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一类具阶段结构的捕食系统正周期解的存在性

Existence of Positive Periodic Solution for a Predator-prey System with Stage Structure

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摘要：建立具有阶段结构和自食现象而且食饵和捕食者均受密度制约的周期捕食系统：

$$\begin{cases} x'(t) = x(t)[\beta_1(t) - a(t)x(t)] - b(t)x(t)z_2(t), \\ z'_1(t) = \beta_2(t)z_2(t) - s(t)z_1(t) - c(t)z_1(t)z_2(t), \\ z'_2(t) = r(t)z_1(t) - d(t)z_2^2(t) + e(t)z_1(t)z_2(t) + h(t)x(t)z_2(t), \\ x(0) > 0, z_1(0) > 0, z_2(0) > 0, \end{cases}$$

并且利用重合度理论得到正周期解存在的充分条件为 $d^l s^l > e^M \beta_2^M + \frac{b^M}{\beta_1^l}(r^M \beta_2^M + \frac{h^M \beta_1^M s^l}{a^l})$.

关键词：捕食系统　阶段结构　自食　正周期解　重合度

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Abstract: In this paper, a class of predator-prey system with stage structure and cannibalism is established, which is restricted by prey's and predators' density:

$$\begin{cases} x'(t) = x(t)[\beta_1(t) - a(t)x(t)] - b(t)x(t)z_2(t), \\ z'_1(t) = \beta_2(t)z_2(t) - s(t)z_1(t) - c(t)z_1(t)z_2(t), \\ z'_2(t) = r(t)z_1(t) - d(t)z_2^2(t) + e(t)z_1(t)z_2(t) + h(t)x(t)z_2(t), \\ x(0) > 0, z_1(0) > 0, z_2(0) > 0. \end{cases}$$

By means of the theory of coincidence degree, a sufficien condition is obtained for the existence of positive periodic solution: $d^l s^l > e^M \beta_2^M + \frac{b^M}{\beta_1^l}(r^M \beta_2^M + \frac{h^M \beta_1^M s^l}{a^l})$.

Key words: predator-prey system, stage structure, cannibalism, positive periodic solution, coincidence degree

在自然界中，许多种群的个体在一生中具有明显的年龄结构特征，例如，哺乳动物和某些两栖动物就有幼年和成年之分. 因此，近年来在研究捕食系统中，许多学者考虑了种群的年龄结构[1～7]. 文献[7]在捕食系统中，除了考虑捕食者种群的年龄结构，还考虑了成虫的自食功能(食本种群的幼虫)，建立了具阶段结构和自食的捕食系统，并研究了系统的持久性、正平衡点的全局渐近稳定性以及 Hopf 分支问题.

由于生物环境是周期变化的，因此我们在本文中讨论捕食者具阶段结构和自食现象，而且食饵和捕食者均受密度制约的周期捕食系统：

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$$\begin{cases} x'(t)=x(t)[\beta_1(t)-a(t)x(t)]-b(t)x(t)z_2(t), \\ z'_1(t)=\beta_2(t)z_2(t)-s(t)z_1(t)-c(t)z_1(t)z_2(t), \\ z'_2(t)=r(t)z_1(t)-d(t)z_2^2(t)+e(t)z_1(t)z_2(t)+ \\ \quad h(t)x(t)z_2(t), \\ x(0)>0, z_1(0)>0, z_2(0)>0, \end{cases} \tag{1}$$

其中 $d(t)$ 是捕食者成虫密度制约系数，$s(t)$ 表示捕食者幼虫的死亡率和成熟率的和；$a,b,c,d,e,h,s,r,\beta_1,\beta_2\in C(\mathbf{R},\mathbf{R}^+)$，是 t 的 $\omega(>0)$ 周期函数. 利用重合度理论得到该系统正周期解存在的充分条件.

