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二元五次 C^2 样条函数空间 $S_5^2(\triangle_W)$ 的局部基*

A Local Basis for C^2 Quintic Spline Space over $S_5^2\triangle_W$ Triangulation

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摘要:利用 Hermite插值条件,给出 Wang型加密三角剖分下二元五次 C^2 样条函数空间 $S_5^2(\triangle_W)$具有局部支集的 11个基底表达式.

关键词:加密三角剖分 二元五次 C^2 样条函数 Hermite插值 局部基

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Abstract $S_5^2(\triangle_W)$ is the bivariate C^2-quintic spline space based on Wang's refined triangulation. In this paper, representations of eleven locally supported the basis of $S_5^2(\triangle_W)$ given by using the Hermite interpolation conditions.

Key words refined triangulation, bivariate C^2-quintic spline function, Hermite interpolation, local basis

假设 K是平面上任一单连通多边形区域,$\triangle = \{T^{(l)}\}_{l=1}^{|T|}$是它的任一正规三角剖分,对满足 $0 \leqslant r < d$的非负整数 d、r,定义二元 d次 r阶光滑的样条函数空间为

$$S_d^r(\triangle) = \{s \in C^r(D): s|_{T^{(l)}} \in P_d, l = 1,2,\cdots,|T|\}, \tag{1}$$

其中 P_d 表示二元 d次多项式空间,$s|_{T^{(l)}}$ 表示二元样条函数 $s(x,y)$ 在 $T^{(l)}$ 上的表达式.

加密三角剖分下的二元样条函数空间,很早就受到学者们的关注,如 Ciarlet[1]和 Percell[2]研究了加密三角剖分 $\triangle_{CT}$ 下的样条函数空间 $S_3^1(\triangle_{CT})$. Powell和 Sabin[3]研究了 PSI 型和 PSII 型加密三角剖分下的样条函数空间 $S_2^1(\triangle_{PS1})$ 与 $S_2^1(\triangle_{PS2})$. Liu和 Hong[4]利用三角化四边形剖分上的 Hermite插值条件,给出三角化的四边形剖分上的样条函数空间 $S_3^1(\diamond)$ 的具有局部支集的基的表达式. Wang[5]给出了加密三角剖分 $\triangle_W$ 下的 C^2 光滑的二元五次样条函数空间 $S_5^2(\triangle_W)$ 的维数和 Hermite插值条件.

本文利用 Hermite插值条件,构造 Wang型加密三角剖分 $\triangle_W$ 下二元五次样条函数空间 $S_5^2(\triangle_W)$ 的具有局部支集的基底.

1 基础知识

1.1 混合偏导数、方向导数、法向导数与重心坐标

对于 K上任意的三角剖分 $\triangle$,设 $v_i (i = 1,2,\cdots,|V|)$ 表示 $\triangle$ 的顶点,$e_j (j = 1,2,\cdots,|E|)$ 表示 $\triangle$ 的边,并且 $T^{(l)} = [v_1^{(l)}, v_2^{(l)}, v_3^{(l)}] (l = 1,2,\cdots,|T|)$ 表示三角剖分 $\triangle$ 的三角形.根据文献[5],把 $T^{(l)}$ 经过加密细分后得到的三角形记为 $T_W^{(l)}$,如图 1所示.

给定一非退化的三角形 $T = [v_1, v_2, v_3]$,顶点坐标分别为 $v_i = (x_i, y_i)$, $i = 1,2,3$. 设 $\lambda_i = y_j - y_k$, $\mu_i = -(x_j - x_k)$, $\gamma_i = x_j y_k - y_j x_k$, $e_j = v_j - v_i$, $e_{jk} = v_k - v_j$, $e_{ik} = v_k - v_i$,其中 (i,j,k)按照 $1 \to 2 \to 3 \to 1$的循环顺序选取.

定义在三角形 $T = [v_1, v_2, v_3]$上的二元五次多项式 $p(x,y)$ 表示为 $p(x,y) = p(T,U,V) =$

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$\sum_{i+j+k=5} c_{ijk}\frac{5!}{i!j!k!}T^iU^jV^k$,其中 T,U,V是点 v 相对于三角形 T 的重心坐标,且 $T+U+V=1$.

对 $p(x,y)$ 求一阶、二阶偏导数,由文献[4]有:

$$p|_{v_1}=c_{500}, \quad (2)$$

$$D_x p|_{v_1}=\frac{5}{A}[\lambda_1 c_{500}+\lambda_2 c_{410}+\lambda_3 c_{401}], \quad (3)$$

$$D_y p|_{v_1}=\frac{5}{A}[\_{1}c_{500}+\_{2}c_{410}+\_{3}c_{401}], \quad (4)$$

$$D_{xx} p|_{v_1}=\frac{20}{A^2}[\lambda_1^2 c_{500}+\lambda_2^2 c_{320}+\lambda_3^2 c_{302}+2\lambda_1\lambda_3 c_{401}+2\lambda_2\lambda_1 c_{410}+2\lambda_3\lambda_2 c_{311}], \quad (5)$$

$$D_{yy} p|_{v_1}=\frac{20}{A^2}[\_{1}^2 c_{500}+\_{2}^2 c_{320}+\_{3}^2 c_{302}+2\_{1}\_{3}c_{401}+2\_{2}\_{1}c_{410}+2\_{3}\_{2}c_{311}], \quad (6)$$

$$D_{xy} p|_{v_1}=\frac{20}{A^2}[\lambda_1\_{1}c_{500}+\lambda_2\_{2}c_{320}+\lambda_3\_{3}c_{302}+(\lambda_1\_{3}+\lambda_3\_{1})c_{401}+(\lambda_2\_{1}+\lambda_1\_{2})c_{410}+(\lambda_3\_{2}+\lambda_2\_{3})c_{311}]. \quad (7)$$

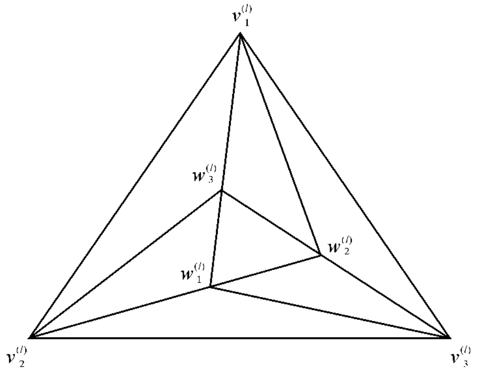

图 1 三角形 $T^{(l)}$ 被细分后得 $T_W^{(l)}$

为了方便后面的计算,给出一个求 n 阶方向导数的公式[6]

$$\frac{\partial^n p}{\partial e^n}=\frac{5!}{(5-n)!}\sum_{|l|=5-n}\sum_{|\_|=n} c_{l+\_}B_{\_}^{n}(Y)B_{5-n}^{l}(f), \quad (8)$$

其中 Y为边 e 的方向向量, f为重心坐标.以边 e_{12} 为例求顶点 v_1 处一阶方向导数

$$\frac{\partial p(v_1)}{\partial e_{12}}=5\sum_{|l|=4}(c_{i+1,j,k}\frac{\partial T}{\partial e_{12}}+c_{i,j+1,k}\frac{\partial U}{\partial e_{12}}+c_{i,j,k+1}\frac{\partial V}{\partial e_{12}})\frac{4!}{i!j!k!}T^iU^jV^k=5(-c_{500}+c_{410}). \quad (9)$$

