# 一类具有时滞和基于比率的 $N$ 种群捕食-被捕食扩散系统的正周期解的存在性*

# The Existence of Positive Periodic Solution of A $N$-Species Predator-Prey Dispersion System with Delay and Ratio-Dependence

陈春涛，罗桂烈

Chen Chuntao，Luo Guilie

（广西师范大学数学与计算机科学学院，广西桂林 541004）

（Dept. of Math. & Comp. Sci.，Guangxi Normal Univ.，Guilin，Guangxi，541004，China）

**摘要**：运用重合度理论的连续性引理，得到一类具有时滞和基于比率的 $N$ 种群食物链捕食-被捕食扩散系统正周期解存在性的充分条件.

**关键词**：扩散系统 捕食-被捕食 正周期解 时滞 基于比率 重合度

**中图法分类号**：O175 **文献标识码**：A **文章编号**：1002-7378(2005)01-0001-06

**Abstract**：The continuation theorem of coincidence degree theory is used to obtain sufficient conditions that guarantee the existence of a positive periodic solution for a $N$-species ratio-dependent predator-prey diffusion model with time delay.

**Key words**：dispersion system，predator-prey，positive periodic solutions，time delay，ratio-dependent，coincidence degree

在人口动力学中，空间因素起到很重要的作用，很多学者进行了深入而细致的研究[1~5]. 此外，过去的历史对系统的稳定性也起着重要的作用. 近来，带时滞的系统的持续性和稳定性也被许多学者所讨论[6~9]，并得到一些充分条件来保证系统的持久性.

本文利用重合度理论的连续性引理，研究了 $N$ 种群捕食-被捕食扩散系统(1)正周期解的存在性，系统(1)中 $x_i(t)(i=1,3,\cdots,n+1)$ 表示 $t$ 时刻第 $i$ 个种群 $X_i$ 在斑块 Ⅰ 的密度，$x_2(t)$ 是 $t$ 时刻种群 $X_1$ 在斑块 Ⅱ 的密度. 种群 $X_i(i=3,\cdots,n+1)$ 只在斑块 Ⅰ 中，而种群 $X_1$ 可在两斑块间迁移，$D_i(t)(i=1,2)$ 是种群 $X_1$ 的扩散系数. $a_i(t),b_i(t)(i=1,2,\cdots,n+1)$ 和 $c_i(t),d_i(t)(i=1,2,\cdots,n-1)$ 及 $r_i(t)$，$D_i(t)(i=1,2)$ 均为连续的严格正 $\omega$- 周期函数；$\tau>0$ 为常数，表示怀孕期；在 $[-\tau,0](0\leqslant\tau<\infty)$ 上 $k_i(s)\geqslant 0(i=1,2)$，且 $k_i(s)$ 为分段连续的函数，满足 $\int_{-\tau}^{0}k_i(s)\mathrm{d}s=1$. 种群 $X_1$ 是种群 $X_3$ 的食饵，$X_3$ 又是 $X_4$ 的食饵，…，如此继续下去，$X_{i-1}$ 是 $X_i$ 的食饵，$i=$

$$
\begin{cases}
\dot{x}_1(t)=x_1(t)[a_1(t)-b_1(t)x_1(t)-\\
\quad r_1(t)\int_{-\tau}^{0}k_1(s)x_1(t+s)\mathrm{d}s-\\
\quad \dfrac{d_1(t)x_3(t)}{m_1(t)x_3(t)+x_1(t)}]+D_1(t)(x_2(t)-\\
\quad x_1(t)),\\
\dot{x}_2(t)=x_2(t)[a_2(t)-b_2(t)x_2(t)-\\
\quad r_2(t)\int_{-\tau}^{0}k_2(s)x_2(t+s)\mathrm{d}s]+\\
\quad D_2(t)(x_1(t)-x_2(t)),\\
\dot{x}_3(t)=x_3(t)[-a_3(t)-b_3(t)x_3(t)+\\
\quad \dfrac{c_1(t)x_1(t-\tau)}{m_1(t)x_3(t-\tau)+x_1(t-\tau)}-\\
\quad \dfrac{d_2(t)x_4(t)}{m_2(t)x_4(t)+x_3(t)}],\\
\dot{x}_i(t)=x_i(t)[-a_i(t)-b_i(t)x_i(t)+\\
\quad \dfrac{c_{i-2}(t)x_{i-1}(t-\tau)}{m_{i-2}(t)x_i(t-\tau)+x_{i-1}(t-\tau)}-\\
\quad \dfrac{d_{i-1}(t)x_{i+1}(t)}{m_{i-1}(t)x_{i+1}(t)+x_i(t)}],\\
\quad i=4,\cdots,n\\
\dot{x}_{n+1}(t)=x_{n+1}(t)[-a_{n+1}(t)-b_{n+1}(t)x_{n+1}(t)+\\
\quad \dfrac{c_{n-1}(t)x_n(t-\tau)}{m_{n-1}(t)x_{n+1}(t-\tau)+x_n(t-\tau)}].
\end{cases}
\tag{1}
$$

收稿日期：2004-07-19

修回日期：2004-09-13

作者简介：陈春涛(1980-)，女，广西武宣人，硕士研究生，主要从事微分方程的研究。

* 国家数学天元基金(A0324644)和广西自然科学基金(桂科青 O339021)资助项目。

$4,\cdots,n+1$,这样形成了一个食物链.

## 1 主要引理

为了得到系统(1)存在周期解的结果,本文先作如下准备.

