Decomposition of the Prime Order Complete Graph K_{313} into Circulant Graphs*
把素数阶完全图 K_{313} 分解为循环图

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Abstract Various colorings for edges of the complete graph K_{313} were studied using constructive method. New lower bounds of three 3-color classical Ramsey numbers were obtained: $R(3,3,17) \geqslant 314, R(3,4,14) \geqslant 314, R(3,6,9) \geqslant 314$.
Key words Ramsey number, lower bound, circulant graph
摘要 用构造的方法研究了多色完全图 K_{313} 的边的各种染色方法，得到了 3 个经典 3 色 Ramsey 数的新下界：$R(3,3,17) \geqslant 314, R(3,4,14) \geqslant 314, R(3,6,9) \geqslant 314$.
关键词 Ramsey 数 下界 循环图
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1 The main results

The computation of Ramsey numbers is a very hard problem in Combinatorics[1]. So far the best known lower bounds $R(3,k,l)$ in the dynamic survey[2] are as follows: $R(3,3,4) \geqslant 30$[8], $R(3,3,5) \geqslant 45$[4,5], $R(3,3,6) \geqslant 60$[6], $R(3,3,7) \geqslant 74$[7], $R(3,3,9) \geqslant 110$[8], $R(3,4,4) \geqslant 55$[5], $R(3,4,5) \geqslant 80$[9].

Based on References [8] and [10] to [15] we have studied various colorings for edges of a complete graph K_p of prime number order, and three new lower bounds for $p = 313$ were obtained:

Theorem 1 $R(3,3,17) \geqslant 314, R(3,4,14) \geqslant 314, R(3,6,9) \geqslant 314$.

These three results have no previous records.

2 An algorithm for the computation of the clique number of the circulant graph $G_p(A)$

Let p be a prime number greater than 5. Let $Z_p = \{(1-p)/2, \cdots, -1, 0, 1, \cdots, (p-1)/2\}$ denote the finite field of p elements. We define a total order in Z_p in the natural way, i. e., $(1-p)/2 < \cdots < -1 < 0 < 1 < \cdots < (p-1)/2$. The absolute value of an element in Z_p is defined in the usual sense. Let $Z_p^+ = \{1, 2, \cdots, (p-1)/2\}$.

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Definition 1 Let A be a subset of Z_p^+. The graph $G_p(A)$ is defined as follows: the set of vertices of $G_p(A)$ is Z_p and there is an edge from a vertex x to another vertex y if and only if $|y-x| \in A$. We call $G_p(A)$ the circulant graph of order p associated with the parameter set A.

Lamma 1 Let $a=1$ or $a=-1$. Let $b \in Z_p$. Then for any parameter set $A \subset Z_p^+$ the transformation $f: x \mapsto ax+b$ gives rise to an automorphism of the graph $G_p(A)$.

Proof Obviously the map f is injective and surjective from Z_p to Z_p. The equality $|f(x)-f(y)| = |ax-ay| = |x-y|$ implies that $\{x,y\}$ is an edge of $G_p(A)$ if and only if $\{f(x), f(y)\}$ is an edge of $G_p(A)$. □

Lemma 1 demonstrates the fundamental property of $G_p(A)$. We are going to investigate other properties.

Definition 2 Let A be a subset of Z_p^+ and let $B=\{x \in Z_p \mid |x| \in A\}$. Let $G[B]$ denote the subgraph of $G_p(A)$ whose vertex set is B and $\{x,y\}$ is an edge of $G[B]$ if and only if $x,y \in B$ and $|x-y| \in A$. We call $G[B]$ the derived graph of $G_p(A)$.

Theorem 2 Let $[A]$ and $[B]$ denote the clique numbers of $G_p(A)$ and $G[B]$ respectively. Then $[A]=[B]+1$.

Proof It follows from Definition 2 that if the vertex 0 is added to a clique of $G[B]$ with $[B]$ vertices then a clique of $G_p(A)$ with $[B]+1$ vertices is obtained. This proves $[A] \geqslant [B]+1$. It remains to show $[A] \leqslant [B]+1$.

