bmatrix} < 0, j \neq i, \quad (26)$$

$$\begin{bmatrix} -\bar{\xi}_1 & \bar{\xi}_1 & \bar{\xi}_1\Upsilon \\ * & -\bar{\xi}_2 & 0 \\ * & * & -\mathscr{Y} \end{bmatrix} < 0, \quad (27)$$

where

$\bar{\Phi}_i = C_iX_i + X_iC_i^{\mathrm{T}} + B_i\bar{K}_i + \bar{K}_i^{\mathrm{T}}B_i^{\mathrm{T}} + \pi_{ii}X_i$, $\Gamma_i = X_i - X_0^{\mathrm{T}} + \varepsilon_{1i}B_i\bar{K}_i$,

$$\bar{\Psi}_i = \begin{bmatrix} -\bar{\xi}_2 I & \bar{\xi}_2 L_i^{\mathrm{T}} & \bar{\xi}_2 \bar{G}_{2i}^{\mathrm{T}} \\ * & -\bar{\alpha}_{2i}I & 0 \\ * & * & -\bar{\beta}_i I \end{bmatrix},$$

$\mathscr{G}_i = [\bar{G}_{1i}^{\mathrm{T}} \quad L_i^{\mathrm{T}} \quad I]$, $\Lambda_i = \mathrm{diag}(\bar{\beta}_i I, \bar{\alpha}_{1i}I, \bar{\xi}_1)$,
$\chi = \mathrm{diag}(X_1, \cdots, X_{i-1}, X_{i+1}, \cdots, X_N)$,
$\gamma = [\mathscr{P}_1 \quad \cdots \quad \mathscr{P}_i \quad \cdots \quad \mathscr{P}_N]$,

$\Upsilon_i = [\sqrt{\pi_{i1}} \quad \cdots \quad \sqrt{\pi_{i,i-1}} \quad \sqrt{\pi_{i,i+1}} \quad \cdots \quad \sqrt{\pi_{iN}}$,

$\mathscr{P}_i = [\sqrt{\mathrm{P}_{1,i}(\delta)} \quad \cdots \quad \sqrt{\mathrm{P}_{i-1,i}(\delta)} \quad \sqrt{\mathrm{P}_{i+1,i}(\delta)} \quad \cdots \quad \sqrt{\mathrm{P}_{N,i}(\delta)}]$,

$\mathscr{Y} = \mathrm{diag}(y_1, y_2, \cdots, y_N)$, $y_i = \mathrm{diag}(\bar{c}_{i,1}, \cdots, \bar{c}_{i,i-1}, \bar{c}_{i,i+1}, \cdots, \bar{c}_{i,N})$,

$\Omega_{ij} = -2\kappa_i X_i + \kappa_i^2 \bar{c}_{ij} I + B_i(\bar{K}_j - \bar{K}_i) + (\bar{K}_j^{\mathrm{T}} - \bar{K}_i^{\mathrm{T}})B_i^{\mathrm{T}}$,

then the admissible controller (15) with $K_i = \bar{K}_iX_0^{-1}$ is mean-square exponentially stabilized system (14).

Proof Define $P_i = X_i^{-1}$, $\bar{K}_i = K_iX_0$, $\beta_i = \bar{\beta}_i^{-1}$, $\alpha_{li} = \bar{\alpha}_{li}^{-1}$, $\xi_l = \bar{\xi}_l^{-1}$, $i \in \mathscr{M}$, $l = 1,2$. It is easy to verify that (24) and (27) are equivalent to (16) and (13), respectively. Applying Schur complement and Lemma 1, the matrix inequality (25) implies

$$\hat{\Xi}_i \triangleq \begin{bmatrix} \hat{\Phi}_i & 0 & A_{0,i}\bar{\alpha}_{1i} & A_{1,i}\bar{\alpha}_{2i} \\ * & \hat{\Psi}_i & 0 & 0 \\ * & * & -\bar{\alpha}_{1i}I & 0 \\ * & * & * & -\bar{\alpha}_{2i}I \end{bmatrix} < 0, i \in \mathscr{M}, \quad (28)$$

where $\hat{\Phi}_i = (C_i + B_iK_i)X_i + X_i(C_i + B_iK_i)^{\mathrm{T}} + X_i\beta_iG_{1i}X_i + X_i\alpha_{1i}L_i^{\mathrm{T}}L_iX_i + X_i\xi_1X_i + \sum_{j=1}^{N}\pi_{ij}X_iP_jX_i$, $\hat{\Psi}_i = -\bar{\xi}_2 I + \bar{\xi}_2\beta_iG_{2i}\bar{\xi}_2 + \bar{\xi}_2\alpha_{1i}L_i^{\mathrm{T}}L_i\bar{\xi}_2$. Then pre-and post-multiplying the both sides of (28) by diag $\{P_i, \xi_2 I, \alpha_{1i}I, \alpha_{2i}I\}$, (28) is changed equivalently to (17).