设 $X,Y$ 为实 $Banach$ 空间,$L:DomL\subset X\to Y$ 是指标为零的 Fredholm 算子,$P:X\to X$,$Q:Y\to Y$ 是连续的投影算子,使得 $\mathrm{Im}P=KerL$,$KerQ=\mathrm{Im}L$,且 $X=KerL\oplus KerP$,$Y=\mathrm{Im}L\oplus\mathrm{Im}Q$. 定义 $L_p$ 为 $L$ 在 $DomL\cap KerP$ 上的限制,$K_p:\mathrm{Im}L\to KerP\cap DomL$ 是 $L_p$ 的逆,$J:\mathrm{Im}Q\to KerL$ 是 $\mathrm{Im}Q$ 到 $KerL$ 的同构. 后面本文将用到 Mawhin 的结果[10]:

**引理1** 设 $\Omega$ 是 $X$ 中的有界开集,$N:X\to Y$ 是连续算子且在 $\overline{\Omega}$ 上是 $L$ 紧的(即 $QN:\overline{\Omega}\to Y$ 及 $K_p(I-Q)N:\overline{\Omega}\to Y$ 是紧的). 假设

(i) 对任意的 $\lambda\in(0,1)$,$x\in\partial\Omega\cap DomL$,$Lx\neq\lambda Nx$;

(ii) 对任意的 $x\in\partial\Omega\cap KerL$,$QNx\neq 0$;

(iii) $\deg\{JQN,\Omega\cap KerL,0\}\neq 0$,

则 $Lx=Nx$ 在 $\overline{\Omega}\cap DomL$ 中至少存在一个解.

## 2 主要结果及证明

对连续的正 $\omega$- 周期函数 $f(t)$,本文定义:

$$\overline{f}=\frac{1}{\omega}\int_0^{\omega}f(t)\mathrm{d}t,\ f^L=\min_{t\in[0,\omega]}|f(t)|,\ f^M=\max_{t\in[0,\omega]}|f(t)|. \tag{2}$$

**定理1** 假设系统(1)满足以下条件:

(B1) $\bar{c}_{i-2}-\bar{a}_i>0,c_{i-2}^M-\bar{a}_i>0,i=3,4,\cdots,n+1$.

(B2) $a_1^L-r_1^MH_1>\dfrac{d_1^M}{m_1^L},a_2^L-r_2^MH_2>\dfrac{d_2^M}{m_2^L}$,

(B3) $\dfrac{c_1^Lh_1}{m_1^MH_3+H_1}-a_3^M>\dfrac{d_2^M}{m_2^L},\dfrac{c_{i-2}^Lh_{i-1}}{m_{i-2}^MH_i+H_{i-1}}-a_i^M>\dfrac{d_{i-1}^M}{m_{i-1}^L},i=4,\cdots,n+1$,

其中,$h_1,\cdots,h_n$ 及 $H_1,\cdots,H_{n+1}$ 由证明过程给出. 则系统(1)至少存在一个正 $\omega$- 周期解.

**证明** 考虑系统(3),系统(3)中,$a_i(t),b_i(t)(i=1,2,\cdots,n+1)$ 和 $c_i(t),d_i(t)(i=1,2,\cdots,n-1)$ 及 $r_i(t),D_i(t),k_i(s)(i=1,2)$ 同系统(1). 显然,若系统(3)有一正 $\omega$- 周期解 $(u_1^*(t),u_2^*(t),\cdots,u_{n+1}^*(t))^T$, 则 $(\exp[u_1^*(t)],\exp[u_2^*(t)],\cdots,\exp[u_{n+1}^*(t)])^T$ 就是系统(1)的一个正 $\omega$- 周期解,故只须证明(3)有一正 $\omega$- 周期解.

$$\begin{cases}\dot{u}_1(t)=a_1(t)-D_1(t)-b_1(t)e^{u_1(t)}-\\ \quad r_1(t)\displaystyle\int_{-\tau}^{0}k_1(s)e^{u_1(t+s)}ds-\\ \quad\dfrac{d_1(t)e^{u_3(t)}}{m_1(t)e^{u_3(t)}+e^{u_1(t)}}+D_1(t)e^{u_2(t)-u_1(t)},\\ \dot{u}_2(t)=a_2(t)-D_2(t)-b_2(t)e^{u_2(t)}-\\ \quad r_2(t)\displaystyle\int_{-\tau}^{0}k_2(s)e^{u_2(t+s)}ds+D_2(t)e^{u_1(t)-u_2(t)},\\ \dot{u}_3(t)=-a_3(t)-b_3(t)e^{u_3(t)}+\\ \quad\dfrac{c_1(t)e^{u_1(t-\tau)}}{m_1(t)e^{u_3(t-\tau)}+e^{u_1(t-\tau)}}-\\ \quad\dfrac{d_2(t)e^{u_4(t)}}{m_2(t)e^{u_4(t)}+e^{u_3(t)}},\\ \dot{u}_i(t)=-a_i(t)-b_i(t)e^{u_i(t)}+\\ \quad\dfrac{c_{i-2}(t)e^{u_{i-1}(t-\tau)}}{m_{i-2}(t)e^{u_i(t-\tau)}+e^{u_{i-1}(t-\tau)}}-\\ \quad\dfrac{d_{i-1}(t)e^{u_{i+1}(t)}}{m_{i-1}(t)e^{u_{i+1}(t)}+e^{u_i(t)}},\\ \quad i=4,\cdots,n\\ \dot{u}_{n+1}(t)=-a_{n+1}(t)-b_{n+1}(t)e^{u_{n+1}(t)}+\\ \quad\dfrac{c_{n-1}(t)e^{u_n(t-\tau)}}{m_{n-1}(t)e^{u_{n+1}(t-\tau)}+e^{u_n(t-\tau)}},\end{cases} \tag{3}$$

为了使用重合度理论的连续性引理来证明系统(3)的 $\omega$- 周期解的存在性,本文令

$X=Y=\{(u_1(t),u_2(t),\cdots,u_{n+1}(t))^T\in C(R,R^{n+1}):u_i(t+\omega)=u_i(t),i=1,2,\cdots,n+1\}$

及 $\|(u_1(t),u_2(t),\cdots,u_{n+1}(t))^T\|=\sum_{i=1}^{n+1}\max_{t\in[0,\omega]}|u_i(t)|$.