Assume that $k=[A] \geqslant 2$. Then there is a clique of $G_p(A)$ with k vertices $\{x_1, x_2, \cdots, x_k\}$. Let $f(x)=x-x_1$ for every $x \in G_p(A)$. Then $\{f(x_2), \cdots, f(x_k)\}$ is a clique of $G[B]$ by Definition 2. This shows $[B] \geqslant k-1$. □

Definition 2 implies that $y \in B, |y-a| \in A$ if and only if $-y \in B, |-y+a| \in A$. Thus we have

Lemma 2 For $a \in B$, let $d(a)$ denote the number of elements of the set $\{y \in B \mid |y-a| \in A\}$. Then $d(a)=d(-a)$.

Definition 3 An order in B is defined as follows: Let $x,y \in B$.

1) If $d(x)<d(y)$, then $x \prec y$;

2) If $d(x)=d(y)$ and $|x|<|y|$, then $x \prec y$;

3) If $d(x)=d(y), x=-y$ and $x>0$, then $x \prec y$.

It is easy to see that $\prec$ is a total order of B. We say that x is a predecessor of y (or y is a successor of x) if $x \prec y$.

Definition 4 A chain $x_0 \prec x_1 \prec \cdots \prec x_k$ in B with $k \geqslant 1$ is called a chain of length k initiated from x_0. Moreover, if $|x_i - x_j| \in A$ for all $0 \leqslant i < j \leqslant k$, then this chain is called an A-chain. *The maximal length of all A-chains initiated from x_0 is denoted by* $l(x_0)$. If x_0 has no successor x_1 with $|x_1-x_0| \in A$, then define $l(x_0)=0$.

Theorem 3 The following equality holds:

$$[B]=1+\max\{l(a) \mid a \in A\}. \tag{1}$$

Proof If $[B]=1$ then (1) is obviously true. Henceforth we assume that $[B]>1$.

It follows from definition directly that the $k+1$ vertices of an A-chain $x_0 \prec x_1 \prec \cdots \prec x_k$ form a clique of $G[B]$. Hence $[B] \geqslant k+1$. This shows that the left hand side of (1) is greater than or equal to the right hand side of (1).

Let $[B]=1+k(k \geqslant 1)$. Then there is a clique of $k+1$ vertices in $G[B]$. Arrange these vertices as $x_0 \prec x_1 \prec \cdots \prec x_k$ and we get an A-chain in B. There are two cases for x_0:

Case 1: $x_0 \in A$.

In this case, we have $k \leqslant l(x_0) \leqslant \max\{l(a) \mid a \in A\}$.

Case 2: $x_0 \notin A$.

Then $-x_0 \in A$ and thus $x_0<0$. Lemma 1 implies that the transformation $f: x \mapsto -x$ is an automorphism of $G_p(A)$. Consequently f maps the clique $\{x_0, x_1, \cdots, x_k\}$ to a clique $\{-x_0, -x_1, \cdots, x_k\}$ of $G[B]$. We claim that $-x_0 \prec -x_i$ for $1 \leqslant i \leqslant k$. It follows from Definition 3 that one of the following conditions is satisfied:

1) $d(x_0) < d(x_i)$;

2) $d(x_0) = d(x_i)$ and $|x_0| < |x_i|$.

Lemma 2 implies that one of the following conditions is satistied:

1) $d(-x_0) < d(-x_i)$;

2) $d(-x_0) = d(-x_i)$ and $|-x_0| < |-x_i|$.

This proves our claim. Hence there exists an A- chain of length k initiated from $-x_0$, which implies that $k \leqslant l(-x_0) \leqslant \max\{l(a) | a \in A\}$. Therefore the left hand side of (1) is less than or equal to the right hand side of (1). This concludes the proof of the theorem. □

Note that if $a \in A$ satisfies $|y-a| \notin A$ for every $y \in B$, then $d(a) = 0$ and $l(a) = 0$ by virtue of Lemma 2 and Definition 4.

Corollay 1 The equality $\max\{d(a) | a \in A\} = 0$ holds if and only if $[B] = 1$.