Furthermore, applying Lemma 1 again, and the inequalities $-c_{i,j}X_iX_i \leqslant -2\kappa_iX_i + \kappa_i^2\bar{c}_{i,j}I$ which is derived from Lemma 2 to (26), we obtain

$B_i(K_j - K_i)X_i + X_i(K_j^{\mathrm{T}} - K_i^{\mathrm{T}})B_i^{\mathrm{T}} - c_{i,j}X_iX_i < 0$.

Multiplying the above inequality to the left and the right by P_i yields (18).

4 Numerical examples

Example 1 Consider the dynamic reliability problem of multiplexed control system[31]:

Controller 1: $\dot{x}_1 = u_1$,

Plant: $\dot{x}_2 = 1.5x_1 + a_{22}x_2 + 1.5x_3$,

Controller 2: $\dot{x}_3 = u_2$,

where $a_{22} = 0$, and $u_1 = -k_1x_1 - k_2x_2$, $u_2 = -k_2x_2 - k_1x_3$. In practice, the controllers may be failure which need to maintenance. Therefore, it is assumed that the failure rate is λ and the repair rate is $\mu(>\lambda)$, and the failure process and the repair process are both exponentially distributed. Then the system can be modeled as (14) with following parameters (see [31-32]):

$$C_1 = \begin{bmatrix} 0 & 0 & 0 \\ 1.5 & 0 & 1.5 \\ 0 & 0 & 0 \end{bmatrix}, K_1 = -\begin{bmatrix} k_1 & k_2 & 0 \\ 0 & k_2 & k_1 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1.5 \\ 0 & 0 & 0 \end{bmatrix}, K_2 = -\begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & k_1 \end{bmatrix},$$

$$C_3 = \begin{bmatrix} 0 & 0 & 0 \\ 1.5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, K_3 = -\begin{bmatrix} k_1 & k_2 & 0 \\ 0 & 0 & k_1 \end{bmatrix},$$

$$C_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, K_4 = -\begin{bmatrix} k_1 & 0 & 0 \\ 0 & 0 & k_1 \end{bmatrix}, B_i = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix},$$

$f_i = 0, g_i = 0, i = 1,2,3,4$,

where the first mode corresponds to the case where both controllers are good, and the second and third modes correspond to the case where one of the controllers fails, and the fourth mode corresponds to the case where both controllers are failure. The failure process and the repair process form a Markov chain with generator

$$\Pi=\begin{bmatrix} -2\lambda & \lambda & \lambda & 0 \\ \mu & -(\lambda+\mu) & 0 & \lambda \\ \mu & 0 & -(\lambda+\mu) & \lambda \\ 0 & \mu & \mu & -2\mu \end{bmatrix}.$$

The authors of [17] showed that when $\lambda=0.4$, $\mu=0.55$, $k_1=2.85$ and $k_2=0.33$, the synchronous state feedback controller $u(t)=K_{\sigma(t)}x(t)$ stabilized the system in mean square. Here, we are interested in that how large detected delay of the controller can be tolerate to preserve the stability of the system.

Applying Theorem 2 with the same gain matrices and Markov generator, it has been found that maximum mode delay δ is 0.61. By using the Euler-Maruyama method[33] with step size 0.001 and setting initial value $\sigma_0=3$, $\varphi(s)=[-1,-0.3,0.5]^T$, the time response of the system is depicted in Fig. 1. Observe that the asynchronous switching controller with $\delta=0.61$ can guarantee the stability of the system.