这里 $|\cdot|$ 代表欧几里德范数,在 $\|\cdot\|$ 下,$X$ 与 $Y$ 为 $Banach$ 空间.

令 $Lu=u'$,$DomL=\{(u_1(t),u_2(t),\cdots,u_{n+1}(t))^T\in C^1(R,R^{n+1})\}$,再令 $N:X\to X$ 得到(4)式.

定义算子 $P,Q$ 为

$Pu=Qu=\dfrac{1}{\omega}\displaystyle\int_0^{\omega}u(t)\mathrm{d}t,u(t)=(u_1(t),u_2(t),\cdots,u_{n+1}(t))^T$.

显然,$KerL=R^{n+1}$,$\mathrm{Im}L=\{(u_1(t),u_2(t),\cdots,u_{n+1}(t))^T\in X:\int_0^{\omega}u_i(t)\mathrm{d}t=0,i=1,2,\cdots,n+1\}$ 是 $X$ 的闭子空间,且 $\dim KerL=co\dim\mathrm{Im}L=n+1$,因此 $L$ 是一个指标为0的 Fredholm 算子. 通过计算,$L_p$ 的逆 $K_p$ 具有形式

$K_p:\mathrm{Im}L\to DomL\cap KerP$,

$K_p(u)=\displaystyle\int_0^t u(s)\mathrm{d}s-\frac{1}{\omega}\int_0^{\omega}\int_0^{\eta}u(t)\mathrm{d}t\mathrm{d}\eta$,

$$N\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ \vdots \\ u_n \\ u_{n+1} \end{bmatrix} = \begin{bmatrix} a_1(t) - D_1(t) - b_1(t)\mathrm{e}^{u_1(t)} - r_1(t)\int_{-\tau}^{0} k_1(s)\mathrm{e}^{u_1(t+s)}ds - \dfrac{d_1(t)\mathrm{e}^{u_3(t)}}{m_1(t)\mathrm{e}^{u_3(t)} + \mathrm{e}^{u_1(t)}} + D_1(t)\mathrm{e}^{u_2(t)-u_1(t)} \\ a_2(t) - D_2(t) - b_2(t)\mathrm{e}^{u_2(t)} - r_2(t)\int_{-\tau}^{0} k_2(s)\mathrm{e}^{u_2(t+s)}\mathrm{d}s + D_2(t)\mathrm{e}^{u_1(t)-u_2(t)} \\ -a_3(t) - b_3(t)\mathrm{e}^{u_3(t)} + \dfrac{c_1(t)\mathrm{e}^{u_1(t-\tau)}}{m_1(t)\mathrm{e}^{u_3(t-\tau)} + \mathrm{e}^{u_1(t-\tau)}} - \dfrac{d_2(t)\mathrm{e}^{u_4(t)}}{m_2(t)\mathrm{e}^{u_4(t)} + \mathrm{e}^{u_3(t)}} \\ -a_4(t) - b_4(t)\mathrm{e}^{u_4(t)} + \dfrac{c_2(t)\mathrm{e}^{u_3(t-\tau)}}{m_2(t)\mathrm{e}^{u_4(t-\tau)} + \mathrm{e}^{u_3(t-\tau)}} - \dfrac{d_3(t)\mathrm{e}^{u_5(t)}}{m_3(t)\mathrm{e}^{u_5(t)} + \mathrm{e}^{u_4(t)}} \\ \vdots \\ -a_n(t) - b_n(t)\mathrm{e}^{u_n(t)} + \dfrac{c_{n-2}(t)\mathrm{e}^{u_{n-1}(t-\tau)}}{m_{n-2}(t)\mathrm{e}^{u_n(t-\tau)} + \mathrm{e}^{u_{n-1}(t-\tau)}} - \dfrac{d_{n-1}(t)\mathrm{e}^{u_{n+1}(t)}}{m_{n-1}(t)\mathrm{e}^{u_{n+1}(t)} + \mathrm{e}^{u_n(t)}} \\ -a_{n+1}(t) - b_{n+1}(t)\mathrm{e}^{u_{n+1}(t)} + \dfrac{c_{n-1}(t)\mathrm{e}^{u_n(t-\tau)}}{m_{n-1}(t)\mathrm{e}^{u_{n+1}(t-\tau)} + \mathrm{e}^{u_n(t-\tau)}} \end{bmatrix} \tag{4}$$

**因此**

$$K_p(I-Q)Nu = \int_0^t \dot{u}(s)\mathrm{d}s - \frac{1}{\omega}\int_0^{\omega}\int_0^{\eta}\dot{u}(s)\mathrm{d}s\mathrm{d}\eta - \left(\frac{t}{\omega} - \frac{1}{2}\right)\int_0^{\omega}\dot{u}(s)\mathrm{d}s,$$

**其中**，$u(s) = (u_1(s), u_2(s), \cdots, u_{n+1}(s))^T$ **同系统**(3)**中的一致. 利用** Lebesgue **收敛定理可以证明** $QN$ **及** $K_P(I-Q)N$ **是连续的. 利用** Arzela-Ascoli **定理可证对** $X$ **中的任意有界开子集** $\Omega$，$QN(\overline{\Omega})$ **及** $K_P(I-Q)N(\overline{\Omega})$ **是相对紧的. 对应于算子方程** $Lx = \lambda Nx$，$\lambda \in (0,1)$，**于是得到系统**(5).