设边 e_{ij} 上的法向向量为 n_{ij},则 $n_{ij}=e_{ik}+V_i e_j$,其中 $V_i=\frac{e_{ik}e_{ij}}{e_{ij}e_{ij}}$,$(i,j,k)$ 也是按照 $1\to2\to3\to1$ 的循环顺序选取.以边 v_1v_2 为例,$n_{12}=(-\frac{1}{2},-\frac{1}{2},1)$,则由(8)式得

$$\frac{\partial p}{\partial n_{12}}=5\sum_{|l|=4}(-\frac{1}{2}c_{l+[1,0,0]}-\frac{1}{2}c_{l+[0,1,0]}+c_{l+[0,0,1]})\frac{4!}{i!j!k!}T^iU^jV^k, \quad (10)$$

及

$$\frac{\partial^2 p}{\partial n_{12}^2}=20\sum_{|l|=3}((V_1+1)^2c_{l+[2,0,0]}-2V_1(V_1+1)c_{l+[1,1,0]}-2(V_1+1)c_{l+[1,0,1]}+V_1^2c_{l+[0,2,0]}+2V_1c_{l+[0,1,1]}+c_{l+[0,0,2]})\frac{3!}{i!j!k!}T^iU^jV^k. \quad (11)$$

1.2 空间 $S_5^2(\triangle_W)$ 的插值条件

文献[5]给出了加密三角剖分 $\triangle_W$ 下的 C^2 光滑的样条函数空间 $S_5^2(\triangle_W)$ 的 Hermite插值条件.在三角形 $T_W^{(l)}=[v_1^{(l)},v_2^{(l)},v_3^{(l)}]$ 上(如图 2),设 $f\in C^2(K)$,考虑插值条件:找一个二元五次多项式 $s\in P_5$,使它满足

$$\begin{cases} s(v_i^{(l)})=f(v_i^{(l)}),\\ \frac{\partial s}{\partial x}|(v_i^{(l)})=\frac{\partial f}{\partial x}|(v_i^{(l)}),\\ \frac{\partial s}{\partial y}|(v_i^{(l)})=\frac{\partial f}{\partial y}|(v_i^{(l)}),\\ \frac{\partial^2 s}{\partial x^2}|(v_i^{(l)})=\frac{\partial^2 f}{\partial x^2}|(v_i^{(l)}),\\ \frac{\partial^2 s}{\partial y^2}|(v_i^{(l)})=\frac{\partial^2 f}{\partial y^2}|(v_i^{(l)}),\\ \frac{\partial^2 s}{\partial x\partial y}|(v_i^{(l)})=\frac{\partial^2 f}{\partial x\partial y}|(v_i^{(l)}),\\ \frac{\partial}{\partial n_i}s(v_{i,\frac{1}{2}}^{(l)})=\frac{\partial}{\partial n_i}f(v_{i,\frac{1}{2}}^{(l)}),\\ \frac{\partial^2}{\partial n_i^2}s(v_{i,\frac{1}{3}}^{(l)})=\frac{\partial^2}{\partial n_i^2}f(v_{i,\frac{1}{3}}^{(l)}),\\ \frac{\partial^2}{\partial n_i^2}s(v_{i,\frac{2}{3}}^{(l)})=\frac{\partial^2}{\partial n_i^2}f(v_{i,\frac{2}{3}}^{(l)}),\\ s(w_i^{(l)})=f(w_i^{(l)}),\\ D_{(w_{m,i+1}-w_{m,i})}s(w_{m,i})=\\ \qquad D_{(w_{m,i+1}-w_{m,i})}f(w_{m,i}), \end{cases} \quad (12)$$

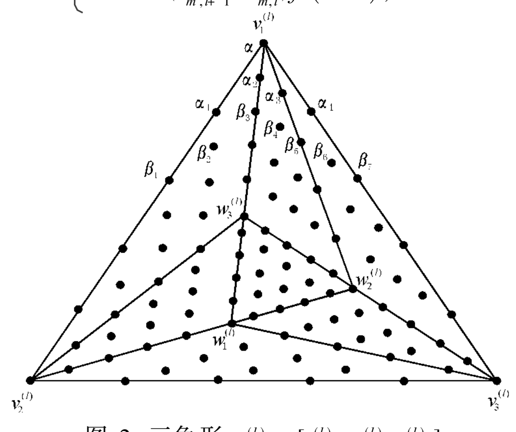

图 2 三角形 $T_W^{(l)}=[v_1^{(l)},v_2^{(l)},v_3^{(l)}]$

其中 $i=1,2,3$,$v_{i,\frac{1}{2}}^{(l)}$、$v_{i,\frac{1}{3}}^{(l)}$、$v_{i,\frac{2}{3}}^{(l)}$ 分别表示边 $v_i^{(l)}v_{i+1}^{(l)}$ 的中点、三分之一点、三分之二点,$\frac{\partial}{\partial n_i}s(v_{i,\frac{1}{2}}^{(l)})$ 是点 $v_{i,\frac{1}{2}}^{(l)}$

处的一阶法向导数, $\frac{\partial^2}{\partial n_i^2}s(v_{i,\frac{1}{3}}^{(l)})$ 是点 $v_{i,\frac{1}{3}}^{(l)}$ 处的二阶法向导数, $\frac{\partial^2}{\partial n_i^2}s(v_{i,\frac{2}{3}}^{(l)})$ 是点 $v_{i,\frac{2}{3}}^{(l)}$ 处的二阶法向导数.

定理 1 设 $f\in C^2(K)$, 则满足插值问题(12)的函数 $s\in S_5^2(\triangle_W)$ 是存在唯一的.

证明 考虑插值问题(12)相应的齐次问题, 则得到

$$c_{500}^{[v_{k+2}^{(l)},w_{k+1}^{(l)},w_k^{(l)}]}=0,\ c_{410}^{[v_{k+2}^{(l)},w_{k+1}^{(l)},w_k^{(l)}]}=0,\ c_{401}^{[v_{k+2}^{(l)},w_{k+1}^{(l)},w_k^{(l)}]}=0,\ c_{320}^{[v_{k+2}^{(l)},w_{k+1}^{(l)},w_k^{(l)}]}=0,\ c_{302}^{[v_{k+2}^{(l)},w_{k+1}^{(l)},w_k^{(l)}]}=0,\ c_{311}^{[v_{k+2}^{(l)},w_{k+1}^{(l)},w_k^{(l)}]}=0,\ c_{122}^{[v_{k+2}^{(l)},v_k^{(l)},w_{k+1}^{(l)}]}=0,\ c_{221}^{[v_{k+2}^{(l)},v_k^{(l)},w_{k+1}^{(l)}]}=0,\ c_{212}^{[v_{k+2}^{(l)},v_k^{(l)},w_{k+1}^{(l)}]}=0,\ c_{500}^{[w_k^{(l)},w_{k+1}^{(l)},w_{k+2}^{(l)}]}=0,\ c_{410}^{[w_k^{(l)},w_{k+1}^{(l)},w_{k+2}^{(l)}]}=0,\ c_{050}^{[w_k^{(l)},w_{k+1}^{(l)},w_{k+2}^{(l)}]}=0,\ c_{041}^{[w_k^{(l)},w_{k+1}^{(l)},w_{k+2}^{(l)}]}=0,\ c_{005}^{[w_k^{(l)},w_{k+1}^{(l)},w_{k+2}^{(l)}]}=0,\ c_{104}^{[w_k^{(l)},w_{k+1}^{(l)},w_{k+2}^{(l)}]}=0,\ k=1,2,3.$$

由 C^1、C^2 光滑条件确定出剩余点所对应的 B-网坐标全部都为零, 所以满足插值问题(12)的函数 s 是唯一存在的.