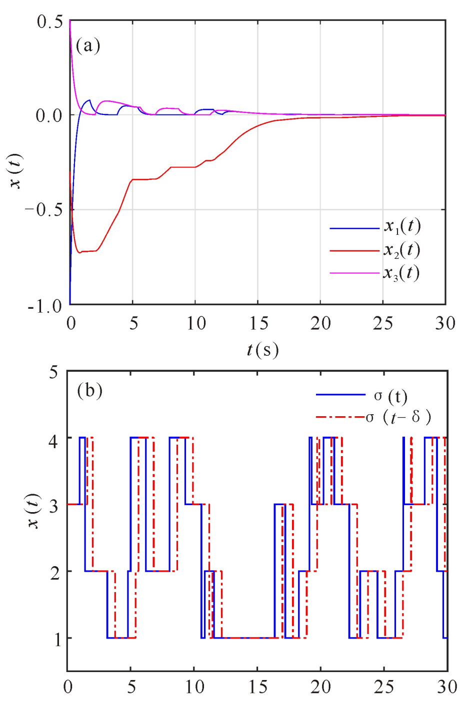

Fig. 1 Simulation of Example 1 with $k_1=2.85$, $k_2=0.33$ and $\delta=0.61$; (a) Sample path trajectories; (b) Switching signals

Furthermore, in order to show the effect of the maximum delay of asynchronous Markovian switching on the stability clearly, assigning that $\lambda=0.4\pi_0$, $\mu=0.55\pi_0$, $k_1=2.85$ and $k_2=0.33$, then for different values of π_0, we calculate the maximum values of the delay δ. The results are listed in Table 1. It can be seen from the table that when the Markov switching is slowly switching, the admissible delay of asynchronous switching is large, otherwise, the allowable delay is relatively small. This is consistent with our intuition. Next, assume that $k_1=k_2=k$, we compute the minimum control intensity k for various δ. From Table 2, the calculation results show that the control intensity increases with the increase of the delay of asynchronous switching.

Table 1 The maximum values of δ for different π_0

π_0	δ	π_0	δ
0.1	5.72	1.6	0.39
0.3	1.95	2.0	0.31
0.7	0.86	3.0	0.21
1.0	0.61	5.0	0.13

Table 2 The minimum values of $k(k_1=k_2)$ for different δ

δ	k	δ	k
0.05	0.09	0.25	0.93
0.10	0.21	0.30	1.70
0.15	0.36	0.35	5.48
0.20	0.57	0.40	—

Example 2 Consider a two dimensional switched system (14) with following parameters

$$C_1=\begin{bmatrix} 2.23 & -1 \\ 3 & -4.92 \end{bmatrix}, A_{01}=0,$$
$$A_{11}=\begin{bmatrix} 0.25 & 0.16 \\ -0.2 & 0.51 \end{bmatrix}, B_1=\begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$\bar{G}_{11}=\begin{bmatrix} 0.51 & 1.23 \\ 0.46 & -0.4 \end{bmatrix}, C_2=\begin{bmatrix} -1 & 1 \\ -2 & 5 \end{bmatrix}, A_{02}=0,$$
$$A_{12}=\begin{bmatrix} 0.1 & -0.3 \\ 0.27 & 0.18 \end{bmatrix},$$
$$B_2=\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \bar{G}_{12}=\begin{bmatrix} 0.21 & 0.43 \\ 0.40 & -0.22 \end{bmatrix},$$
$$\Pi=\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix},$$

$f_i(x,t)=x$, $g_i(x,y,t)=\bar{G}_{1i}x$, $i=1,2$, and the time delay of asynchronous switching $\delta=0.1$. This hybrid stochastic system is not mean-square exponentially stable without control input. Our purpose is to design a mode-dependent state-feedback controller (14) to stabilize the system with any constant state delay. Therefore, applying Theorem 3 with the choices of $\kappa_1=18$, $\kappa_2=12$, $\varepsilon_{11}=0.06$, $\varepsilon_{21}=0.05$, $\varepsilon_{12}=$

$\varepsilon_{22}=0.01$, it has been found that LMIs (24)—(27) are feasible, and the corresponding control gain matrices are

$$K_1=[-7.67 \quad -3.39], K_2=[0.75 \quad -7.56]. \tag{29}$$

For simulation studies, we use the Euler-Maruyama method[33] with step size 0.001 and initial values $\sigma_0=1$, $\varphi(s)=[4,-3]^{\mathrm{T}}$, $s\in[-\tau,0]$. Then, the sample path of the solution is plotted in Fig. 2a—b for $\tau=1$ and $\tau=10$, respectively. The corresponding switching signal is also given in Fig. 2c. It is seen that the asynchronous switching controller (14) with gain matrices (29) indeed asymptotically stabilizes the system with different delays.