**设** $(u_1(t), u_2(t), \cdots, u_{n+1}(t))^T$ **是系统**(5)**对应于某一** $\lambda \in (0,1)$ **的一个解，选取** $t_i \in [0,\omega]$，**使得**

$$u_i(t_i) = \max_{t\in[0,\omega]} u_i(t), i = 1,2,\cdots,n+1,$$

**则由系统**(5)**可得到**(6)～(10)**式.**

$$\begin{cases} \dot{u}_1(t) = \lambda\Big[a_1(t) - D_1(t) - b_1(t)\mathrm{e}^{u_1(t)} - r_1(t)\int_{-\tau}^{0} k_1(s)\mathrm{e}^{u_1(t+s)}\mathrm{d}s - \dfrac{d_1(t)^{u_3(t)}}{m_1(t)\mathrm{e}^{u_3(t)} + \mathrm{e}^{u_1(t)}} + D_1(t)\mathrm{e}^{u_2(t)-u_1(t)}\Big], \\ \dot{u}_2(t) = \lambda\Big[a_2(t) - D_2(t) - b_2(t)\mathrm{e}^{u_2(t)} - r_2(t)\int_{-\tau}^{0} k_2(s)\mathrm{e}^{u_2(t+s)}\mathrm{d}s + D_2(t)\mathrm{e}^{u_1(t)-u_2(t)}\Big], \\ \dot{u}_3(t) = \lambda\Big[-a_3(t) - b_3(t)\mathrm{e}^{u_3(t)} + \dfrac{c_1(t)\mathrm{e}^{u_1(t-\tau)}}{m_1(t)\mathrm{e}^{u_3(t-\tau)} + \mathrm{e}^{u_1(t-\tau)}} - \dfrac{d_2(t)\mathrm{e}^{u_4(t)}}{m_2(t)\mathrm{e}^{u_4(t)} + \mathrm{e}^{u_3(t)}}\Big], \\ \dot{u}_i(t) = \lambda\Big[-a_i(t) - b_i(t)\mathrm{e}^{u_i(t)} + \dfrac{c_{i-2}(t)\mathrm{e}^{u_{i-1}(t-\tau)}}{m_{i-2}(t)\mathrm{e}^{u_i(t-\tau)} + \mathrm{e}^{u_{i-1}(t-\tau)}} - \dfrac{d_{i-1}(t)\mathrm{e}^{u_{i+1}(t)}}{m_{i-1}(t)\mathrm{e}^{u_{i+1}(t)} + \mathrm{e}^{u_i(t)}}\Big], \\ \quad i = 4,\cdots,n \\ \dot{u}_{n+1}(t) = \lambda\Big[-a_{n+1}(t) - b_{n+1}(t)\mathrm{e}^{u_{n+1}(t)} + \dfrac{c_{n-1}(t)\mathrm{e}^{u_n(t-\tau)}}{m_{n-1}(t)\mathrm{e}^{u_{n+1}(t-\tau)} + \mathrm{e}^{u_n(t-\tau)}}\Big], \end{cases} \tag{5}$$

$$a_1(t_1) - D_1(t_1) - b_1(t_1)\mathrm{e}^{u_1(t_1)} - r_1(t_1)\int_{-\tau}^{0} k_1(t_1)\mathrm{e}^{u_1(t_1+s)}\mathrm{d}s - \frac{d_1(t_1)\mathrm{e}^{u_3(t_1)}}{m_1(t_1)\mathrm{e}^{u_3(t_1)} + \mathrm{e}^{u_1(t_1)}} + D_1(t_1)\mathrm{e}^{u_2(t_1)-u_1(t_1)} = 0, \tag{6}$$

$$a_2(t_2) - D_2(t_2) - b_2(t_2)\mathrm{e}^{u_2(t_2)} - r_2(t_2)\int_{-\tau}^{0} k_2(s)\mathrm{e}^{u_2(t_2+s)}\mathrm{d}s + D_2(t_2)\mathrm{e}^{u_1(t_2)-u_2(t_2)} = 0, \tag{7}$$

$$-a_3(t_3) - b_3(t_3)\mathrm{e}^{u_3(t_3)} + \frac{c_1(t_3)\mathrm{e}^{u_1(t_3-\tau)}}{m_1(t_3)\mathrm{e}^{u_3(t_3-\tau)} + \mathrm{e}^{u_1(t_3-\tau)}} - \frac{d_2(t_3)\mathrm{e}^{u_4(t_3)}}{m_2(t_3)\mathrm{e}^{u_4(t_3)} + \mathrm{e}^{u_3(t_3)}} = 0, \tag{8}$$

$$-a_i(t_i) - b_i(t_i)\mathrm{e}^{u_i(t_i)} + \frac{c_{i-2}(t_i)\mathrm{e}^{u_{i-1}(t_i-\tau)}}{m_{i-2}(t_i)\mathrm{e}^{u_i(t_i-\tau)} + \mathrm{e}^{u_{i-1}(t_i-\tau)}} - \frac{d_{i-1}(t_i)\mathrm{e}^{u_{i+1}(t_i)}}{m_{i-1}(t_i)\mathrm{e}^{u_{i+1}(t_i)} + \mathrm{e}^{u_i(t_i)}} = 0, i = 4,\cdots,n, \tag{9}$$

$$-a_{n+1}(t_{n+1}) - b_{n+1}(t_{n+1})\mathrm{e}^{u_{n+1}(t_{n+1})} + \frac{c_{n-1}(t_{n+1})\mathrm{e}^{u_n(t_{n+1}-\tau)}}{m_{n-1}(t_{n+1})\mathrm{e}^{u_{n+1}(t_{n+1}-\tau)} + \mathrm{e}^{u_n(t_{n+1}-\tau)}} = 0. \tag{10}$$