Now we describe an algorithm to calculate the clique number of $G_p(A)$ based on the above results.

Algorithm 1

Step 1) Generate the parameter set A for a given prime number p.

Step 2) Let $B = \{x \mid |x| \in A\}$.

Step 3) For every $a \in A$, compute $d(a)$. If max $\{d(a) | a \in A\} = 0$, let $[B] = 1$ and jump to Step 7.

Step 4) Rearrange the elements of B in terms of the order defined in Definition 3: $a_1, -a_1, a_2, -a_2, \cdots, a_r, -a_r$ where r is the cardinality of A. Set $i = 1$.

Step 5) List all A- chains initiated from a_i and compute $l(a_i)$ in terms of Definition 4.

Step 6) Increase i by 1. If $i < r$ jump to Step 5, otherwise we obtain $[B] = 1 + \max\{l(a_i) | 1 \leqslant i \leqslant r\}$ according to Theorem 3.

Step 7) $[A] = [B] + 1$ according to Theorem 2 and terminate the process.

Remark We may use the standard depth-first search technique to find the longest A- chain initiated from a_i in Step 5. Since the search is restricted to the chains intiated from a point in A and since the order we have introduced in B can significantly reduce the amout of calculation during the process of backtracking, our algorithm is rather effective.

3 Clique numbers of some circulant graphs G_{313} (A)

In this section we assume that $p = 313$. For some parameter sets A_i we have computed the clique numbers of the corresponding graphs $G_p(A_i)$.

Lemma 3 For the parameter sets

$A_1 = \{1, 5, 7, 19, 23, 27, 33, 36, 44, 48, 58, 61, 64, 73, 79, 82, 93, 95, 103, 111, 113, 124, 133, 135, 148, 150\}$,

$A_2 = \{2, 10, 13, 14, 17, 38, 43, 46, 47, 54, 65, 66, 72, 87, 88, 91, 96, 107, 116, 122, 123, 127, 128, 146, 149, 155\}$,

$A_3 = \{10, 14, 17, 30, 40, 41, 42, 43, 46, 47, 51, 53, 56, 62, 65, 68, 74, 89, 90, 91, 101, 109, 110, 118, 120, 122, 123, 125, 126, 127, 129, 134, 138, 141, 145, 146, 149, 153, 154\}$,

$A_4 = \{9, 11, 12, 16, 18, 22, 24, 29, 30, 31, 32, 37, 40, 42, 45, 50, 51, 53, 55, 56, 60, 63, 68, 70, 75, 77, 78, 80, 83, 84, 85, 98, 100, 102, 104, 105, 106, 112, 117, 118, 119, 125, 129, 134, 136, 138, 140, 141, 142, 143, 147, 156\}$,

$A_5 = \{2, 3, 4, 6, 8, 10, 13, 14, 15, 17, 20, 21, 25, 26, 28, 34, 35, 38, 39, 41, 43, 46, 47, 49, 52, 54, 57, 59, 62, 65, 66, 67, 69, 71, 72, 74, 76, 81, 86, 87, 88, 89, 90, 91, 92, 94, 96, 97, 99, 101, 107, 108, 109, 110, 114, 115, 116, 120, 121, 122, 123, 126, 127, 128, 130, 131, 132, 137, 139, 144, 145, 146, 149, 151, 152, 153, 154, 155},

A_6 = {2, 3, 4, 6, 8, 9, 11, 12, 13, 15, 16, 18, 20, 21, 22, 24, 25, 26, 28, 29, 31, 32, 34, 35, 37, 38, 39, 45, 49, 50, 52, 54, 55, 57, 59, 60, 63, 66, 67, 69, 70, 71, 72, 75, 76, 77, 78, 80, 81, 83, 84, 85, 86, 87, 88, 92, 94, 96, 97, 98, 99, 100, 102, 104, 105, 106, 107, 108, 112, 114, 115, 116, 117, 119, 121, 128, 130, 131, 132, 136, 137, 139, 140, 142, 143, 144, 147, 151, 152, 155, 156},