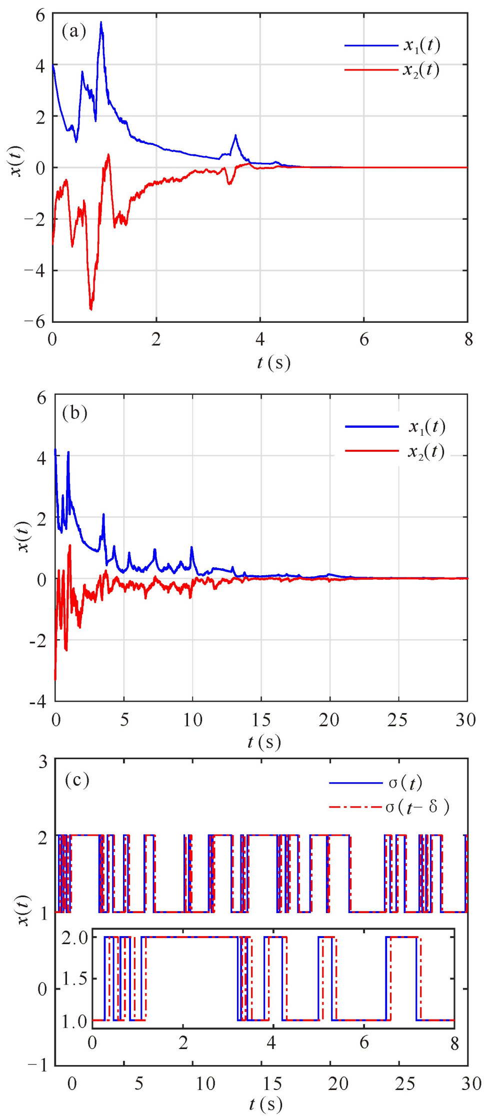

Fig. 2 Simulation of Example 2 under asynchronous switching control; (a) Sample path trajectories for $\tau=1$; (b) Sample path trajectories for $\tau=10$; (c) Switching signals

5 Conclusion

By exploring the relationship between the sizes of detected delay of switching signal and the generator of the Markov chain, a novel integral-inequality-estimation technique is developed to deal with time-varying delay, and some state-delay-independent stability criteria have been established for hybrid stochastic retarded systems with asynchronous Markovian switching. It has shown that if the synchronous switched controlled system is pth-moment exponentially stable, then sufficiently small delays of asynchronous switching will not destroy the stability of the system. Compared with some existing results on the synchronous switching, our Theorem 1 does not impose any restriction on the derivative of state delay. As its application, the obtained results have been applied to design stabilizing mode-dependent controller for a class of time-delay stochastic systems under asynchronous Markovian switching. Simulation results have verified the effectiveness of the theoretical results.

References

[1] LIBERZON D. Switching in systems and control[M]. Boston: Springer, 2003.
[2] LIN H, ANTSAKLIS P J. Stability and stabilizability of switched linear systems: A survey of recent results[J]. IEEE Transactions on Automatic Control, 2009, 54(2): 308-322.
[3] CHEN W H, ZHENG W X. Delay-independent minimum dwell time for exponential stability of uncertain switched delay systems[J]. IEEE Transactions on Automatic Control, 2010, 55(10): 2406-2413.
[4] SUN Z D, GE S S. Stability theory of switched dynamical systems[M]. London: Springer, 2011.
[5] ZHAO X D, KAO Y G, NIU B, et al. Control synthesis of switched systems[M]. Cham: Springer, 2017.
[6] ZHANG L X, SHI P. Stability, L_2-gain and asynchronous H_∞ control of discrete-time switched systems with average dwell time[J]. IEEE Transactions on Automatic Control, 2009, 54(9): 2193-2200.
[7] ZHANG L X, GAO H J. Asynchronously switched control of switched linear systems with average dwell time [J]. Automatica, 2010, 46(5): 953-958.
[8] XIE G M, WANG L. Stabilization of switched linear systems with time-delay in detection of switching signal [J]. Journal of Mathematical Analysis and Applications, 2005, 305(1): 277-290.
[9] XIANG Z R, WANG R H. Robust control for uncertain

switched non-linear systems with time delay under asynchronous switching[J],Iet Control Theory & Applications,2007,3(8):1041-1050.

[10] ZHAO X D,SHI P,ZHANG L X. Asynchronously switched control of a class of slowly switched linear systems[J]. Systems & Control Letters,2012,61(12):1151-1156.