**若** $u_1(t_1) \geqslant u_2(t_2)$，**则** $u_1(t_1) \geqslant u_2(t_2) \geqslant u_2(t_1)$，**由**(6)**式有**

$$\mathrm{e}^{u_1(t_1)} < \frac{a_1^M}{b_1^L}, \tag{11}$$

**若** $u_1(t_1) < u_2(t_2)$，**则** $u_1(t_2) \leqslant u_1(t_1) < u_2(t_2)$，**由**(7)**式有**

$$\mathrm{e}^{u_2(t_2)} < \frac{a_2^M}{b_2^L}, \tag{12}$$

**从而由**(11)，(12)**式有**

$$\mathrm{e}^{u_i(t)} \leqslant \mathrm{e}^{u_i(t_i)} < \max\left\{\frac{a_1^M}{b_1^L}, \frac{a_2^M}{b_2^L}\right\} := H_i, i = 1,2, \tag{13}$$

**此外，由**(8)～(10)**式，均有**

$$e^{u_i(t)} \leqslant e^{u_i(t_i)} \frac{c_{i-2}^M - a_i^L}{b_i^L} := H_i, i = 3,4,\cdots,n+1. \quad (14)$$

**选取** $\tau_i \in [0,\omega]$，**使得**

$$u_i(\tau_i) = \min_{t\in[0,\omega]} u_i(t), i = 1,2,\cdots,n+1,$$

**则由系统**(5)**有**

$$a_1(\tau_1) - D_1(\tau_1) - b_1(\tau_1)e^{u_1(\tau_1)} - r_1(\tau_1)\int_{-\tau}^0 k_1(s)e^{u_1(\tau_1+s)}ds - \frac{d_1(\tau_1)e^{u_3(\tau_1)}}{m_1(\tau_1)e^{u_3(\tau_1)} + e^{u_1(\tau_1)}} + D_1(\tau_1)e^{u_2(\tau_1)-u_1(\tau_1)} = 0, \quad (15)$$

$$a_2(\tau_2) - D_2(\tau_2) - b_2(\tau_2)e^{u_2(\tau_2)} - r_2(\tau_2)\int_{-\tau}^0 k_2(s)e^{u_2(\tau_2+s)}ds + D_2(\tau_2)e^{u_1(\tau_2)-u_2(\tau_2)} = 0, \quad (16)$$

$$-a_3(\tau_3) - b_3(\tau_3)e^{u_3(\tau_3)} + \frac{c_1(\tau_3)e^{u_1(\tau_3-\tau)}}{m_1(\tau_3)e^{u_3(\tau_3-\tau)} + e^{u_1(\tau_3-\tau)}} - \frac{d_2(\tau_3)e^{u_4(\tau_3)}}{m_2(\tau_3)e^{u_4(\tau_3)} + e^{u_3(\tau_3)}} = 0, \quad (17)$$

$$-a_i(\tau_i) - b_i(\tau_i)e^{u_i(\tau_i)} + \frac{c_{i-2}(\tau_i)e^{u_{i-1}(\tau_i-\tau)}}{m_{i-2}(\tau_i)e^{u_i(\tau_i-\tau)} + e^{u_{i-1}(\tau_i-\tau)}} - \frac{d_{i-1}(\tau_i)e^{u_{i+1}(\tau_i)}}{m_{i-1}(\tau_i)e^{u_{i+1}(\tau_i)} + e^{u_i(\tau_i)}} = 0, i = 4,\cdots,n, \quad (18)$$

$$-a_{n+1}(\tau_{n+1}) - b_{n+1}(\tau_{n+1})e^{u_{n+1}(\tau_{n+1})} + \frac{c_{n-1}(\tau_{n+1})e^{u_n(\tau_{n+1}-\tau)}}{m_{n-1}(\tau_{n+1})e^{u_{n+1}(\tau_{n+1}-\tau)} + e^{u_n(\tau_{n+1}-\tau)}} = 0, \quad (19)$$

**若** $u_1(\tau_1) \leqslant u_2(\tau_2)$，**则有** $u_1(\tau_1) \leqslant u_2(\tau_2) \leqslant u_2(\tau_1)$，**由**(13)**和**(15)**式，有**

$$b_1(\tau_1)e^{u_1(\tau_1)} > a_1(\tau_1) - r_1(\tau_1)\int_{-\tau}^0 k_1(s)H_1ds - \frac{d_1(\tau_1)}{m_1(\tau_1)},$$

**即**
$$e^{u_1(\tau_1)} > \frac{a_1^L - r_1^M H_1 - \frac{d_1^M}{m_1^L}}{b_1^M}, \quad (20)$$

**若** $u_1(\tau_1) > u_2(\tau_2)$，**则有** $u_1(\tau_2) \geqslant u_1(\tau_1) > u_2(\tau_2)$，**由**(13)**和**(16)**式有**

$$b_2(\tau_2)e^{u_2(\tau_2)} > a_2(\tau_2) - r_2(\tau_2)\int_{-\tau}^0 k_2(s)H_2ds,$$

**即**
$$e^{u_2(\tau_2)} > \frac{a_2^L - r_2^M H_2}{b_2^M}, \quad (21)$$

**从而由**(20)，(21)**式，有**

$$e^{u_i(t)} \geqslant e^{u_i(\tau_i)} > \min\left\{\frac{a_1^L - r_1^M H_1 - \frac{d_1^M}{m_1^L}}{b_1^M}, \frac{a_2^L - r_2^M H_2}{b_2^M}\right\} := h_i, i = 1,2. \quad (22)$$

**由**(17)**式，结合**(14)**及**(22)**式有**

$$0 > -a_3^M - b_3^M e^{u_3(\tau_3)} + \frac{c_1^L h_1}{m_1^M H_3 + H_1} - \frac{d_2^M}{m_2^L},$$