A_7 = {3, 4, 6, 8, 9, 11, 12, 15, 16, 18, 20, 21, 22, 24, 25, 26, 28, 29, 30, 31, 32, 34, 35, 37, 39, 40, 41, 42, 45, 49, 50, 51, 52, 53, 55, 56, 57, 59, 60, 62, 63, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 80, 81, 83, 84, 85, 86, 89, 90, 92, 94, 97, 98, 99, 100, 101, 102, 104, 105, 106, 108, 109, 110, 112, 114, 115, 117, 118, 119, 120, 121, 125, 126, 129, 130, 131, 132, 134, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 147, 151, 152, 153, 154, 156 },

the clique numbers of the graph $G_p(A_i)$ are: $[A_1] = [A_2] = 2, [A_3] = 3, [A_4] = 5, [A_5] = 8, [A_6] = 13, [A_7] = 16$ respectively.

Proof.

(1) Let $A = A_1$ and $B = \{x \mid |x| \in A\}$. We obtain $\max\{d(a); a \in A\} = 0 \Rightarrow [A_1] = 2$. Similarly we may prove that $[A_2] = 2$.

(2) Let $A = A_3$ and $B = \{x \mid |x| \in A\}$. We obtain $d(10) = d(14) = d(17) = d(30) = d(40) = d(41) = d(42) = d(43) = d(46) = d(47) = d(51) = d(53) = d(56) = d(62) = d(65) = d(68) = d(74) = d(89) = d(90) = d(91) = d(101) = d(109) = d(110) = d(118) = d(120) = d(122) = d(123) = d(125) = d(126) = d(127) = d(129) = d(134) = d(138) = d(141) = d(145) = d(146) = d(149) = d(153) = d(154) = 14$ by computation.

Hence the elements of B can be arranged in the ascending order:

$(B_3, \prec)$ = {10, −10, 14, −14, 17, −17, 30, −30, 40, −40, 41, −41, 42, −42, 43, −43, 46, −46, 47, −47, 51, −51, 53, −53, 56, −56, 62, −62, 65, −65, 68, −68, 74, −74, 89, −89, 90, −90, 91, −91, 101, −101, 109, −109, 110, −110, 118, −118, 120, −120, 122, −122, 123, −123, 125, −125, 126, −126, 127, −127, 129, −129, 134, −134, 138, −138, 141, −141, 145, −145, 146, −146, 149, −149, 153, −153, 154, −154}.

For $a = 10$, the set $\{y \in B \mid |y - a| \in A\}$ is equal to {−30, 40, −41, −43, −46, 51, 53, 56, −91, 101, −110, 120, −149, −154},

which is obtained during tbe computation of $d(a)$. Hence the longest A-chain initiated from a is $10 \prec 30$. Thus $l(a) = 1$.

Similarly we may prove that $l(a) \leqslant 1$ for every $a \in A$. Therefore we have

$$[B] = 1 + \max\{l(a) \mid a \in A\} = 2 \text{ and } [A_3] = 3.$$

(3) Let $A = A_4$ and $B = \{x \mid |x| \in A\}$. We obtain $d(9) = d(11) = d(12) = d(16) = d(50) = d(70) = d(78) = d(83) = d(85) = d(98) = d(104) = d(119) = d(142) = 24, d(31) = d(37) = d(45) = d(55) = d(60) = d(63) = d(77) = d(80) = d(84) = d(102) = d(106) = d(112) = d(136) = 31, d(18) = d(22) = d(24) = d(29) = d(32) = d(75) = d(100) = d(105) = d(117) = d(140) = d(143) = d(147) = d(156) = 35, d(30) = d(40) = d(42) = d(51) = d(53) = d(56) = d(68) = d(118) = d(125) = d(129) = d(134) = d(138) = d(141) = 36$ by computation.