[11] WANG Y E,SUN X M,SHI P,et al. Input-to-state stability of switched nonlinear systems with time delays under asynchronous switching[J]. IEEE Transactions on Cybernetics,2013,43(6):2261-2265.

[12] WANG Y E,ZHAO J,JIANG B. Stabilization of a class of switched linear neutral systems under asynchronous switching[J]. IEEE Transactions on Automatic Control,2013,58(8):2114-2119.

[13] MAO X R,YUAN C G. Stochastic differential equations with Markovian switching[M]. Imperial College Press,2006.

[14] CHEN W H,XU J X,GUAN Z H. Guaranteed cost control for uncertain Markovian jump systems with mode-dependent time-delays[J]. IEEE Transactions on Automatic Control,2003,48(12):2270-2277.

[15] WU L G,SHI P,GAO H J. State estimation and sliding-mode control of Markovian jump singular systems[J]. IEEE Transactions on Automatic Control,2010,55(5):1213-1219.

[16] MAO X R,MATASOV A,PIUNOVSKIY A B. Stochastic differential delay equations with Markovian switching[J]. Bernoulli,2000,6(1):73-90.

[17] ZHOU W N,YANG J,YANG X Q,et al. pth moment exponential stability of stochastic delayed hybrid systems with Lévy noise[J]. Applied Mathematical Modelling,2015,39(18):5650-5658.

[18] CHEN W M,XU S Y,ZOU Y. Stabilization of hybrid neutral stochastic differential delay equations by delay feedback control[J]. Systems & Control Letters,2016,88:1-13.

[19] MAO X R. Stability of stochastic differential equations with Markovian switching[J]. Stochastic Processes and their Applications,1999,79(1):45-67.

[20] HUANG L R,MAO X R. On input-to-state stability of stochastic retarded systems with Markovian switching[J]. IEEE Transactions on Automatic Control,2009,54(8):1898-1902.

[21] HUANG L L R,MAO X R. On almost sure stability of hybrid stochastic systems with mode-dependent interval delays[J]. IEEE Transactions on Automatic Control,2010,55(8):1946-1952.

[22] PENG S G,YANG L P. The pth moment boundedness of stochastic functional differential equations with markovian switching[J]. Journal of the Franklin Institute,2017,354(1):345-359.

[23] ZHAI D H,KANG Y,ZHAO P,et al. Stability of a class of switched stochastic nonlinear systems under asynchronous switching[J]. International Journal of Control Automation & Systems,2012,10(6):1182-1192.

[24] WANG Y E,SUN X M,ZHAO J. Stabilization of a class of switched stochastic systems with time delays under asynchronous switching[J]. Circuits Systems & Signal Processing,2013,32(1):347-360.

[25] REN W,XIONG J L. Stability and stabilization of switched stochastic systems under asynchronous switching[J]. Systems & Control Letters,2016,97:184-192.

[26] XIONG J L. LAM J. Stabilization of discrete-time Markovian jump linear systems via time-delayed controllers[J]. Automatica,2006,42(5):747-753.

[27] LIU M,HO D W,NIU Y G. Stabilization of Markovian jump linear system over networks with random communication delay[J]. Automatica,2009,45(2):416-421.

[28] KANG Y,ZHAI D H,LIU G P,et al. Stability analysis of a class of hybrid stochastic retarded systems under asynchronous switching[J]. IEEE Transactions on Automatic Control,2014,59(6):1511-1523.

[29] CHEN W H,WEI C H,LU X M. Stochastic finite-time stabilization for a class of nonlinear Markovian jump stochastic systems with impulsive effects[J]. Journal of Dynamic Systems,Measurement,and Control,2015:137(4):044501-044506.

[30] CHEN W H,ZHENG W X. Robust stabilization of delayed Markovian jump systems subject to parametric uncertainties[J]. Conference on Decision and Control,2007:3054-3059.

[31] LADDE G S,SLLJAK D. Multiplex control systems:Stochastic stability and dynamic reliability[J]. International Journal of Control,1983,38(3):515-524.

[32] FANG Y G,LOPARO K A. Stabilization of continuous-time jump linear systems[J]. IEEE Transactions on Automatic Control,2002,47(10):1590-1603.

[33] YUAN C G,MAO X R. Convergence of the Euler-Maruyama method for stochastic differential equations with markovian switching[J]. Mathematics and Computers in Simulation,2004,64(2):223-235.

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