为了行文方便，本文采用如下记号：$\mathbf{R}$ 表示实数集，R^3 为三维欧氏空间，$R^3_+=\{(x_1,x_2,x_3)^T\in R^3\,|\,x_1>0,x_2>0,x_3>0\}$，对以 ω 为周期的连续函数 $g(t)$，有 $\bar{g}(t)=\frac{1}{\omega}\int_0^\omega g(t)\mathrm{d}t$，$g^M=\max\limits_{t\in[0,\omega]}g(t)$，$g^l=\min\limits_{t\in[0,\omega]}g(t)$.

1 预备知识

引理1 R^3_+ 是系统(1)的正不变集.

为了证明系统(1)周期解的存在性，我们再引入重合度的延拓定理.

设 X,Z 是实Banach空间，$L:DomL\subset X\to Z$ 为线性映射，$N:X\to Z$ 为连续映射，若满足 $dimKerL=codimImL<+\infty$ 且 ImL 为 Z 中的闭子集，则映射 L 称为一个指标为零的Fredholm算子. 若 L 是指标为零的Fredholm算子且存在连续投射 $P:X\to X$，$Q:Z\to Z$ 使得 $ImP=KerL$，$ImL=KerQ=Im(I-Q)$，则 $L|_{DomL\cap KerP}:(I-P)X\to ImL$ 可逆，逆映射记为 K_P. 设 Ω 是 X 中的有界开集，若 $QN(\bar{\Omega})$ 有界且 $K_P(I-Q)N:\bar{\Omega}\to Z$ 是紧的，则称 N 在 $\bar{\Omega}$ 上是 L-紧的. 因为 ImQ 与 $KerL$ 同构，所以存在同构映射 $J:ImQ\to KerL$.

引理2(延拓定理)[8] 设 L 是指标为零的Fredholm映射，N 在 $\bar{\Omega}$ 上是 L-紧的. 若

(1) 对任意的 $\lambda\in(0,1)$，方程 $Lx=\lambda Nx$ 的解满足 $x\notin\partial\Omega$；

(2) 对任意的 $x\in KerL\cap\partial\Omega$，$QNx\neq0$；

(3) $deg(JQN,\Omega\cap KerL,0)\neq0$，

则方程 $Lx=Nx$ 在 $DomL\cap\bar{\Omega}$ 内至少存在一个解.

2 正周期解的存在性

定理 当 $d^l s^l>e^M\beta_2^M+\frac{b^M}{\beta_1^l}(r^M\beta_2^M+\frac{h^M\beta_1^M s^l}{a^l})$ 时，系统(1)至少存在一个 ω 周期的正周期解.

证明 由引理1知，系统(1)具有正初始条件的解为正，故作变换 $x(t)=\exp\{y_1(t)\}$，$z_1(t)=\exp\{y_2(t)\}$，$z_2(t)=\exp\{y_3(t)\}$，则系统(1)变为

$$\begin{cases} y'_1(t)=\beta_1(t)-a(t)\exp\{y_1(t)\}- \\ \quad b(t)\exp\{y_3(t)\}, \\ y'_2(t)=\beta_2(t)\exp\{y_3(t)-y_2(t)\}- \\ \quad s(t)-c(t)\exp\{y_3(t)\}, \\ y'_3(t)=r(t)\exp\{y_2(t)-y_3(t)\}- \\ \quad d(t)\exp\{y_3(t)\}+e(t)\exp\{y_2(t)\}+ \\ \quad h(t)\exp\{y_1(t)\}. \end{cases} \tag{2}$$