Hence the elements of B can be arranged in the ascending order:

$(B_4, \prec)$ = {9, −9, 11, −11, 12, −12, 16, −16, 50, −50, 70, −70, 78, −78, 83, −83, 85, −85, 98, −98, 104, −104, 119, −119, 142, −142, 31, −31, 37,

$-37, 45, -45, 55, -55, 60, -60, 63, -63, 77, -77, 80, -80, 84, -84, 102, -102, 106, -106, 112, -112, 136, -136, 18, -18, 22, -22, 24, -24, 29, -29, 32, -32, 75, -75, 100, -100, 105, -105, 117, -117, 140, -140, 143, -143, 147, -147, 156, -156, 30, -30, 40, -40, 42, -42, 51, -51, 53, -53, 56, -56, 68, -68, 118, -118, 125, -125, 129, -129, 134, -134, 138, -138, 141, -141\}$.

For $a = 9$, the set $\{y \in B \mid |y-a| \in A\}$ is equal to $\{-9, 31, -31, 60, 77, 84, 18, -22, -75, 143, 147, -147, 156, 40, -42, 51, -51, -68, -125, -129, 134, -134, 138, -138\}$,

which is obtained during the computation of $d(a)$. Hence the longest A -chain initiated from a is $9 \prec -9 \prec 147 \prec -134$. Thus $l(a) = 3$.

Similarly we may prove that $l(a) \leqslant 3$ for every $a \in A$. Therefore we have

$$[B] = 1 + \max\{l(a) \mid a \in A\} = 4 \text{ and } [A_4] = 5.$$

(4) Let $A = A_5$ and $B = \{x \mid |x| \in A\}$. We obtain $d(3) = d(4) = d(26) = d(57) = d(76) = d(81) = d(99) = d(108) = d(121) = d(132) = d(137) = d(139) = d(144) = 65, d(10) = d(14) = d(17) = d(43) = d(46) = d(47) = d(65) = d(91) = d(122) = d(123) = d(127) = d(146) = d(149) = 72, d(41) = d(62) = d(74) = d(89) = d(90) = d(101) = d(109) = d(110) = d(120) = d(126) = d(145) = d(153) = d(154) = 76, d(6) = d(8) = d(15) = d(20) = d(21) = d(25) = d(28) = d(34) = d(35) = d(39) = d(49) = d(52) = d(59) = d(67) = d(69) = d(71) = d(86) = d(92) = d(94) = d(97) = d(114) = d(115) = d(130) = d(131) = d(151) = d(152) = 77, d(2) = d(13) = d(38) = d(54) = d(66) = d(72) = d(87) = d(88) = d(96) = d(107) = d(116) = d(128) = d(155) = 80$ by computation.

Hence the elements of B can be arranged in the ascending order:

$(B_5, \prec) = \{3, -3, 4, -4, 26, -26, 57, -57, 76, -76, 81, -81, 99, -99, 108, -108, 121, -121, 132, -132, 137, -137, 139, -139, 144, -144, 10, -10, 14, -14, 17, -17, 43, -43, 46, -46, 47, -47, 65, -65, 91, -91, 122, -122, 123, -123, 127, -127, 146, -146, 149, -149, 41, -41, 62, -62, 74, -74, 89, -89, 90, -90, 101, -101, 109, -109, 110, -110, 120, -120, 126, -126, 145, -145, 153, -153, 154, -154, 6, -6, 8, -8, 15, -15, 20, -20, 21, -21, 25, -25, 28, -28, 34, -34, 35, -35, 39, -39, 49, -49, 52, -52, 59, -59, 67, -67, 69, -69, 71, -71, 86, -86, 92, -92, 94, -94, 97, -97, 114, -114, 115, -115, 130, -130, 131, -131, 151, -151, 152, -152, 2, -2, 13, -13, 38, -38, 54, -54, 66, -66, 72, -72, 87, -87, 88, -88, 96, -96, 107, -107, 116, -116, 128, -128, 155, -155\}$.

For $a = 3$, the set $\{y \in B \mid |y-a| \in A\}$ is equal to $\{-3, 57, 99, -10, -14, 17, -17, -43, 46, -46, 65, 91, -91, 123, -123, -127, -146, 149, -149, 41, 62, -62, 74, 89, -89, 90, 110, -120, 126, 154, 6, 20, -25, 28, -35, 49, -49, 52, -59, 69, -69, -71, -86, 92, 94, -94, 97, 130, 131, -151, 152, -152, 13, 38, -38, -54, -66, 72, -87, -88, -96, -107, -128, 155, -155\}$,

which is obtained during the computation of $d(a)$. Hence the longest A -chain initiated from a is $3 \prec -3 \prec 46 \prec -123 \prec 89 \prec -69 \prec 155$. Thus $l(a) = 6$.