2 空间 $S_5^2(\triangle_W)$ 具有局部支集的基底

设 $V=\{v\}$ 表示原三角剖分 $\triangle$ 的顶点集, $E=\{e\}$ 表示原三角剖分 $\triangle$ 的边的集合, $W=\{w\}$ 表示三角形 $T_W^{(l)}=[v_1^{(l)},v_2^{(l)},v_3^{(l)}]$ 的内点集, $T_W^{(l)}\in\triangle_W$, 设 $v_i^{(l)}=(x_i,y_i)$, $w_i^{(l)}=(x_i',y_i')$, $i=1,2,3$. 我们构造 $S_5^2(\triangle_W)$ 的具有局部支集的基底, 包括 6个顶点样条 $V_{v,s}$, 其中 $s=(s_1,s_2)\in\{(0,0),(1,0),(0,1),(2,0),(1,1),(0,2)\}$, 3个边样条 V_{e1},V_{e2},V_{e3} ($e\in E$, $i=1,2,3$), 两个样条 V_{w1},V_{w2} ($w\in W$).

顶点样条 $V_{v,s}$ 由以下条件决定:

$$\begin{cases} D^tV_{v,s}(u)=\delta_{v,u}\delta_{t,s},\ t\in\{(0,0),(1,0),(0,1),(2,0),(1,1),(0,2)\},\ u\in V;\\ \frac{\partial V_{v,s}}{\partial n_j}\Big|_{E_{j,\frac12}}=\frac{\partial^2 V_{v,s}}{\partial n_j^2}\Big|_{E_{j,\frac13}}=\frac{\partial^2 V_{v,s}}{\partial n_j^2}\Big|_{E_{j,\frac23}}=0,\ j=1,2,3;\\ V_{v,s}(w)=0,\ w\in W;\\ D_{(w_{i+1}^{(l)}-w_i^{(l)})}V_{v,s}(w_i^{(l)})=0,\ i=1,2,3.\end{cases}\qquad(13)$$

边样条 V_{e1} 由以下条件决定:

$$\begin{cases} D^tV_{e1}(u)=0,\ t\in\{(0,0),(1,0),(0,1),(2,0),(1,1),(0,2)\},\ u\in V;\\ \frac{\partial V_{e1}}{\partial n_j}\Big|_{E_{j,\frac12}}=1,\ j=1;\ \frac{\partial V_{e1}}{\partial n_j}\Big|_{E_{j,\frac12}}=0,\ j=2,3;\\ \frac{\partial^2 V_{e1}}{\partial n_i^2}\Big|_{E_{j,\frac13}}=\frac{\partial^2 V_{e1}}{\partial n_i^2}\Big|_{E_{j,\frac23}}=0,\ j=1,2,3;\\ V_{e1}(w)=0,\ w\in W;\\ D_{(w_{i+1}^{(l)}-w_i^{(l)})}V_{e1}(w_i^{(l)})=0,\ i=1,2,3.\end{cases}\qquad(14)$$

边样条 V_{e2} 由以下条件决定:

$$\begin{cases} D^tV_{e2}(u)=0,\ t\in\{(0,0),(1,0),(0,1),(2,0),(1,1),(0,2)\},\ u\in V;\\ \frac{\partial V_{e2}}{\partial n_j}\Big|_{E_{j,\frac12}}=\frac{\partial^2 V_{e2}}{\partial n_j^2}\Big|_{E_{j,\frac23}}=0,\ j=1,2,3;\\ \frac{\partial^2 V_{e2}}{\partial n_j^2}\Big|_{E_{j,\frac13}}=1,\ j=1;\\ \frac{\partial^2 V_{e2}}{\partial n_j^2}\Big|_{E_{j,\frac13}}=0,\ j=2,3;\\ V_{e2}(w)=0,\ w\in W;\\ D_{(w_{i+1}^{(l)}-w_i^{(l)})}V_{e2}(w_i^{(l)})=0,\ i=1,2,3.\end{cases}\qquad(15)$$

边样条 V_{e3} 由以下条件决定:

$$\begin{cases} D^tV_{e3}(u)=0,\ t\in\{(0,0),(1,0),(0,1),(2,0),(1,1),(0,2)\},\ u\in V;\\ \frac{\partial V_{e3}}{\partial n_j}\Big|_{E_{j,\frac12}}=\frac{\partial^2 V_{e3}}{\partial n_j^2}\Big|_{E_{j,\frac13}}=0,\ j=1,2,3;\\ \frac{\partial^2 V_{e3}}{\partial n_j^2}\Big|_{E_{j,\frac23}}=1,\ j=1;\\ \frac{\partial^2 V_{e3}}{\partial n_j^2}\Big|_{E_{j,\frac23}}=0,\ j=2,3;\\ V_{e2}(w)=0,\ w\in W;\\ D_{(w_{i+1}^{(l)}-w_i^{(l)})}V_{e3}(w_i^{(l)})=0,\ i=1,2,3.\end{cases}\qquad(16)$$

样条 V_{w1} 由以下条件决定:

$$\begin{cases} D^tV_{w1}(u)=0,\ t\in\{(0,0),(1,0),(0,1),(2,0),(1,1),(0,2)\},\ u\in V;\\ \frac{\partial V_{w1}}{\partial n_j}\Big|_{E_{j,\frac12}}=\frac{\partial^2 V_{w1}}{\partial n_j^2}\Big|_{E_{j,\frac13}}=\frac{\partial^2 V_{w1}}{\partial n_j^2}\Big|_{E_{j,\frac23}}=0,\\ j=1,2,3;\\ V_{w1}(w)=\delta_{\omega,w},\ \omega\in W;\\ D_{(w_{i+1}^{(l)}-w_i^{(l)})}V_{w1}(w_i^{(l)})=0,\ i=1,2,3.\end{cases}\qquad(17)$$

样条 V_{w2} 由以下条件决定:

$$\begin{cases} D^tV_{w2}(u)=0,\ t\in\{(0,0),(1,0),(0,1),(2,0),(1,1),(0,2)\},\ u\in V;\\ \frac{\partial V_{w2}}{\partial n_j}\Big|_{E_{j,\frac12}}=\frac{\partial V_{w2}}{\partial n_j^2}\Big|_{E_{j,\frac13}}=\frac{\partial V_{w2}}{\partial n_j^2}\Big|_{E_{j,\frac23}}=0,\\ j=1,2,3;\\ V_{w2}(w)=0,\ w\in W;\\ D_{(w_{i+1}^{(l)}-w_i^{(l)})}V_{w2}(w_i^{(l)})=1,\ i=1;\\ D_{(w_{i+1}^{(l)}-w_i^{(l)})}V_{w2}(w_i^{(l)})=0,\ i=2,3.\end{cases}\qquad(18)$$

其中 $D^tf(x,y)=D_x^{t_1}D_y^{t_2}f(x,y)$, $t=(t_1,t_2)\in Z_+^2$, $\delta_{v,u}$, $\delta_{t,s}$, $\delta_{\omega,w}$ 是 Kronecker delta.