**即**
$$e^{u_3(t)} \geqslant e^{u_3(\tau_3)} > \frac{\frac{c_1^L h_1}{m_1^M H_3 + H_1} - a_3^M - \frac{d_2^M}{m_2^L}}{b_3^M} := h_3. \quad (23)$$

**对于** $i = 4,\cdots,n$，**根据**(18)**式，类似地有**

$$0 > -a_i^M - b_i^M e^{u_i(\tau_i)} + \frac{c_{i-2}^L h_{i-1}}{m_{i-2}^M H_i + H_{i-1}} - \frac{d_{i-1}^M}{m_{i-1}^L},$$

**即**
$$e^{u_i(t)} \geqslant e^{u_i(\tau_i)} > \frac{\frac{c_{i-2}^L h_{i-1}}{m_{i-2}^M H_i + H_{i-1}} - a_i^M - \frac{d_{i-1}^M}{m_{i-1}^L}}{b_i^M} := h_i. \quad (24)$$

**同理，由**(19)**式可得**

$$e^{u_{n+1}(t)} \geqslant e^{u_{n+1}(\tau_{n+1})} > \frac{\frac{c_{n-1}^L h_n}{m_{n-1}^M H_{n+1} + H_n} - a_{n+1}^M}{b_{n+1}^M} := h_{n+1}. \quad (25)$$

**因此，由**(13)，(14)，(22)～(25)**式得**

$$|u_i(t)| < \max\{|\ln H_i|, |\ln h_i|\} := R_i, i = 1,2,\cdots n+1. \quad (26)$$

**显然，**$R_i$ **与** $\lambda$ **无关. 根据中值定理可知存在某点** $\xi_i \in [0,\omega](i = 1,2,\cdots,n+1)$，**使得当** $(u_1(t),u_2(t),\cdots,u_{n+1}(t))^T$ **是常向量时有**

$$QN\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ \cdots \\ u_n \\ u_{n+1} \end{bmatrix} = \begin{bmatrix} \bar{a}_1 - (\bar{b}_1 + \bar{r}_1)e^{u_1} - \bar{D}_1 - \frac{\bar{d}_1 e^{u_3}}{m_1(\xi_1)e^{u_3} + e^{u_1}} + \bar{D}_1 e^{u_2 - u_1} \\ \bar{a}_2 - (\bar{b}_2 + \bar{r}_2)e^{u_2} - \bar{D}_2 + \bar{D}_2 e^{u_1 - u_2} \\ -\bar{a}_3 + \frac{\bar{c}_1 e^{u_1}}{m_1(\xi_3)e^{u_3} + e^{u_1}} - \bar{b}_3 e^{u_3} - \frac{\bar{d}_2 e^{u_4}}{m_2(\xi_3)e^{u_4} + e^{u_3}} \\ -\bar{a}_4 + \frac{\bar{c}_2 e^{u_3}}{m_2(\xi_4)e^{u_4} + e^{u_3}} - \bar{b}_4 e^{u_4} - \frac{\bar{d}_3 e^{u_5}}{m_3(\xi_4)e^{u_5} + e^{u_4}} \\ \vdots \\ -\bar{a}_n + \frac{c_{n-2}e^{u_{n-1}}}{m_{n-2}(\xi_n)e^{u_n} + e^{u_{n-1}}} - \bar{b}_n e^{u_n} - \frac{\bar{d}_{n-1}e^{u_{n+1}}}{m_{n-1}(\xi_n)e^{u_{n+1}} + e^{u_n}} \\ -\bar{a}_{n+1} + \frac{\bar{c}_{n-1}e^{u_n}}{m_{n-1}(\xi_{n+1})e^{u_{n+1}} + e^{u_n}} - \bar{b}_{n+1}e^{u_{n+1}} \end{bmatrix}$$

**定义** $M = \sum_{i=1}^{n+1} R_i + R_0$，**这里** $R_0$ **充分大，使得系统**(27)**的每个解** $(v_1,v_2,\cdots,v_{n+1})^T$ **满足** $\|(v_1,v_2,\cdots,v_{n+1})^T\| = |v_1| + |v_2| + \cdots + |v_{n+1}| < M$. **令** $\Omega = \{u \in X : \|u\| < M\}$，**则** $\Omega$ **满足引理**1**的条件**(i)，**当** $(u_1(t),u_2(t),\cdots,u_{n+1}(t))^T \in \partial\Omega \cap KerL = \partial\Omega \cap R^{n+1}$，**此时** $(u_1,u_2,\cdots,u_{n+1})^T$ **是** $R^{n+1}$ **中的常向量且** $\sum_{i=1}^{n+1}|u_i| = M$，**那么**

$$QN(u_1,u_2,\cdots,u_{n+1})^T \neq (0,0,\cdots,0)^T.$$

$$\begin{cases} \bar{a}_1 - (\bar{b}_1 + \bar{r}_1)e^{v_1} - \overline{D}_1 - \dfrac{\overline{d}_1 e^{v_3}}{m_1(\xi_1)e^{v_3} + e^{v_1}} + \overline{D}_1 e^{v_2 - v_1} = 0, \\ \bar{a}_2 - (\bar{b}_2 + \bar{r}_2)e^{v_2} - \overline{D}_2 + \overline{D}_2 e^{v_1 - v_2} = 0, \\ -\bar{a}_3 + \dfrac{\bar{c}_1 e^{v_1}}{m_1(\xi_3)e^{v_3} + e^{v_1}} - \bar{b}_3 e^{v_3} - \dfrac{\overline{d}_2 e^{v_4}}{m_2(\xi_3)e^{v_4} + e^{v_3}} = 0, \\ -\bar{a}_i + \dfrac{c_{i-2} e^{v_{i-1}}}{m_{i-2}(\xi_i)e^{v_i} + e^{v_{i-1}}} - \bar{b}_i e^{v_i} - \dfrac{\overline{d}_{i-1} e^{v_{i+1}}}{m_{i-1}(\xi_i)e^{v_{i+1}} + e^{v_i}} = 0, \\ \quad i = 4,\cdots,n \\ -\bar{a}_{n+1} + \dfrac{\bar{c}_{n-1} e^{v_n}}{m_{n-1}(\xi_{n+1})e^{v_{n+1}} + e^{v_n}} - \bar{b}_{n+1} e^{v_{n+1}} = 0 \end{cases} \tag{27}$$

因此引理 1 中的条件(ii) 满足.