Similarly we may prove that $l(a) \leqslant 6$ for every $a \in A$. Therefore we have

$$[B] = 1 + \max\{l(a) \mid a \in A\} = 7 \text{ and } [A_5] = 8.$$

(5) Let $A = A_6$ and $B = \{x \mid |x| \in A\}$. We obtain $d(6) = d(8) = d(25) = d(35) = d(39) = d(49) = d(52) = d(71) = d(97) = d(114) = d(115) = d(151) = d(152) = 103, d(2) = d(13) = d(38) = d(54) = d(66) = d(72) = d(87) = d(88) = d(96) = d(107) = d(116) = d(128) = d(155) = 104, d(18) = d(22) = d(24) = d(29) = d(32) = d(75) = d(100)$

$= d(105) = d(117) = d(140) = d(143) = d(147) = d(156) = 105, d(3) = d(4) = d(9) = d(11) = d(12) = d(16) = d(26) = d(50) = d(57) = d(70) = d(76) = d(78) = d(81) = d(83) = d(85) = d(98) = d(99) = d(104) = d(108) = d(119) = d(121) = d(132) = d(137) = d(139)zd(142) = d(144) = 107, d(15) = d(20) = d(21) = d(28) = d(34) = d(59) = d(67) = d(69) = d(86) = d(92) = d(94) = d(130) = d(131) = 108, d(31) = d(37) = d(45) = d(55) = d(60) = d(63) = d(77) = d(80) = d(84) = d(102) = d(106) = d(112) = d(136) = 110$ by computation.

Hence the elements of B can be arranged in the ascending order:

$(B_6, \prec) = \{6, -6, 8, -8, 25, -25, 35, -35, 39, -39, 49, -49, 52, -52, 71, -71, 97, -97, 114, -114, 115, -115, 151, -151, 152, -152, 2, -2, 13, -13, 38, -38, 54, -54, 66, -66, 72, -72, 87, -87, 88, -88, 96, -96, 107, -107, 116, -116, 128, -128, 155, -155, 18, -18, 22, -22, 24, -24, 29, -29, 32, -32, 75, -75, 100, -100, 105, -105, 117, -117, 140, -140, 143, -143, 147, -147, 156, -156, 3, -3, 4, -4, 9, -9, 11, -11, 12, -12, 16, -16, 26, -26, 50, -50, 57, -57, 70, -70, 76, -76, 78, -78, 81, -81, 83, -83, 85, -85, 98, -98, 99, -99, 104, -104, 108, -108, 119, -119, 121, -121, 132, -132, 137, -137, 139, -139, 142, -142, 144, -144, 15, -15, 20, -20, 21, -21, 28, -28, 34, -34, 59, -59, 67, -67, 69, -69, 86, -86, 92, -92, 94, -94, 130, -130, 131, -131, 31, -31, 37, -37, 45, -45, 55, -55, 60, -60, 63, -63, 77, -77, 80, -80, 84, -84, 102, -102, 106, -106, 112, -112, 136, -136\}$.

For $a = 6$, the set $\{y \in B \mid |y - a| \in A\}$ is equal to $\{-6, 8, -25, 35, -39, -49, -71, 114, -115, -151, -152, 2, -2, 38, -54, 66, -66, 72, -72, 87, -88, -96, -155, 18, -18, 22, -22, 24, -29, 32, -32, 75, -75, 100, -100, -105, 143, -156, 3, -3, 4, 9, -9, 12, -12, -16, 26, -26, -57, -70, 76, 78, -78, 81, -81, 83, 98, -98, -99, 104, 108, -108, 121, 137, -137, 142, 15, -15, -20, 21, 28, -28, 34, 69, -69, 86, -86, 92, -92, 94, -94, -130, -131, 31, -31, 37, 45, 55, 60, -60, 63, -63, 77, -77, -80, 84, 102, -102, 106, -106, 112, 136, -136\}$,

which is obtained during tbe computation of $d(a)$. Hence the longest A- chain initiated from a is $6 \prec -6 \prec 66 \prec 72 \prec 75 \prec 3 \prec -3 \prec 9 \prec -9 \prec 12 \prec -12 \prec 69$. Thus $l(a) = 11$.