取 $X=Z=\{y(t)=(y_1(t),y_2(t)y_3(t))^T\in C(\mathbf{R},R^3):y(t+\omega)=y(t)\}$，记 $\|y\|=\|(y_1(t),y_2(t),y_3(t))^T\|=\max\limits_{t\in[0,\omega]}|y_1(t)|+\max\limits_{t\in[0,\omega]}|y_2(t)|+\max\limits_{t\in[0,\omega]}|y_3(t)|$，则 X,Z 在范数 $\|\cdot\|$ 下为Banach空间. 令

$$Ny=\begin{pmatrix} \beta_1(t)-a(t)\exp\{y_1(t)\}-b(t)\exp\{y_3(t)\} \\ \beta_2(t)\exp\{y_3(t)-y_2(t)\}-s(t)- \\ \quad c(t)\exp\{y_3(t)\} \\ r(t)\exp\{y_2(t)-y_3(t)\}-d(t)\exp\{y_3(t)\}+ \\ \quad e(t)\exp\{y_2(t)\}+h(t)\exp\{y_1(t)\} \end{pmatrix} := \begin{pmatrix} N_1(t) \\ N_2(t) \\ N_3(t) \end{pmatrix}.$$

$$Ly=\frac{dy}{dt},\ Py=\frac{1}{\omega}\int_0^\omega y(t)\mathrm{d}t,\ y\in X,\ Qz=\frac{1}{\omega}\int_0^\omega z(t)\mathrm{d}t,\ z\in Z,$$

则

$$KerL=\{(y_1,y_2,y_3)^T\in X\,|\,(y_1,y_2,y_3)^T=(h_1,h_2,h_3)^T\in R^3\},$$

$$ImL=\{(y_1,y_2,y_3)^T\in Z\,|\,\int_0^\omega y_1(t)\mathrm{d}t=0,\int_0^\omega y_2(t)\mathrm{d}t=0,\int_0^\omega y_3(t)\mathrm{d}t=0\}.$$

$dimKerL=3=codimImL$，ImL 为 Z 中的闭集，故 L 是指标为零的Fredholm算子. 易证 P,Q 为连续的投影算子且使得 $ImP=KerL$，$ImL=KerQ=Im(I-Q)$，因此 L 逆映射 $K_P:ImL\to KerP\cap DomL$ 存在，且 $K_P(Z)=\int_0^t Z(u)\mathrm{d}u-\frac{1}{\omega}\int_0^\omega\int_0^t Z(u)\mathrm{d}u\mathrm{d}t$. 于是，

$$QNy=\begin{pmatrix}\frac{1}{\omega}\int_0^{\omega}[\beta_1(t)-a(t)\exp\{y_1(t)\}-b(t)\exp\{y_3(t)\}]\mathrm{d}t\\ \frac{1}{\omega}\int_0^{\omega}[\beta_2(t)\exp\{y_3(t)-y_2(t)\}-s(t)-c(t)\exp\{y_3(t)\}]\mathrm{d}t\\ \frac{1}{\omega}\int_0^{\omega}[r(t)\exp\{y_2(t)-y_3(t)\}-d(t)\exp\{y_3(t)\}+e(t)\exp\{y_2(t)\}+h(t)\exp\{y_1(t)\}]\mathrm{d}t\end{pmatrix};$$

$$K_P(I-Q)Ny=\begin{pmatrix}\int_0^t N_1(s)\mathrm{d}s-\frac{1}{\omega}\int_0^{\omega}\int_0^t N_1(s)\mathrm{d}s\mathrm{d}t-(\frac{1}{\omega}-\frac{1}{2})\int_0^{\omega}N_1(t)\mathrm{d}t\\ \int_0^{\omega}N_2(s)\mathrm{d}s-\frac{1}{\omega}\int_0^{\omega}\int_0^t N_2(s)\mathrm{d}s\mathrm{d}t-(\frac{1}{\omega}-\frac{1}{2})\int_0^{\omega}N_2(t)\mathrm{d}t\\ \int_0^{\omega}N_3(s)\mathrm{d}s-\frac{1}{\omega}\int_0^{\omega}\int_0^t N_3(s)\mathrm{d}s\mathrm{d}t-(\frac{1}{\omega}-\frac{1}{2})\int_0^{\omega}N_3(t)\mathrm{d}t\end{pmatrix},$$

显然,QN 及 $K_P(I-Q)N$ 连续,设 Ω 为 X 中的有界开集,则 $QN(\overline{\Omega})$ 有界,利用 Arzela-Ascoli 定理容易证明 $\overline{K_P(I-Q)N(\overline{\Omega})}$ 是紧致集,因此 N 在 $\overline{\Omega}$ 上是 L- 紧的.