接下来证明引理 1 中的条件(iii) 成立. 定义 $F(u_1,u_2,\cdots,u_n,\mu): DomL \times [0,1] \to R^{n+1}$:

$$F(u_1,\cdots,u_{n+1},\mu) = \begin{pmatrix} \bar{a}_1 - (\bar{b}_1 + \bar{r}_1)e^{u_1} \\ \bar{a}_2 - (\bar{b}_2 + \bar{r}_2)e^{u_2} \\ -\bar{a}_3 + \dfrac{\bar{c}_1 e^{u_1}}{m_1(\xi_3)e^{u_3} + e^{u_1}} \\ -\bar{a}_4 + \dfrac{\bar{c}_2 e^{u_3}}{m_2(\xi_4)e^{u_4} + e^{u_3}} \\ \vdots \\ -\bar{a}_n + \dfrac{\bar{c}_{n-2} e^{u_{n-1}}}{m_{n-2}(\xi_n)e^{u_n} + e^{u_{n-1}}} \\ -\bar{a}_{n+1} + \dfrac{\bar{c}_{n-1} e^{u_n}}{m_{n-1}(\xi_{n+1})e^{u_{n+1}} + e^{u_n}} \end{pmatrix} + \mu \begin{pmatrix} -\overline{D}_1 - \dfrac{\overline{d}_1 e^{u_3}}{m_1(\xi_1)e^{u_3} + e^{u_1}} + \overline{D}_1 e^{u_2 - u_1} \\ -\overline{D}_2 + \overline{D}_2 e^{u_1 - u_2} \\ -\bar{b}_3 e^{u_3} - \dfrac{\overline{d}_2 e^{u_4}}{m_2(\xi_3)e^{u_4} + e^{u_3}} \\ -\bar{b}_4 e^{u_4} - \dfrac{\overline{d}_3 e^{u_5}}{m_3(\xi_4)e^{u_5} + e^{u_4}} \\ \vdots \\ -\bar{b}_n e^{u_n} - \dfrac{\overline{d}_{n-1} e^{u_{n+1}}}{m_{n-1}(\xi_n)e^{u_{n+1}} + e^{u_n}} \\ -\bar{b}_{n+1} e^{u_{n+1}} \end{pmatrix}$$

其中 $\mu \in [0,1]$ 为参数. 当 $(u_1,u_2,\cdots,u_{n+1})^T \in \partial\Omega \cap KerL = \partial\Omega \cap R^{n+1}$, $(u_1,u_2,\cdots,u_{n+1})^T$ 是 $R^{n+1}$ 中的常向量, 且 $\sum_{i=1}^{n+1}|u_i| = M$, 此时有 $F(u_1,u_2,\cdots,u_{n+1},\mu) \neq 0$. 倘若不然, 即存在某个 $u^* = (u_1^*,u_2^*,\cdots,u_{n+1}^*)^T \in \partial\Omega \cap KerL$, 且 $\sum_{i=1}^{n+1}|u_i^*| = M$, $\mu^* \in [0,1]$ 满足 $F(u_1^*,u_2^*,\cdots,u_{n+1}^*,\mu^*) = 0$. 则由前面的讨论知最终有(26) 式的结果, 即

$$|u_i^*(t)| < \max\{|\ln H_i|, |\ln h_i|\} := R_i, i = 1,2,\cdots,n+1,$$

于是 $\sum_{i=1}^{n+1}|u_i^*| \leqslant \sum_{i=1}^{n+1}R_i < M$ 与 $\sum_{i=1}^{n+1}|u_i^*| = M$ 矛盾.

因此对 $(u_1,u_2,\cdots,u_{n+1})^T \in \partial\Omega \cap KerL$, 有 $F(u_1,u_2,\cdots,u_{n+1},\mu) \neq 0$.

令 $J = I: \mathrm{ImL} \to KerL, (u_1,u_2,\cdots,u_{n+1})^T \to (u_1,u_2,\cdots,u_{n+1})^T$, 根据同伦不变性, 有

$$\deg(JQN(u_1,u_2,\cdots,u_{n+1})^T, \Omega \cap KerL, (0,0,\cdots,0)^T) = \deg(F(u_1,u_2,\cdots,u_{n+1},1), \Omega \cap KerL, (0,0,\cdots,0)^T) = \deg(F(u_1,u_2,\cdots,u_{n+1},0), \Omega \cap KerL, (0,0,\cdots,0)^T).$$

我们知道 $F(u_1,u_2,\cdots,u_{n+1},0) = 0$ 有唯一的解 $v^* = (v_1^*,v_2^*,\cdots,v_{n+1}^*)^T$,

$$v_1^* = \ln\frac{\bar{a}_1}{\bar{b}_1 + \bar{r}_1}, v_2^* = \ln\frac{\bar{a}_2}{\bar{b}_2 + \bar{r}_2},$$

$$v_3^* = \ln\frac{\bar{a}_1(\bar{c}_1 - \bar{a}_3)}{\bar{a}_3 m_1(\bar{t}_3)(\bar{b}_1 + \bar{r}_1)}, v_i^* = v_{i-1} + \ln\frac{\bar{c}_{i-2} - \bar{a}_i}{\bar{a}_i m_{i-2}(\bar{t}_i)}, i = 4,\cdots,n+1.$$