Similarly we may prove that $l(a) \leqslant 11$ for every $a \in A$. Therefore we have

$$[B] = 1 + \max\{l(a) \mid a \in A\} = 12 \text{ and } [A_6] = 13.$$

(6) Let $A = A_7$ and $B = \{x \mid |x| \in A\}$. We obtain $d(3) = d(4) = d(26) = d(57) = d(76) = d(81) = d(99) = d(108) = d(121) = d(132) = d(137) = d(139) = d(144) = 136, d(9) = d(11) = d(12) = d(16) = d(18) = d(22) = d(24) = d(29) = d(32) = d(50) = d(70) = d(75) = d(78) = d(83) = d(85) = d(98) = d(100) = d(104) = d(105) = d(117) = d(119) = d(140) = d(142) = d(143) = d(147) = d(156) = 137, d(6) = d(8) = d(25) = d(30) = d(35) = d(39) = d(40) = d(42) = d(49) = d(51) = d(52) = d(53) = d(56) = d(68) = d(71) = d(97) = d(114) = d(115) = d(118) = d(125) = d(129) = d(134) = d(138) = d(141) = d(151) = d(152) = 139, d(15) = d(20) = d(21) = d(28) = d(34) = d(59) = d(67) = d(69) = d(86) = d(92) = d(94) = d(130) = d(131) = 142, d(31) = d(37) = d(41) = d(45) = d(55) = d(60) = d(62) = d(63) = d(74) = d(77) = d(80) = d(84) = d(89) = d(90) = d(101) = d(102) = d(106) = d(109) = d(110) = d(112) = d(120) = d(126) = d(136) = d(145) = d(153) = d(154) = 143$ by computation.

Hence the elements of B can be arranged in the ascending order:

$(B_7, \prec) = \{3, -3, 4, -4, 26, -26, 57, -57, 76, -76, 81, -81, 99, -99,$

108, −108, 121, −121, 132, −132, 137, −137, 139, −139, 144, −144, 9, −9, 11, −11, 12, −12, 16, −16, 18, −18, 22, −22, 24, −24, 29, −29, 32, −32, 50, −50, 70, −70, 75, −75, 78, −78, 83, −83, 85, −85, 98, −98, 100, −100, 104, −104, 105, −105, 117, −117, 119, −119, 140, −140, 142, −142, 143, −143, 147, −147, 156, −156, 6, −6, 8, −8, 25, −25, 30, −30, 35, −35, 39, −39, 40, −40, 42, −42, 49, −49, 51, −51, 52, −52, 53, −53, 56, −56, 68, −68, 71, −71, 97, −97, 114, −114, 115, −115, 118, −118, 125, −125, 129, −129, 134, −134, 138, −138, 141, −141, 151, −151, 152, −152, 15, −15, 20, −20, 21, −21, 28, −28, 34, −34, 59, −59, 67, −67, 69, −69, 86, −86, 92, −92, 94, −94, 130, −130, 131, −131, 31, −31, 37, −37, 41, −41, 45, −45, 55, −55, 60, −60, 62, −62, 63, −63, 74, −74, 77, −77, 80, −80, 84, −84, 89, −89, 90, −90, 101, −101, 102, −102, 106, −106, 109, −109, 110, −110, 112, −112, 120, −120, 126, −126, 136, −136, 145, −145, 153, −153, 154, −154}.