2.1 顶点样条 $V_{v,(0,0)}$

条件(13)中的第一个等式等价于: $V_{v,(0,0)}(v_1^{(l)})=1$, $D_xV_{v,(0,0)}(v_1^{(l)})=0$, $D_yV_{v,(0,0)}(v_1^{(l)})=0$,

$D_{xx}V_{v,(0,0)}(v_1^{(l)}) = 0, D_{xy}V_{v,(0,0)}(v_1^{(l)}) = 0, D_{yy}V_{v,(0,0)}(v_1^{(l)}) = 0.$

如果把 $V_{v,(0,0)}$限制在$\triangle v_1^{(l)}v_2^{(l)}w_3^{(l)}$上,则由(2)~(7)式得到

$T_1 = 1,$

$(y_2 - y_3')T_1 + (y_3' - y_1)T + (y_1 - y_2)T_2 = 0,$

$(x_3' - x_2)T_1 + (x_1 - x_3')T + (x_2 - x_1)T_2 = 0,$

$(y_2 - y_3')^2T_1 + (y_3' - y_1)^2U_1 + (y_1 - y_2)^2U_3 + 2(y_2 - y_3')(y_1 - y_2)T_2 + 2(y_3' - y_1)(y_2 - y_3')T + 2(y_1 - y_2)(y_3' - y_1)U_2 = 0,$

$(x_3' - x_2)^2T_1 + (x_1 - x_3')^2U_1 + (x_2 - x_1)^2U_3 + 2(x_3' - x_2)(x_2 - x_1)T_2 + 2(x_1 - x_3')(x_3' - x_2)T + 2(x_2 - x_1)(x_1 - x_3')U_2 = 0,$

$(y_2 - y_3')(x_3' - x_2)T_1 + (y_3' - y_1)(x_1 - x_3')U_1 + (y_1 - y_2)(x_2 - x_1)U_3 + [(y_2 - y_3')(x_2 - x_1) + (y_1 - y_2)(x_3' - x_2)]T_2 + [(y_3' - y_1)(x_3' - x_2) + (y_2 - y_3')(x_1 - x_3')]T + [(y_1 - y_2)(x_1 - x_3') + (y_3' - y_1)(x_2 - x_1)]U_2 = 0.$

由上述 6个方程可解出 $T = T_i = 1, i = 1, 2; U_j = 1, j = 1, 2, 3$. 如果把 $V_{v,(0,0)}$限制在$\triangle v_1^{(l)}w_3^{(l)}w_2^{(l)}$上,由(2)~(7)式同理可得 $T_1 = T_2 = T_3 = 1, U_3 = U_4 = U_5$;如果把 $V_{v,(0,0)}$限制在$\triangle v_1^{(l)}w_2^{(l)}v_3^{(l)}$上,由(2)~(7)式同理可得 $T_1 = T_3 = T_4 = 1, U_5 = U_6 = U_7$. 以上 $T, T_i, U_j, i = 1, 2, 3, 4; j = 1, 2, 3, 4, 5, 6, 7$,如图 2所示.

由(10)、(11)两式,条件(13)中的第二个等式等价于

$$c_{122}^{[v_1^{(l)},v_2^{(l)},w_3^{(l)}]} = 0, c_{212}^{[v_1^{(l)},v_2^{(l)},w_3^{(l)}]} = 0, c_{221}^{[v_1^{(l)},v_2^{(l)},w_3^{(l)}]} = 0,$$
$$c_{122}^{[v_1^{(l)},w_2^{(l)},v_3^{(l)}]} = 0, c_{212}^{[v_1^{(l)},w_2^{(l)},v_3^{(l)}]} = 0, c_{221}^{[v_1^{(l)},w_2^{(l)},v_3^{(l)}]} = 0,$$
$$c_{122}^{[w_1^{(l)},v_2^{(l)},v_3^{(l)}]} = 0, c_{212}^{[w_1^{(l)},v_2^{(l)},v_3^{(l)}]} = 0, c_{221}^{[w_1^{(l)},v_2^{(l)},v_3^{(l)}]} = 0.$$

由(2)式,条件(13)中的第三个等式等价于

$$c_{500}^{[w_1^{(l)},w_2^{(l)},w_3^{(l)}]} = 0, c_{050}^{[w_1^{(l)},w_2^{(l)},w_3^{(l)}]} = 0, c_{005}^{[w_1^{(l)},w_2^{(l)},w_3^{(l)}]} = 0.$$

由(9)式,条件(13)中的第四个等式等价于

$$c_{410}^{[w_1^{(l)},w_2^{(l)},w_3^{(l)}]} = 0, c_{041}^{[w_1^{(l)},w_2^{(l)},w_3^{(l)}]} = 0, c_{104}^{[w_1^{(l)},w_2^{(l)},w_3^{(l)}]} = 0.$$

其余点所对应的 B-网坐标可由文献[5]中(2.7)~(2.14)式计算出来(见图 3),其中

$a_1 = \frac{31}{26}, a_2 = \frac{124}{63}, a_3 = \frac{1}{2}, a_4 = \frac{1}{126}, a_5 = -\frac{61}{126}, b_1 = -\frac{31}{126}, b_2 = -\frac{47}{126}, b_3 = -\frac{1}{2}, b_4 = -\frac{64}{126}, b_5 = -\frac{8}{126}, c_1 = \frac{61}{126}, c_2 = -\frac{1}{126}, c_3 = -\frac{1}{2}, c_4 = \frac{128}{63}, c_5 = \frac{1}{2}, d_1 = -\frac{8}{63}, d_2 = -2, d_3 = -\frac{244}{63}, d_4 = \frac{32}{63}, e_1 = \frac{4}{126}, e_2 = \frac{8}{126}, e_3 = -\frac{4}{126}, u_1 = \frac{1}{2}, u_2 = 2, u_3 = \frac{47}{126}$.

图 3中◦表示的区域点的 B-网坐标为零(下同).

用同样的方法可以分别求出顶点样条 $V_{v,(1,0)}$、$V_{v,(0,1)}$、$V_{v,(2,0)}$、$V_{v,(1,1)}$、$V_{v,(0,2)}$的 B-网坐标.

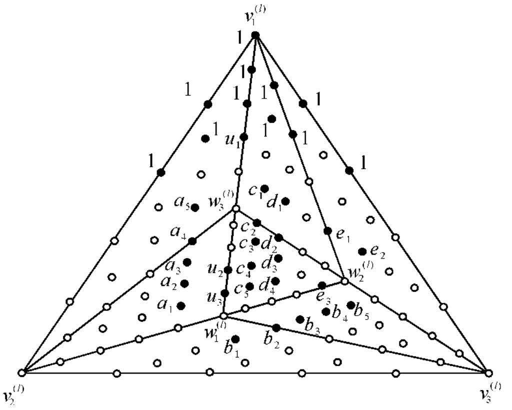

图 3 顶点样条 $V_{v,(0,0)}$的 B-网坐标

2.2 顶点样条 $V_{v,(1,0)}$

顶点样条 $V_{v,(1,0)}$的 B-网坐标见图 4,其中

$T_1 = 0, T_2 = \frac{x_3' - x_1}{5}, T_3 = \frac{x_2' - x_1}{5}, T_4 = \frac{x_3 - x_1}{5}, T = \frac{x_2 - x_1}{5}, U_1 = \frac{2}{5}(x_2 - x_1), U_2 = \frac{1}{5}(x_2 - x_1) + \frac{1}{5}(x_3' - x_1), U_3 = \frac{2}{5}(x_3' - x_1), U_4 = \frac{1}{5}(x_3' - x_1) + \frac{1}{5}(x_2' - x_1), U_5 = \frac{2}{5}(x_2' - x_1), U_6 = \frac{1}{5}(x_2' - x_1) + \frac{1}{5}(x_3 - x_1), U_7 = \frac{2}{5}(x_3 - x_1), V_1 = \frac{3}{35}x_3' + \frac{2}{35}x_2' - \frac{1}{7}x_1, V_2 = -\frac{16}{35}x_3' + \frac{8}{35}x_2' + \frac{8}{35}x_1, V_3 = -\frac{8}{35}x_3' + \frac{4}{35}x_2' + \frac{4}{35}x_1, V_4 = -\frac{4}{35}x_3' + \frac{2}{5}x_2' + \frac{2}{35}x_1, l_1 = \frac{-16V_1 - 22V_2 + 16V_3 - 63V_4}{252}, l_2 = \frac{-2V_1 + 13V_2 + 2V_3}{126},$