对应于算子方程 $Ly=\lambda Ny,\lambda\in(0,1)$,有

$$\begin{cases}y'_1(t)=\lambda[\beta_1(t)-a(t)\exp\{y_1(t)\}-b(t)\exp\{y_3(t)\}],\\ y'_2(t)=\lambda[\beta_2(t)\exp\{y_3(t)-y_2(t)\}-s(t)-c(t)\exp\{y_3(t)\}],\\ y'_3(t)=\lambda[r(t)\exp\{y_2(t)-y_3(t)\}-d(t)\exp\{y_3(t)\}+e(t)\exp\{y_2(t)\}+h(t)\exp\{y_1(t)\}],\end{cases}\tag{3}$$

设 $y(t)\in X$ 为系统(3)对应于某个 $\lambda\in(0,1)$ 的解,选择 $\eta_i\in[0,\omega]$,使得 $y_i(\eta_i)=\max\limits_{t\in[0,\omega]}y_i(t),i=1,2,3$,有

$$\begin{cases}y'_1(\eta_1)=\lambda[\beta_1(\eta_1)-a(\eta_1)\exp\{y_1(\eta_1)\}-b(\eta_1)\exp\{y_3(\eta_1)\}]=0,\\ y'_2(\eta_2)=\lambda[\beta_2(\eta_2)\exp\{y_3(\eta_2)-y_2(\eta_2)\}-s(\eta_2)-c(\eta_2)\exp\{y_3(\eta_2)\}]=0,\\ y'_3(\eta_3)=\lambda[r(\eta_3)\exp\{y_2(\eta_3)-y_3(\eta_3)\}-d(\eta_3)\exp\{y_3(\eta_3)\}+e(\eta_3)\exp\{y_2(\eta_3)\}+h(\eta_3)\exp\{y_1(\eta_3)\}]=0,\end{cases}\tag{4}$$

由系统(4)的第1个式子得

$$a(\eta_1)\exp\{y_1(\eta_1)\}=\beta_1(\eta_1)-b(\eta_1)\exp\{y_3(\eta_1)\},$$

$$\exp\{y_1(\eta_1)\}<\frac{\beta_1(\eta_1)}{a(\eta_1)}\leqslant\frac{\beta_1^M}{a^l}:=\delta_1.\tag{5}$$

由系统(4)的第2个和第3个式子得

$$s(\eta_2)\exp\{y_2(\eta_2)\}<\beta_2(\eta_2)\exp\{y_3(\eta_2)\},$$

$$\exp\{y_2(\eta_2)\}<\frac{\beta_2^M}{s^l}\exp\{y_3(\eta_2)\}\leqslant\frac{\beta_2^M}{s^l}\exp\{y_3(\eta_3)\}.\tag{6}$$

$$d(\eta_3)\exp\{2y_3(\eta_3)\}=r(\eta_3)\exp\{y_2(\eta_3)\}+e(\eta_3)\exp\{y_2(\eta_3)+y_3(\eta_3)\}+h(\eta_3)\exp\{y_1(\eta_3)+y_3(\eta_3)\}d^l\exp\{2y_3(\eta_3)\}\leqslant d(\eta_3)\exp\{2y_3(\eta_3)\}\leqslant r^M\exp\{y_2(\eta_2)\}+e^M\exp\{y_2(\eta_2)\}\exp\{y_3(\eta_3)\}+h^M\exp\{y_1(\eta_1)\}\exp\{y_3(\eta_3)\}\leqslant\frac{r^M\beta_2^M}{s^l}\exp\{y_3(\eta_3)\}+\frac{e^M\beta_2^M}{s^l}\exp\{2y_3(\eta_3)\}+\frac{h^M\beta_1^M}{a^l}\exp\{y_3(\eta_3)\}(d^l-\frac{e^M\beta_2^M}{s^l})\exp\{y_3(\eta_3)\}\leqslant\frac{r^M\beta_2^M}{s^l}+\frac{h^M\beta_1^M}{a^l}.$$