For $a = 3$, the set $\{y \in B \mid |y - a| \in A\}$ is equal to {−3, −26, −57, 81, −81, −99, 108, 121, 132, 137, −137, 139, −139, 144, −144, 9, −9, 11, 12, −12, 18, −18, −22, 24, 29, −29, 32, −32, −50, 70, −75, 78, −78, 83, −83, −98, 100, 104, 105, −105, 117, −117, 140, −140, 142, −142, 143, 147, 156, −156, 6, −6, −8, 25, −25, 35, −39, 40, 42, −42, −49, 52, −52, 53, −53, 56, −56, −68, 71, −71, 97, −97, −114, 115, −115, 118, −118, 129, −129, 134, −134, −138, 141, −141, −151, 15, −15, 21, −21, 28, −28, 34, −34, 59, −59, −67, 86, −86, 92, −94, −191, 31, −31, 37, −37, 45, 55, 60, −60, 62, 63, 74, −74, 77, −77, 80, −80, 84, 89, −89, 101, −101, 102, −102, −106, 109, −109, 112, −112, 120, −126, −136, 145, −153, 154, −154},

which is obtained during the computation of $d(a)$. Hence the longest A- chain initiated from a is $3 \prec -3 \prec 137 \prec -144 \prec -29 \prec -32 \prec 52 \prec 141 \prec -141 \prec 86 \prec 31 \prec 60 \prec -74 \prec 102 \prec -109$. Thus $l(a) = 14$.

Similarly we may prove that $l(a) \leqslant 14$ for every $a \in A$. Therefore we have

$$[B] = 1 + \max\{l(a) \mid a \in A\} = 15 \text{ and } [A_7] = 16.$$

4 Proof of Theorem 1

Let p be a prime number. Let $Z_p^+ = S_1 \cup \cdots \cup S_n$ be a partition of the set Z_p^+. Let Z_p be the vertex set of the complete graph K_p. Let $E_i = \{\{x,y\} \in Z_p \times Z_p \mid |x - y| \in S_i\}$ for $1 \leqslant i \leqslant n$. Then $E_1 \cup \cdots \cup E_n$ is a partition of the edge set E of K_p. We say that the edges in E_i are colored in i. Thus we obtained a coloring of the edges of K_p using n colors $1, \cdots, n$.

The subgraph $G_p(S_i) = (Z_p, E_i)$ is exactly the circulant graph associated with the parameter set S_i as defined in Definition 1. Let $[S_i]$ denote the clique number of $G_p(S_i)$. It follows from Ramsey's Theorem that

Lemma 4 $R([S_1] + 1, [S_2] + 1, \cdots, [S_n] + 1) \geqslant p + 1$.

Proof of Theorem 1: Let $p = 313$.

1) Let $S_1 = A_1, S_2 = A_2, S_3 = A_7$. Then $\{S_1, S_2, S_3\}$ is a partition of Z_p^+. Lemma 3 implies that $[S_1] = [S_2] = 2, [S_3] = 16$. It follows from Lemma 4 that $R(3,3,17) \geqslant 314$.

2) Let $S_1 = A_1, S_2 = A_3, S_3 = A_6$. Then $\{S_1, S_2, S_3\}$ is a partition of Z_p^+. Lemma 3 implies that $[S_1] = 2, [S_2] = 3, [S_3] = 13$. It follows from Lemma 4 that $R(3,4,14) \geqslant 314$.

3) Let $S_1 = A_1, S_2 = A_4, S_3 = A_5$. Then $\{S_1, S_2, S_3\}$ is a partition of Z_p^+. Lemma 3 implies that $[S_1] = 2, [S_2] = 5, [S_3] = 8$, and it follows from Lemma 4 that $R(3,6,9) \geqslant 314$. □

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进行描述。同时，对于 Agent 的行为也要进行细化，也有组件的形式描述。这样，既可以便于系统资源的重用，同时又有利于系统的更新换代。而近年发展迅速的、以 CORBA 和 DCOM 为代表的软构件/软总线技术，则为异质组件的开发与“即插即用”提供了规范。

4 结语

本文仅仅对多 Agent 系统的构造技术进行了框架性的研究，工作只是初步的，仍有很多的问题需要解决。应该指出的是，面向多 Agent 系统的体系理论和相关软件技术的发展，将会对计算机应用领域产生深刻的影响。开展面向多 Agent 系统的技术研究是很有意义的。

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