$l_3 = \frac{188V_1 - \frac{113}{2}V_2 + V_3}{252}, a_1 = 2l_3 - V_1 - \frac{1}{8}V_2, a_2 = 4l_2 + 4V_1, a_3 = V_1, a_4 = -l_2, a_5 = -2l_2 - V_1, b_1 = V_1 + \frac{1}{8}V_2 - 2l_3, b_2 = -l_3, b_3 = -V_1 - \frac{1}{8}V_2, b_4 = -4V_1 - \frac{1}{2}V_2 + 4l_3, b_5 = 2l_1 + \frac{1}{4}V_4, c_1 = 2l_2 - V_1, c_2 = V_1, c_3 = 4V_1 + V_2 - 4l_2, c_4 = V_1 + \frac{1}{8}V_2, d_1 = 4l_1 + V_4, d_3 = -4l_1 - V_4, d_4 = 4V_1 + \frac{1}{2}V_2 - 4l_3, e_1 = \frac{1}{4}V_4, e_2 = -l_1, e_3 = -2l_1 - \frac{1}{4}V_4, e_4 = -\frac{1}{4}V_4, u_1 = V_1, u_2 = 4V_1 + \frac{1}{2}V_2.$

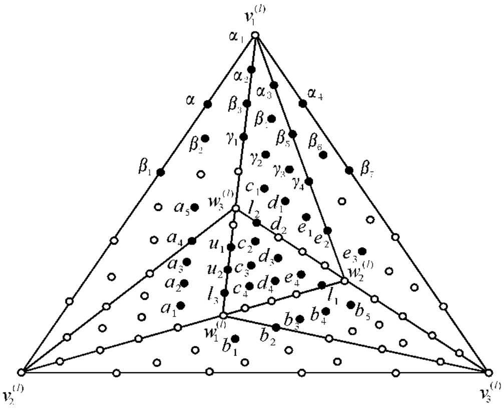

图 4 $V_{v,(1,0)}$的 B-网坐标

2.3 顶点样条 $V_{v,(0,1)}$

顶点样条 $V_{v,(0,1)}$的 B-网坐标见图 5,其中

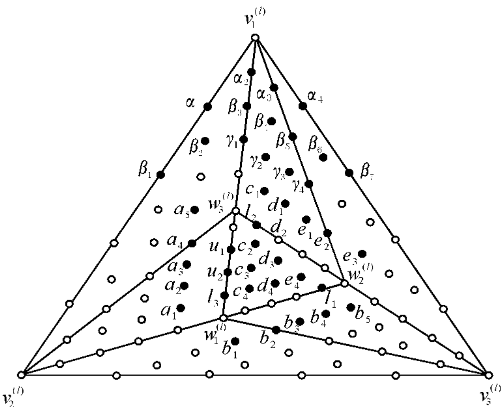

图 5 $V_{v,(0,1)}$的 B-网坐标

$T_1=0$, $T_2=\frac{y'_3-y_1}{5}$, $T_3=\frac{y'_2-y_1}{5}$, $T_4=\frac{y_3-y_1}{5}$, $T=\frac{y_2-y_1}{5}$, $U_1=\frac{2}{5}(y_2-y_1)$, $U_2=\frac{1}{5}(y_2-y_1)+\frac{1}{5}(y'_3-y_1)$, $U_3=\frac{2}{5}(y'_3-y_1)$, $U_4=\frac{1}{5}(y'_3-y_1)+\frac{1}{5}(y'_2-y_1)$, $U_5=\frac{2}{5}(y'_2-y_1)$, $U_6=\frac{1}{5}(y'_2-y_1)+\frac{1}{5}(y_3-y_1)$, $U_7=\frac{2}{5}(y_3-y_1)$, $V_1=\frac{3}{35}y'_3+\frac{2}{35}y'_2-\frac{1}{7}y_1$, $V_2=-\frac{16}{35}y'_3+\frac{8}{35}y'_2+\frac{8}{35}y_1$, $V_3=-\frac{8}{35}y'_3+\frac{4}{35}y'_2+\frac{4}{35}y_1$, $V_4=-\frac{4}{35}y'_3+\frac{2}{5}y'_2+\frac{2}{35}y_1$, $l_1=\frac{-16V_1-22V_2+16V_3-63V_4}{252}$, $l_2=\frac{-2V_1+13V_2+2V_3}{126}$, $l_3=\frac{188V_1-\frac{113}{2}V_2+V_3}{252}$, $a_1=2l_3-V_1-\frac{1}{8}V_2$, $a_2=4l_2+4V_1$, $a_3=V_1$, $a_4=-l_2$, $a_5=-2l_2-V_1$, $b_1=V_1+\frac{1}{8}V_2-2l_3$, $b_2=-l_3$, $b_3=-V_1-\frac{1}{8}V_2$, $b_4=-4V_1-\frac{1}{2}V_2+4l_3$, $b_5=2l_1+\frac{1}{4}V_4$, $c_1=2l_2-V_1$, $c_2=V_1$, $c_3=4V_1+V_2-4l_2$, $c_4=V_1+\frac{1}{8}V_2$, $d_1=4l_1+V_4$, $d_3=-4l_1-V_4$, $d_4=4V_1+\frac{1}{2}V_2-4l_3$, $e_1=-\frac{1}{4}V_4$, $e_2=-l_1$, $e_3=-2l_1-\frac{1}{4}V_4$, $e_4=-\frac{1}{4}V_4$, $u_1=V_1$, $u_2=4V_1+\frac{1}{2}V_2$.

2.4 顶点样条 $V_{v,(1,1)}$

顶点样条 $V_{v,(1,1)}$的 B-网坐标见图 6,其中

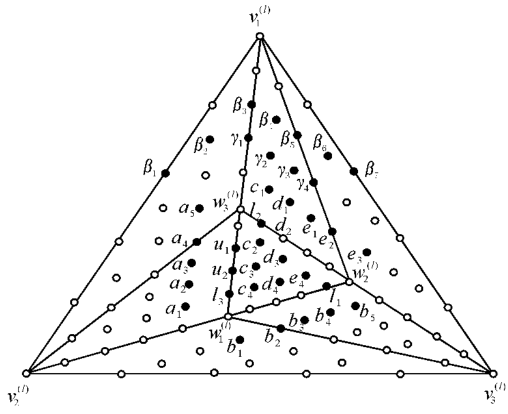

图 6 $V_{v,(1,1)}$的 B-网坐标

$T_1=0$, $T_2=0$, $T_3=0$, $T_4=0$, $T=0$, $U_1=-\frac{1}{10}(x_2-x_1)(y_1-y_2)$, $U_2=\frac{1}{20}[(x_2-x_1)(y'_3-y_1)+(y_1-y_2)(x_1-x'_3)]$, $U_3=-\frac{1}{10}(x_1-x'_3)(y'_3-y_1)$, $U_4=\frac{1}{20}[(x'_3-x_1)(y'_2-y_1)+(y_1-y'_3)(x_1-x'_2)]$, $U_5=-\frac{1}{10}(x_1-x'_2)(y'_2-y_1)$, $U_6=\frac{1}{20}[(x'_2-x_1)(y_3-y_1)+(y_1-y'_2)(x_1-x_3)]$, $U_7=-\frac{1}{10}(x_1-x_3)(y_3-y_1)$, $V_1=\frac{1}{140}(x'_3-x_1)(y'_3-2y'_2+y_1)+\frac{1}{70}(y_1-y'_3)(x_1-x'_2)$, $V_2=\frac{2}{35}(x_1-x'_3)(4y'_3-y'_2-3y_1)+\frac{2}{35}(y_1-y'_3)(x_1-x'_2)$, $V_3=\frac{1}{35}(x_1-x'_3)(4y'_3-y'_2-3y_1)+\frac{1}{35}(y_1-y'_3)(x_1-x'_2)$, $V_4=\frac{1}{70}(x_1-x'_3)(4y'_3-y'_2-3y_1)+\frac{1}{70}(y_1-y'_3)(x_1-x'_2)$, $l_1=\frac{-16V_1-22V_2+16V_3-63V_4}{252}$, $l_2=\frac{-2V_1+13V_2+2V_3}{126}$, $l_3=\frac{188V_1-\frac{113}{2}V_2+V_3}{252}$, $a_1=2l_3-V_1-\frac{1}{8}V_2$, $a_2=4l_2+4V_1$, $a_3=V_1$, $a_4=-l_2$, $a_5=-2l_2-V_1$, $b_1=V_1+\frac{1}{8}V_2-2l_3$, $b_2=-l_3$, $b_3=-V_1-\frac{1}{8}V_2$, $b_4=-4V_1-\frac{1}{2}V_2+4l_3$, $b_5=2l_1+\frac{1}{4}V_4$, $c_1=2l_2-V_1$, $c_2=V_1$, $c_3=4V_1+V_2-4l_2$, $c_4=V_1+\frac{1}{8}V_2$, $d_1=4l_1+V_4$, $d_3=$