当 $d^ls^l>e^M\beta_2^M+\frac{b^M}{\beta_1^l}(r^M\beta_2^M+\frac{h^M\beta_1^Ms^l}{a^l})$ 时,则有

$$d^l-\frac{e^M\beta_2^M}{s^l}>0,$$

$$\exp\{y_3(\eta_3)\}\leqslant\frac{\frac{r^M\beta_2^M}{s^l}+\frac{h^M\beta_1^M}{a^l}}{d^l-\frac{e^M\beta_2^M}{s^l}}:=\delta_3.\tag{7}$$

由(6),(7)式得

$$\exp\{y_2(\eta_2)\}<\frac{\beta_2^M}{s^l}\delta_3:=\delta_2.\tag{8}$$

另一方面,选择 $\zeta_i\in[0,\omega]$,使得 $y_i(\zeta_i)=\min\limits_{t\in[0,\omega]}y_i(t),i=1,2,3$,有

$$\begin{cases} y'_1(\zeta_1) = \lambda[\beta_1(\zeta_1) - a(\zeta_1)\exp\{y_1(\zeta_1)\} - \\ \quad b(\zeta_1)\exp\{y_3(\zeta_1)\}] = 0, \\ y'_2(\zeta_2) = \lambda[\beta_2(\zeta_2)\exp\{y_3(\zeta_2) - y_2(\zeta_2)\} - \\ \quad s(\zeta_2)\} - c(\zeta_2)\exp\{y_3(\zeta_2)\}] = 0, \\ y'_3(\zeta_3) = \lambda[r(\zeta_3)\exp\{y_2(\zeta_3) - y_3(\zeta_3)\} - \\ \quad d(\zeta_3)\exp\{y_3(\zeta_3)\} + e(\zeta_3)\exp\{y_2(\zeta_3)\} + \\ \quad h(\zeta_3)\exp\{y_1(\zeta_3)\}] = 0. \end{cases} \tag{9}$$

由系统(9)的第 1 个式子和定理的条件得

$$a^M\exp\{y_1(\zeta_1)\} \geqslant a(\zeta_1)\exp\{y_1(\zeta_1)\} = \beta_1(\zeta_1) - b(\zeta_1)\exp\{y_3(\zeta_1)\},$$

$$\exp\{y_1(\zeta_1)\} \geqslant \frac{\beta_1^l - b^M\delta_3}{a^M} := \rho_1 > 0. \tag{10}$$

由系统(9)的第 3 个和第 2 个式子得

$$r^l\exp\{y_2(\zeta_2)\} \leqslant r(\zeta_3)\exp\{y_2(\zeta_3)\} < d(\zeta_3)\exp\{2y_3(\zeta_3)\},$$

$$\exp\{y_2(\zeta_2)\} < \frac{d^M\exp\{2y_3(\zeta_3)\}}{r^l}. \tag{11}$$

$$\beta_2(\zeta_2)\exp\{y_3(\zeta_2)\} = s(\zeta_2)\exp\{y_2(\zeta_2)\} + c(\zeta_2)\exp\{y_2(\zeta_2)\}\exp\{y_3(\zeta_2)\} \leqslant s^M\exp\{y_2(\zeta_2)\} + c^M\delta_3\exp\{y_2(\zeta_2)\} \leqslant (s^M + c^M\delta_3)\exp\{y_2(\zeta_2)\}. \tag{12}$$

将(11)式代入(12)式得

$$\beta_2^l\exp\{y_3(\zeta_3)\} \leqslant (s^M + c^M\delta_3)\exp\{y_2(\zeta_2)\} < (s^M + c^M\delta_3)\frac{d^M\exp\{2y_3(\zeta_3)\}}{r^l}\exp\{y_3(\zeta_3)\} > \frac{\beta_2^l r^l}{(s^M + c^M\delta_3)d^M} := \rho_3. \tag{13}$$