显然 $v_i^* > 0, i = 1,2,\cdots,n+1$, 因此

$$\deg(JQN(u_1,u_2,\cdots,u_{n+1})^T, \Omega \cap KerL, (0,0,\cdots,0)^T) = \mathrm{sgn} \begin{vmatrix} a_{22} & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & a_{23} & 0 & 0 & 0 & \cdots & 0 & 0 \\ a_{42} & 0 & a_{44} & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & a_{54} & a_{55} & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & a_{n(n-1)} & a_{nn} \end{vmatrix} = \begin{cases} 1, & n \text{ 为奇数}, \\ -1, & n \text{ 为偶数}, \end{cases}$$

其中, $a_{22} = -(\bar{b}_1 + \bar{r}_1)e^{v_1^*}$; $a_{33} = -(\bar{b}_2 + \bar{r}_2)e^{v_2^*}$;

$$a_{42} = \frac{\bar{c}_1 m_1(\xi_3)e^{v_1^* + v_3^*}}{(m_1(\xi_3)e^{v_3^*} + e^{v_1^*})^2}; a_{44} = \frac{-\bar{c}_1 m_1(\xi_3)e^{v_1^* + v_3^*}}{(m_1(\xi_3)e^{v_3^*} + e^{v_1^*})^2}; a_{54} = \frac{\bar{c}_2 m_2(\xi_4)e^{v_3^* + v_4^*}}{(m_2(\xi_4)e^{v_4^*} + e^{v_3^*})^2};$$

$$a_{55} = \frac{-\bar{c}_2 m_2(\xi_4)e^{v_3^* + v_4^*}}{(m_2(\xi_4)e^{v_4^*} + e^{v_3^*})^2}; a_{n(n-1)} =$$

$$\frac{\bar{c}_{n-1}m_{n-1}(\xi_{n+1})e^{v_n^*+v_{n+1}^*}}{(m_{n-1}(\xi_{n+1})e^{v_{n+1}^*}+e^{v_n^*})^2};a_{nn}=$$

$$\frac{-\bar{c}_{n-1}m_{n-1}(\xi_{n+1})e^{v_n^*+v_{n+1}^*}}{(m_{n-1}(\xi_{n+1})e^{v_{n+1}^*}+e^{v_n^*})^2}.$$

因此,引理1的条件(iii)成立,从而定理1证毕.

**参考文献:**

[1] Allen L J S. Persistence, extinction, and critical patch number for island populations[J]. Math Biosci, 1987, 24:617-625.

[2] Freedman H I, Rai B, Waltman P. Mathematical model of population interactions with dispersal II: Differential survival in a change of habitat[J]. Math Anal Appl, 1986, 115:140-154.

[3] Hastings A. Dynamics of a single species in a spatially varying environment: The stabilizing role of high dispersal rates[J]. Math Biol, 1982, 16:49-55.

[4] Holt R D. Population dynamics in two-patch environments: Some anomalous consequences of an optional habitat distribution[J]. Theoret Population Biol, 1985, 28:181-208.

[5] Kuang Y, Takeuchi Y. Predator-prey dynamics in models of prey dispersal in two-patch environments[J]. Math Biosci, 1994, 120:77-98.

[6] Beretta E, Solimano F. Global stability and periodic orbits for two-patch predator-prey diffusion delay model [J]. Math Biosci, 1987, 85:153-183.

[7] Beretta E, Takeuchi Y. Global asymptotic stability of Lotka-Volterra diffusion models with continuous time delays[J]. SIAM Appl Math, 1988, 48:276-651.

[8] Kuang Y. Delay Differential Equations with Applications in Population Dynamics[M]. New York: Academic Press, 1993.

[9] Zhang Z Q, Wang Z C. The existence of periodic solution of a two-patch predator-prey dispersion delay models with functional response[J]. Korean Math Soc, 2003, 40(5):869-881.

[10] Gaines R E, Mawhin J L. Coincidence Degree and Nonlinear Differential Equations [M]. Berlin: Springer-Verlag, 1977.

(责任编辑:黎贞崇)

## 电脑病毒最终将会“感染”人类

凯文·瓦威克是英国瑞丁大学的控制论教授,他正在期待再次成为电子人。

瓦威克已经将其神经系统与计算机连在了一起,并在其手臂上植入了一个RFID芯片,他警告说,终有一天,计算机病毒能够像感染PC那样感染人类。

困扰现代计算技术的安全问题与电子人在未来遭遇的安全问题不会有太大区别。瓦威克说,我们看到,软件病毒和生物病毒正在合二为一。安全问题将要大得多。

瓦威克表示,如果人被网络连接起来,被黑客攻击的影响要严重得多,对黑客的态度也将发生极大的变化。如果人与互联网相连接,受到黑客攻击时被攻击的范围将大大扩大。

瓦威克一直在与英国Stoke Mandeville医院进行合作,研究网络化神经系统对于脊柱有疾病的人的意义。例如,研究人员正在探索人是否可以通过神经系统控制轮椅。

瓦威克在谈到其RFID芯片试验时,他说,我受到了许多批评,我不知道这是怎么回事。在手臂上植入RFID芯片已经不是什么新鲜事儿,在西班牙的一家夜总会,消费者可以选择使用RFID芯片支付帐单。一些墨西哥的司法官员也植入了RFID芯片,防止犯罪分子的拉扰腐蚀。美国联邦食品、药品管理已经批准在人身上使用RFID芯片,其中一种可能的应用是让医疗人员利用该芯片获得病人的病历资料。

(据《科学时报》)