$-4l_1-V_4$, $d_4=4V_1+\frac{1}{2}V_2-4l_3$, $e_1=\frac{1}{4}V_4$, $e_2=-l_1$, $e_3=-2l_1-\frac{1}{4}V_4$, $e_4=-\frac{1}{4}V_4$, $u_1=V_1$, $u_2=4V_1+\frac{1}{2}V_2$.

2.5 顶点样条 $V_{v,(2,0)}$

顶点样条 $V_{v,(2,0)}$的 B-网坐标见图 7,其中

$T_1=0$, $T_2=0$, $T_3=0$, $T_4=0$, $T_5=0$, $U_1=\frac{1}{20}(x_2-x_1)^2$, $U_2=-\frac{1}{20}(x_1-x'_3)(x_2-x_1)$, $U_3=\frac{1}{20}(x'_1-x'_3)^2$, $U_4=-\frac{1}{20}(x_1-x'_2)(x'_3-x_1)$, $U_5=\frac{1}{20}(x_1-x'_2)^2$, $U_6=-\frac{1}{20}(x_1-x_3)(x'_2-x_1)$, $U_7=\frac{1}{20}(x_1-x_3)^2$, $V_1=-\frac{1}{280}(x'_1-x'_3)^2-\frac{1}{70}(x_1-x'_2)(x'_3-x_1)$, $V_2=-\frac{4}{35}(x'_1-x'_3)^2-\frac{2}{35}(x_1-x'_2)(x'_3-x_1)$, $V_3=-\frac{2}{35}(x'_1-x'_3)^2-\frac{1}{35}(x_1-x'_2)(x'_3-x_1)$, $V_4=-\frac{1}{35}\circ(x'_1-x'_3)^2-\frac{1}{70}(x_1-x'_2)(x'_3-x_1)$, $l_1=\frac{-16V_1-22V_2+16V_3-63V_4}{252}$, $l_2=\frac{-2V_1+13V_2+2V_3}{126}$, $l_3=\frac{188V_1-\frac{113}{2}V_2+V_3}{252}$, $a_1=2l_3-V_1-\frac{1}{8}V_2$, $a_2=4l_2+4V_1$, $a_3=V_1$, $a_4=-l_2$, $a_5=-2l_2-V_1$, $b_1=V_1+\frac{1}{8}V_2-2l_3$, $b_2=-l_3$, $b_3=-V_1-\frac{1}{8}V_2$, $b_4=-4V_1-\frac{1}{2}V_2+4l_3$, $b_5=2l_1+\frac{1}{4}V_4$, $c_1=2l_2-V_1$, $c_2=V_1$, $c_3=4V_1+V_2-4l_2$, $c_4=V_1+\frac{1}{8}V_2$, $d_1=4l_1+V_4$, $d_3=-4l_1-V_4$, $d_4=4V_1+\frac{1}{2}V_2-4l_3$, $e_1=\frac{1}{4}V_4$, $e_2=-l_1$, $e_3=-2l_1-\frac{1}{4}V_4$, $e_4=-\frac{1}{4}V_4$, $u_1=V_1$, $u_2=4V_1+\frac{1}{2}V_2$.

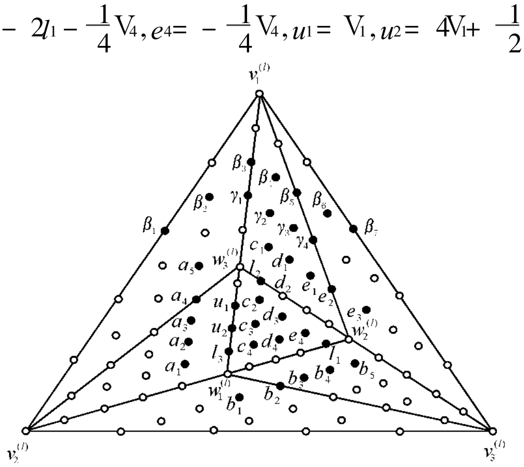

图 7　$V_{v,(2,0)}$的 B-网坐标

2.6 顶点样条 $V_{v,(0,2)}$

顶点样条 $V_{v,(0,2)}$的 B-网坐标见图 8,其中

$T_1=0$, $T_2=0$, $T_3=0$, $T_4=0$, $T_5=0$, $U_1=\frac{1}{20}(y_2-y_1)^2$, $U_2=-\frac{1}{20}(y_1-y'_3)(y_2-y_1)$, $U_3=\frac{1}{20}(y_1-y'_3)^2$, $U_4=-\frac{1}{20}(y_1-y'_2)(y'_3-y_1)$, $U_5=\frac{1}{20}(y_1-y'_2)^2$, $U_6=-\frac{1}{20}(y_1-y_3)(y'_2-y_1)$, $U_7=\frac{1}{20}(y_1-y_3)^2$, $V_1=-\frac{1}{280}(y'_1-y'_3)^2-\frac{1}{70}(y_1-y'_2)(y'_3-y_1)$, $V_2=-\frac{4}{35}\circ(y'_1-y'_3)^2-\frac{2}{35}(y_1-y'_2)(y'_3-y_1)$, $V_3=-\frac{2}{35}(y'_1-y'_3)^2-\frac{1}{35}(y_1-y'_2)(y'_3-y_1)$, $V_4=-\frac{1}{35}(y'_1-y'_3)^2-\frac{1}{70}(y_1-y'_2)(y'_3-y_1)$, $l_1=\frac{-16V_1-22V_2+16V_3-63V_4}{252}$, $l_2=\frac{-2V_1+13V_2+2V_3}{126}$, $l_3=\frac{188V_1-\frac{113}{2}V_2+V_3}{252}$, $a_1=2l_3-V_1-\frac{1}{8}V_2$, $a_2=4l_2+4V_1$, $a_3=V_1$, $a_4=-l_2$, $a_5=-2l_2-V_1$, $b_1=V_1+\frac{1}{8}V_2-2l_3$, $b_2=-l_3$, $b_3=-V_1-\frac{1}{8}V_2$, $b_4=-4V_1-\frac{1}{2}V_2+4l_3$, $b_5=2l_1+\frac{1}{4}V_4$, $c_1=2l_2-V_1$, $c_2=V_1$, $c_3=4V_1+V_2-4l_2$, $c_4=V_1+\frac{1}{8}V_2$, $d_1=4l_1+V_4$, $d_3=-4l_1-V_4$, $d_4=4V_1+\frac{1}{2}V_2-4l_3$, $e_1=\frac{1}{4}V_4$, $e_2=-l_1$, $e_3=-2l_1-\frac{1}{4}V_4$, $e_4=-\frac{1}{4}V_4$, $u_1=V_1$, $u_2=4V_1+\frac{1}{2}V_2$.