由(12),(13)式得

$$\exp\{y_2(\zeta_2)\} \geqslant \frac{\beta_2(\zeta_2)\exp\{y_3(\zeta_2)\}}{s^M + c^M\delta_3} > \frac{\beta_2^l\rho_3}{s^M + c^M\delta_3} := \rho_2. \tag{14}$$

由(5),(7),(8),(10),(13),(14)式有

$$|y_i(t)| \leqslant \max_{t\in[0,\omega]}\{|\ln\delta_i|, |\ln\rho_i|\} := R_i, i = 1,2,3.$$

显然,R_1, R_2, R_3 与 λ 的选取无关. 令 $H = R_1 + R_2 + R_3 + R_0$,其中 R_0 充分大使得代数方程

$$\begin{cases} \bar\beta_1 - \bar a\exp(u_1) - \bar b\exp(u_3) = 0, \\ \bar\beta_2\exp(u_3 - u_2) - \bar s - \bar c\exp(u_3) = 0, \\ \bar r\exp(u_2 - u_3) - \bar d\exp(u_3) + \\ \quad \bar e\exp(u_2) + \bar h\exp(u_1) = 0 \end{cases} \tag{15}$$

有解,则它的每一个解 $(v_1^*, v_2^*, v_3^*)^T$ 满足 $\|(v_1^*, v_2^*, v_3^*)^T\| < H$.

令 $\Omega = \{y = (y_1(t), y_2(t), y_3(t))^T \in X, \|y\| < H\}$,则 Ω 为 X 中有界开集且满足引理 2 的条件(1). 当 $y(t) \in KerL \cap \partial\Omega$ 时,$\{y(t)\}$ 是 R^3 中满足条件 $\|y\| = H$ 的常值向量,若方程(15)有解,则

$$QNy = \begin{bmatrix} \bar\beta_1 - \bar a\exp(y_1) - \bar b\exp(y_3) \\ \bar\beta_2\exp(y_3 - y_2) - \bar s - \bar c\exp(y_3) \\ \bar r\exp(y_2 - y_3) - \bar d\exp(y_3) + \\ \quad \bar e\exp(y_2) + \bar h\exp(y_1) \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

若方程(15)无解,则 $QNy \neq (0\quad 0\quad 0)^T$. 因此引理 2 的条件(2)是满足的.

为了证明引理 2 的条件(3)亦满足. 作映射 ϕ: $DomL \times [0,1] \to Z$ 为

$$\phi(y_1, y_2, y_3, \mu) = \begin{bmatrix} \bar\beta_1 - \bar a\exp(u_1) \\ \bar\beta_2\exp(u_3 - u_2) - \bar s \\ \bar r\exp(u_2 - u_3) - \bar d\exp(u_3) \end{bmatrix} + \mu\begin{bmatrix} -\bar b\exp(u_3) \\ -\bar c\exp(u_3) \\ \bar e\exp(u_2) + \bar h\exp(u_1) \end{bmatrix},$$

其中 $\mu \in [0,1]$ 是一个参数,当 $y \in \partial\Omega \cap KerL$ 时,y 是一个常值向量且满足 $\|y\| = H$. 设 $y \in \partial\Omega \cap KerL$,$\phi(y_1, y_2, y_3, \mu) \neq 0$,若不真,则当 $y \in \partial\Omega \cap KerL$,$\phi(y_1, y_2, y_3, \mu) = 0$. 于是由

$$\begin{cases} \bar\beta_1 - \bar a\exp(y_1) - \bar b\mu\exp(y_3) = 0, \\ \bar\beta_2\exp(y_3 - y_2) - \bar s - \bar c\mu\exp(y_3) = 0, \\ \bar r\exp(y_2 - y_3) - \bar d\exp(y_3) + \\ \quad \bar e\mu\exp(y_2) + \bar h\mu\exp(y_1) = 0, \end{cases}$$