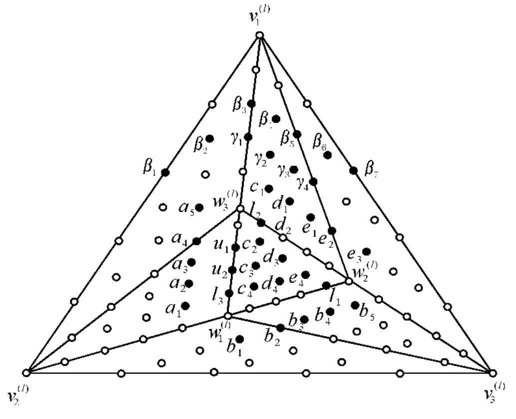

图 8　$V_{v,(0,2)}$的 B-网坐标

2.7 边样条 V_{e1}

条件 (14)中的第一个等式等价于: 顶点 $v_1^{(l)}$的二盘 $D_2(v_1^{(l)})$内的区域点所对应的 B-网坐标全为零,顶点 $v_2^{(l)}$的二盘 $D_2(v_2^{(l)})$内的区域点所对应的 B-网坐标全为零,顶点 $v_3^{(l)}$的二盘 $D_2(v_3^{(l)})$内的区域点所对应的 B-网坐标全为零.

由 (10)、(11)两式,条件 (14)中的第二、第三、第

四个等式等价于

$c_{122}^{[v_1^{(l)},v_2^{(l)},w_3^{(l)}]}=\frac{8}{15}, c_{212}^{[v_1^{(l)},v_2^{(l)},w_3^{(l)}]}=\frac{8}{15}, c_{221}^{[v_1^{(l)},v_2^{(l)},w_3^{(l)}]}=\frac{8}{15}, c_{122}^{[v_1^{(l)},w_2^{(l)},v_3^{(l)}]}=0, c_{3}^{[v_1^{(l)},w_2^{(l)},v_3^{(l)}]}=0, c_{221}^{[v_1^{(l)},w_2^{(l)},v_3^{(l)}]}=0, c_{122}^{[w_1^{(l)},v_2^{(l)},v_3^{(l)}]}=0, c_{212}^{[w_1^{(l)},v_2^{(l)},v_3^{(l)}]}=0, c_{221}^{[w_1^{(l)},v_2^{(l)},v_3^{(l)}]}=0.$

由(2)式,条件(14)中的第五个等式等价于

$c_{500}^{[w_1^{(l)},w_2^{(l)},w_3^{(l)}]}=0, c_{050}^{[w_1^{(l)},w_2^{(l)},w_3^{(l)}]}=0, c_{005}^{[w_1^{(l)},w_2^{(l)},w_3^{(l)}]}=0.$

由(9)式,条件(14)中的最后一个等式等价于

$c_{410}^{[w_1^{(l)},w_2^{(l)},w_3^{(l)}]}=0, c_{041}^{[w_1^{(l)},w_2^{(l)},w_3^{(l)}]}=0, c_{104}^{[w_1^{(l)},w_2^{(l)},w_3^{(l)}]}=0.$

其余点所对应的 B-网坐标由文献[5]中(2.7)~(2.14)式中的关系式计算所得(见图 9),其中

$k_1=\frac{2}{105}, k_2=\frac{8}{105}, k_3=\frac{32}{105}, k_4=\frac{44}{105}, k_5=\frac{8}{15}, k_6=\frac{8}{15}, a_1=\frac{1255}{6615}, a_2=\frac{10040}{6615}, a_3=\frac{37}{105}, a_4=\frac{703}{6615}, a_5=-\frac{925}{6615}, a_6=\frac{8}{15}, b_1=-\frac{1507}{6615}, b_2=-\frac{1919}{6615}, b_3=-\frac{37}{105}, b_4=-\frac{1648}{6615}, b_5=-\frac{584}{6615}, c_1=\frac{48}{105}, c_2=-\frac{2351}{6615}, c_3=-\frac{703}{6615}, c_4=\frac{1}{7}, c_5=\frac{9616}{6615}, c_6=\frac{41}{105}, d_1=\frac{24}{105}, d_2=-\frac{1168}{6615}, d_3=\frac{1168}{6615}, d_4=\frac{3160}{6615}, d_5=\frac{2}{105}, e_1=\frac{12}{105}, e_2=\frac{292}{6615}, e_3=\frac{584}{6615}, e_4=-\frac{292}{6615}, e_5=\frac{12}{105}, u_1=\frac{26}{105}, u_2=\frac{26}{105}, u_3=\frac{156}{105}, u_4=\frac{1919}{6615}.$

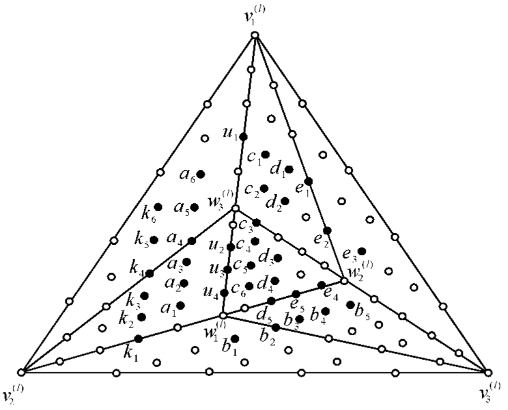

图 9 V_e的 B-网坐标

用同样的方法可以分别求出边样条 V_{e2}、V_{e3}的 B-网坐标.

2.8 边样条 V_{e2}

边样条 V_{e2}的 B-网坐标见图 10,其中

$k_1=-\frac{3}{560}, k_2=-\frac{3}{140}, k_3=-\frac{12}{140}, k_4=-\frac{9}{112}, k_5=-\frac{3}{40}, a_1=-\frac{1149}{70560}, a_2=\frac{30177}{15435}, a_3=\frac{27}{448}, a_4=\frac{18393}{141120}, a_5=-\frac{317037}{987840}, a_6=\frac{3}{20}, b_1=\frac{1905}{7056}, b_2=-\frac{3489}{70560}, b_3=-\frac{141}{1120}, b_4=-\frac{22539}{17640}, b_5=-\frac{45983}{164640}, c_1=\frac{6}{35}, c_2=-\frac{356727}{987840}, c_3=-\frac{18393}{141120}, c_4=\frac{45}{448}, c_5=\frac{428211}{432180}, c_6=\frac{129}{1120}, d_1=\frac{3}{35}, d_2=-\frac{2681}{5145}, d_3=\frac{2681}{5145}, d_4=\frac{4281}{3528}, d_5=-\frac{3}{560}, e_1=\frac{3}{70}, e_2=\frac{701}{4704}, e_3=\frac{45983}{164640}, e_4=-\frac{701}{4704}, e_5=-\frac{9}{280}, e_6=-\frac{21}{1120}, e_7=\frac{21}{1120}, u_1=\frac{9}{112}, u_2=\frac{9}{112}, u_3=\frac{27}{56}, u_4=\frac{3489}{70560}.$

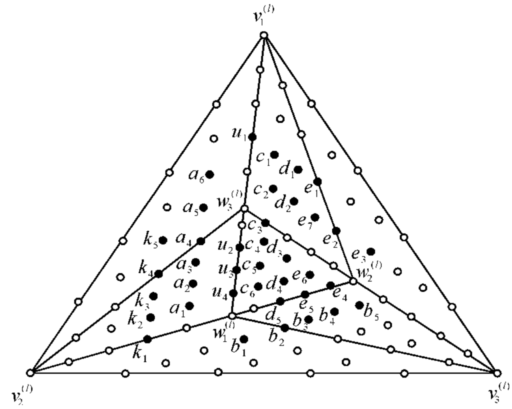

图 10 V_{e2}的 B-网坐标

2.9 边样条 V_{e3}

边样条 V_{e3}的 B-网坐标见图 11,其中

$k_1=\frac{3}{280}, k_2=\frac{3}{70}, k_3=\frac{6}{35}, k_4=\frac{9}{56}, k_5=\frac{3}{20}, a_1=\frac{43267}{1382976}, a_2=-\frac{6125}{172872}, a_3=-\frac{17101}{1728720}, a_4=-\frac{17101}{864360}, a_5=-\frac{3}{40}, b_1=-\frac{60319}{3457440}, b_2=-\frac{60319}{6914880}, b_3=\frac{79}{2240}, b_4=\frac{60319}{1728720}, b_5=\frac{2716}{123480}, c_1=-\frac{3}{35}, c_2=\frac{1211819}{12101040}, c_3=\frac{17101}{1728720}, c_4=-\frac{9}{112}, c_5=\frac{6601}{49392}, c_6=-\frac{31}{2240}, d_1=-\frac{3}{70}, d_2=\frac{1}{882}, d_3=-\frac{1}{882}, d_4=\frac{32389}{345744}, d_5=\frac{3}{280}, e_1=-\frac{3}{140}, e_2=-\frac{2681}{123480}, e_3=-\frac{2716}{123480}, e_4=-\frac{292}{6615}, e_5=\frac{9}{140}, e_6=\frac{3}{140}, e_7=-\frac{3}{140}, u_1=-\frac{9}{224}, u_2=-\frac{9}{224}, u_3=-\frac{11}{112}, u_4=\frac{60319}{6914880}.$

2.10 样条 V_{w1}

条件(17)中的第一个等式等价于: 顶点 $v_1^{(l)}$的二盘 $D_2(v_1^{(l)})$内的区域点所对应的 B-网坐标全为零,

顶点 $v_2^{(l)}$的二盘 $D_2(v_2^{(l)})$内的区域点所对应的 B-网坐标全为零,顶点 $v_3^{(l)}$的二盘 $D_2(v_3^{(l)})$内的区域点所对应的 B-网坐标全为零.

由(10)、(11)两式,条件(17)中的第二个等式等价于:

$$c_{122}^{[v_1^{(l)},v_2^{(l)},w_3^{(l)}]}=0,\ c_{212}^{[v_1^{(l)},v_2^{(l)},w_3^{(l)}]}=0,\ c_{221}^{[v_1^{(l)},v_2^{(l)},w_3^{(l)}]}=0,$$

$$c_{122}^{[v_1^{(l)},w_2^{(l)},v_3^{(l)}]}=0,\ c_{212}^{[v_1^{(l)},w_2^{(l)},v_3^{(l)}]}=0,\ c_{221}^{[v_1^{(l)},w_2^{(l)},v_3^{(l)}]}=0,$$

$$c_{122}^{[w_1^{(l)},v_2^{(l)},v_3^{(l)}]}=0,\ c_{212}^{[w_1^{(l)},v_2^{(l)},v_3^{(l)}]}=0,\ c_{221}^{[w_1^{(l)},v_2^{(l)},v_3^{(l)}]}=0.$$

由(2)式,条件(17)中的第三个等式等价于

$$c_{500}^{[w_1^{(l)},w_2^{(l)},w_3^{(l)}]}=1,\ c_{050}^{[w_1^{(l)},w_2^{(l)},w_3^{(l)}]}=0,$$

$$c_{005}^{[w_1^{(l)},w_2^{(l)},w_3^{(l)}]}=0.$$

由(9)式,条件(17)中的第四个等式等价于

$$c_{410}^{[w_1^{(l)},w_2^{(l)},w_3^{(l)}]}=1,\ c_{041}^{[w_1^{(l)},w_2^{(l)},w_3^{(l)}]}=0,$$

$$c_{104}^{[w_1^{(l)},w_2^{(l)},w_3^{(l)}]}=0.$$

其余点所对应的 B-网坐标可以由文献[5]中(2.7)~(2.14)式中的关系式计算出来(见图 12),其中

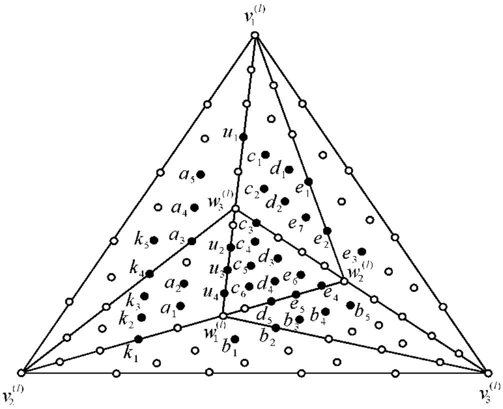

图 11 V_{e3}的 B-网坐标

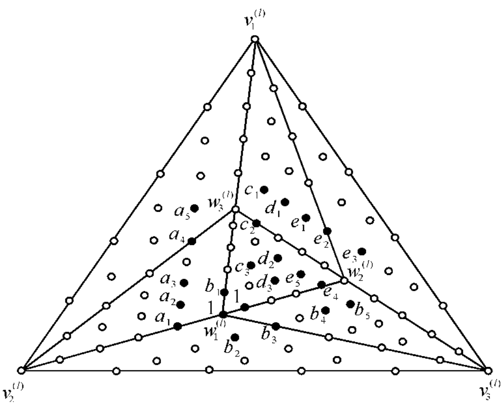

图 12 V_{w1}的 B-网坐标

$a_1=1, a_2=\frac{128}{63}1, a_3=\frac{16}{63}, a_4=-\frac{4}{63}, a_5=-\frac{8}{63}, b_1=\frac{64}{63}, b_2=\frac{124}{63}, b_3=\frac{62}{63}, b_4=-\frac{248}{63}, b_5=-\frac{31}{63}, c_1=\frac{8}{63}, c_2=\frac{4}{63}, c_3=-\frac{16}{63}, d_1=\frac{64}{63}, d_2=-\frac{64}{63}, d_3=-\frac{256}{63}, e_1=1, e_2=\frac{47}{63}, e_3=\frac{31}{63}, e_4=-\frac{47}{63}, e_5=-1.$

用同样的方法可以求出样条 V_{w2}的 B-网坐标.

2.11 样条 V_{w2}

样条 V_{w2}的 B-网坐标见图 13,其中

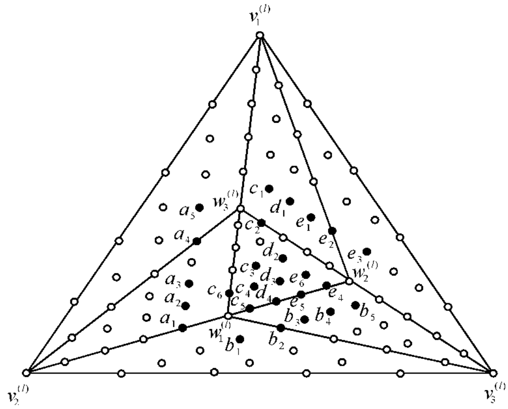

图 13 V_{w2}的 B-网坐标

$a_1=-\frac{1}{5}, a_2=-\frac{71}{315}, a_3=-\frac{1072}{315}, a_4=\frac{268}{315}, a_5=\frac{536}{315}, b_1=\frac{8}{315}, b_2=\frac{130}{315}, b_3=\frac{4}{5}, b_4=\frac{488}{315}, b_5=\frac{61}{315}, c_1=-\frac{536}{315}, c_2=-\frac{268}{315}, c_3=\frac{1072}{315}, c_4=\frac{4}{5}, c_5=\frac{1}{5}, c_6=-\frac{4}{315}, d_1=-\frac{256}{315}, d_2=\frac{256}{315}, d_3=\frac{1124}{315}, d_4=\frac{4}{5}, e_1=-\frac{3}{5}, e_2=-\frac{25}{63}, e_3=-\frac{61}{315}, e_4=\frac{25}{63}, e_5=\frac{12}{5}, e_6=-\frac{3}{5}.$

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