利用与(5),(7),(8),(10),(13),(14)式相类似的证明可得到 $|y_i(t)| \leqslant \max\limits_{t\in[0,\omega]}\{|\ln\delta_i|, |\ln\rho_i|\} := R_i$,$\|y\| < H$,这与 $\|y\| = H$ 矛盾. 因此 $\phi(y_1, y_2, y_3, \mu)$ 是一个同伦映射. 取 $J: ImQ \to KerL$ 为恒同映射,于是

$$deg(JQN, \Omega \cap KerL, 0) = deg(\phi(y_1, y_2, y_3, 1), \Omega \cap KerL, 0) = deg(\phi(y_1, y_2, y_3, 0), \Omega \cap KerL, 0) = deg\{(\bar\beta_1 - \bar a\exp(y_1), \bar\beta_2\exp(y_3 - y_2) - \bar s, \bar r\exp(y_2 - y_3) - \bar d\exp(y_3))^T, \Omega \cap KerL, 0\}.$$

由于代数系统:

$$\begin{cases} \bar\beta_1 - \bar a\exp(u_1) = 0, \\ \bar\beta_2\exp(u_3 - u_2) - \bar s = 0, \\ \bar r\exp(u_2 - u_3) - \bar d\exp(u_3) = 0 \end{cases}$$

有唯一解 $(u_1^*\quad u_2^*\quad u_3^*)^T$,其中

$$u_1^* = \ln\frac{\bar\beta_1}{\bar a}, u_3^* = \ln\frac{\bar\beta_2\bar r}{\bar s\bar d}, u_2^* = \ln\frac{\bar\beta_2^2\bar r}{\bar s^2\bar d}.$$

直接计算

$$deg(JQNy, \Omega \cap KerL, 0) =$$

$deg\{(\overline{\beta}_1-\overline{a}\exp(y_1),\overline{\beta}_2\exp(y_3-y_2)-\overline{s},\overline{r}\exp(y_2-y_3)-\overline{d}\exp(y_3))^T,\Omega\cap KerL,0\}\neq 0$,

显然,引理 2 的条件(3)满足.则 $Ly=Ny$ 在 $DomL\cap\overline{\Omega}$ 中至少有一个 ω 周期解 $y^*=(y_1^*,y_2^*,y_3^*)^T$,使 $x^*=\exp\{y_1^*\}$,$z_1^*=\exp\{y_2^*\}$,$z_2^*=\exp\{y_3^*\}$,为系统(1)的正周期解.

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其中 L_1、L_2 为算子,k_1、k_2 为常量,Γ_1 表示区域的左边,Γ_2 表示区域的右边,f_1、f_2 为表达式常量.整体区域 $\Omega=\Omega_1\cup\Omega_2$.

图 2,3 分析比较表明,采用组合网格法得到了符合精度要求的数值解.

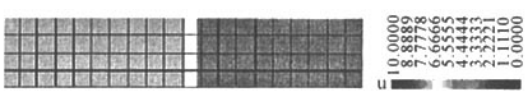

图 2 Galerkin 方法得到的计算结果

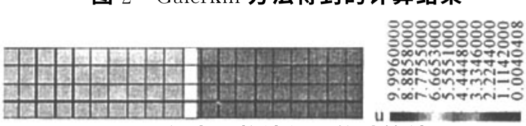

图 3 组合网格法得到的计算结果

3 结束语

与 Galerkin 法相比,本文提出的组合网格法是方便在计算机上编程实现的;另外一个突出的优点就是粗细两套网格都是在各自区域上单独剖分,两套网格互不影响,在粗网格上达到了粗网格的精度,在细网格上达到细网格的精度.该方法不仅适合于不同的椭圆微分算子 L_c、L_f,而且当椭圆微分算子 L_c 和 L_f 相同时,迭代法一步收敛.

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